# INTERNATIONAL ELECTRICAL ENGINEERS HANDBOOK

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# The Electrical Engineer's Handbook

*A REFERENCE BOOK DEALING WITH*

ELECTRICITY AND MAGNETISM, DYNAMOS AND MOTORS, ELECTRIC LIGHTING, INTERIOR WIRING, POWER TRANSMISSION, AND OPERATION AND MAINTENANCE OF ELECTRICAL APPARATUS

COMPILED AND EDITED BY
INTERNATIONAL CORRESPONDENCE SCHOOLS

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# PREFACE

The publishers have not attempted in this work to produce a condensed cyclopedia covering the broad field of electrical engineering, but they have aimed to present to the public a handy reference book convenient to carry in the pocket, containing the rules, formulas, tables, and diagrams of connections that are most generally used and needed by wiremen, dynamo attendants, foremen of construction, managers, engineers, and superintendents of power stations, electric-lighting and electric-railway systems, and manufacturers and dealers in electrical apparatus and supplies.

The aim of the publishers has been to select from the vast amount of material at hand only that portion which is most likely to be used in connection with the daily work or which will be most frequently consulted. While the treatment of some subjects is of necessity brief, it is sufficient for the purpose. More important subjects have been covered in greater detail; for instance, the description of dynamos and motors, the faults to which they are liable, and the methods of locating these faults is very thorough and complete. The tables selected are those most in demand, and the applications of the rules and formulas are shown, in many cases, by practical examples and solutions, together with explanations.

The material is conveniently and logically arranged for ready reference. A large amount of valuable data on weights and measures and on the physical and electrical properties of metals and alloys has been incorporated. The connections of the different types of dynamos and motors are well illustrated by diagrams. Of especial interest is the table of sizes of motors necessary to drive various machine tools. The treatise on the diseases of electrical apparatus is of great value to those engaged in operating machinery.

The treatment of alternators, alternating-current motors, alternating-current measuring instruments, and electric transmission, though concise, contains many valuable diagrams of connections, data tables, and other instruction relating to this branch of the profession, which is constantly increasing in importance. The methods of controlling the speed of electric cars and multiple-unit trains are illustrated and explained. In the section on "First Aid to the Injured," directions are given in cases of electric shock, or other accidents, which if promptly followed may result in the saving of life, or alleviation of suffering. Numerous illustrations show clearly the proper method of procedure in case of accident.

INTERNATIONAL CORRESPONDENCE SCHOOLS
SCRANTON, PA.

February 1, 1908

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# The Electrical Engineer's Handbook

## USEFUL TABLES

## WEIGHTS AND MEASURES

### LINEAR MEASURE

| | | | |
|---|---|---|---|
| 12 | inches (in.) | = 1 foot | ft. |
| 3 | feet | = 1 yard | yd. |
| 5½ | yards | = 1 rod | rd. |
| 40 | rods | = 1 furlong | fur. |
| 8 | furlongs | = 1 mile | mi. |

| *in.* | *ft.* | *yd.* | *rd.* | *fur.* | *mi.* |
|---|---|---|---|---|---|
| 36 = | 3 = | 1 | | | |
| 198 = | 16.5 = | 5.5 = | 1 | | |
| 7,920 = | 660 = | 220 = | 40 = | 1 | |
| 63,360 = | 5,280 = | 1,760 = | 320 = | 8 = | 1 |

### SQUARE MEASURE

| | | | |
|---|---|---|---|
| 144 | square inches (sq. in.) | = 1 square foot | sq. ft. |
| 9 | square feet | = 1 square yard | sq. yd. |
| 30¼ | square yards | = 1 square rod | sq. rd. |
| 160 | square rods | = 1 acre | A. |
| 640 | acres | = 1 square mile | sq. mi |

| *sq. mi.* | *A.* | *sq. rd.* | *sq. yd.* | *sq. ft.* | *sq. in.* |
|---|---|---|---|---|---|
| 1 = | 640 = | 102,400 = | 3,097,600 = | 27,878,400 = | 4,014,489,600 |

## CUBIC MEASURE

| | | |
|---|---|---|
| 1,728 cubic inches (cu. in.).... | = | 1 cubic foot ......cu. ft. |
| 27 cubic feet............. | = | 1 cubic yard .....cu. yd. |
| 128 cubic feet............. | = | 1 cord ..............cd. |
| $24\frac{3}{4}$ cubic feet............. | = | 1 perch ..............P. |

1 cu. yd. = 27 cu. ft. = 46,656 cu. in.

## MEASURE OF ANGLES OR ARCS

| | | |
|---|---|---|
| 60 seconds (″)............. | = | 1 minute............... ′ |
| 60 minutes................. | = | 1 degree.............. ° |
| 90 degrees................. | = | 1 rt. angle or quadrant ∟ |
| 360 degrees................. | = | 1 circle .............cir. |

1 cir. = 360° = 21,600′ = 1,296,000″

## AVOIRDUPOIS WEIGHT

| | | |
|---|---|---|
| 437.5 grains (gr.)............ | = | 1 ounce .............oz. |
| 16 ounces................ | = | 1 pound .............lb. |
| 100 pounds................ | = | 1 hundredweight ....cwt. |
| 20 cwt., or 2,000 lb........ | = | 1 ton.................T. |
| 2,240 lb.................... | = | 1 long ton .........L. T. |

1 T. = 20 cwt. = 2,000 lb. = 32,000 oz. = 14,000,000 gr.

The avoirdupois pound contains 7,000 gr.

## TROY WEIGHT

| | | |
|---|---|---|
| 24 grains (gr.).............. | = | 1 pennyweight .....pwt. |
| 20 pennyweights............ | = | 1 ounce .............oz. |
| 12 ounces.................. | = | 1 pound .............lb. |

1 lb. = 12 oz. = 240 pwt. = 5,760 gr.

## DRY MEASURE

| | | |
|---|---|---|
| 2 pints (pt.)................ | = | 1 quart .............qt. |
| 8 quarts.................... | = | 1 peck ..............pk. |
| 4 pecks.................... | = | 1 bushel .............bu. |

1 bu. = 4 pk. = 32 qt. = 64 pt.

The U. S. bushel contains 2,150.42 cu. in. = approximately $1\frac{1}{4}$ cu. ft. The British bushel contains 2,218.19 cu. in.

## LIQUID MEASURE

| | | |
|---|---|---|
| 4 gills (gi.) | = 1 pint | pt. |
| 2 pints | = 1 quart | qt. |
| 4 quarts | = 1 gallon | gal. |
| 31½ gallons | = 1 barrel | bbl. |
| 2 barrels, or 63 gallons | = 1 hogshead | hhd. |

1 hhd. = 2 bbl. = 63 gal. = 252 qt. = 504 pt. = 2,016 gi.

The U. S. gallon contains 231 cu. in. = .134 cu. ft., nearly, or 1 cu. ft. contains 7.481 gal.

When water is at its maximum density, 1 cu. ft. weighs 62.425 lb. and 1 gallon weighs 8.345 lb.

For approximations, 1 cu. ft. of water is considered equal to 7½ gal., and 1 gal. as weighing 8⅓ lb.

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# THE METRIC SYSTEM

The metric system is based on the meter, which, according to the U. S. Coast and Geodetic Survey Report of 1884, is equal to 39.370432 in. The value commonly used is 39.37 in. and is authorized by the U. S. government.

There are three principal units—the *meter*, the *liter* (pronounced "lee-ter"), and the *gram*, the units of length, capacity, and weight, respectively. Multiples of these units are obtained by prefixing to the names of the principal units the Greek words *deca* (10), *hecto* (100), and *kilo* (1,000); the submultiples, or divisions, are obtained by prefixing the Latin words *deci* ($\frac{1}{10}$), *centi* ($\frac{1}{100}$), and *milli* ($\frac{1}{1000}$). These prefixes form the key to the entire system. The abbreviations of the principal units of these submultiples begin with a small letter, while those of the multiples begin with a capital letter.

## MEASURES OF LENGTH

| | | |
|---|---|---|
| 10 millimeters (mm.) | = 1 centimeter | cm. |
| 10 centimeters | = 1 decimeter | dm. |
| 10 decimeters | = 1 meter | m. |
| 10 meters | = 1 decameter | Dm. |
| 10 decameters | = 1 hectometer | Hm. |
| 10 hectometers | = 1 kilometer | Km. |

## MEASURES OF SURFACE (NOT LAND)

| | | |
|---|---|---|
| 100 square millimeters (sq. mm.) | = | 1 square centimeter....sq. cm. |
| 100 square centimeters... | = | 1 square decimeter.....sq. dm. |
| 100 square decimeters.... | = | 1 square meter.........sq. m. |

## MEASURES OF VOLUME

| | | |
|---|---|---|
| 1,000 cubic millimeters (cu. mm.) | = | 1 cubic centimeter.....cu. cm. |
| 1,000 cubic centimeters... | = | 1 cubic decimeter......cu. dm. |
| 1,000 cubic decimeters... | = | 1 cubic meter..........cu. m. |

## MEASURES OF CAPACITY

| | | |
|---|---|---|
| 10 milliliters (ml.)........... | = | 1 centiliter............cl. |
| 10 centiliters................. | = | 1 deciliter............dl. |
| 10 deciliters.................. | = | 1 liter.................l. |
| 10 liters...................... | = | 1 decaliter............Dl. |
| 10 decaliters.................. | = | 1 hectoliter...........Hl. |
| 10 hectoliters................. | = | 1 kiloliter............Kl. |

The liter is equal to the volume occupied by 1 cu. dm.

## MEASURES OF WEIGHT

| | | |
|---|---|---|
| 10 milligrams (mg.).......... | = | 1 centigram...........cg. |
| 10 centigrams................ | = | 1 decigram...........dg. |
| 10 decigrams................. | = | 1 gram................g. |
| 10 grams..................... | = | 1 decagram..........Dg. |
| 10 decagrams................. | = | 1 hectogram.........Hg. |
| 10 hectograms................ | = | 1 kilogram...........Kg. |
| 1,000 kilograms.............. | = | 1 ton.................T. |

The gram is the weight of 1 cu. cm. of pure distilled water at a temperature of 39.2° F.; the kilogram is the weight of 1 liter of water; the ton is the weight of 1 cu. m. of water.

## METRIC CONVERSION FACTORS

In order to use the following factors for converting from English to metric units, it is necessary to transform the

equations; for example, 1,000 Km. × .621 = 621 mi., but 1,000 mi. ÷ .621 = 1,610 Km.

Km. × .621 = mi.
Km. ÷ 1.609 = mi.
Km. × 3,281 = ft.
m. × 39.37 = in.
m. × 3.281 = ft.
m. × 1.094 = yd.
cm. × .3937 = in.
cm. ÷ 2.54 = in.
mm. × .03937 = in.
mm. ÷ 25.4 = in.
sq. Km. × 247.1 = A.
sq. m. × 10.764 = sq. ft.
sq. cm. × .155 = sq. in.
sq. cm. ÷ 6.451 = sq. in.
sq. mm. × .00155 = sq. in.
sq. mm. ÷ 645.1 = sq. in.
cu. m. × 35.315 = cu. ft.
cu. m. × 1.308 = cu. yd.
cu. m. × 264.2 = gal. (U.S.)
cu. cm. ÷ 16.383 = cu. in.
l. × 61.022 = cu. in.
l. × .2642 = gal. (U. S.)
l. ÷ 3.78 = gal. (U. S.)
l. ÷ 28.316 = cu. ft.
g. × 15.432 = gr.
g. × 981 = dynes
g. ÷ 28.35 = oz. (avoir.)
kilograms per sq. cm. × 14.22 = lb. per sq. in.
Kg. × 2.205 = lb.
Kg. × 35.3 = oz. (avoir.)
Kg. × .0011023 = tons (2,000 lb.)
Kg. per sq. cm. × 14,223 = lb. per sq. in.
Kg.-m. × 7.233 = ft.-lb.
kilowatts (k. w.) × 1.34 = H. P.
watts ÷ 746 = H. P.
watts × .7373 = ft.-lb. per sec.
Joules × .7373 = ft.-lb.
Calorie (kilogram-degree) × 3.968 = B. T. U.
Calorie (kilogram-degree) ÷ .252 = B. T. U.
Joules × .24 = gram-calories
gram-calories × 4.19 = Joules
gravity (Paris) = 981 cm. per sec. per sec.
(Degrees centigrade × 1.8) + 32° = degrees F.

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# WEIGHT AND SPECIFIC GRAVITY OF VARIOUS SUBSTANCES

The specific gravity of a substance is the ratio of the weight of any volume of the substance to the weight of an equal volume of some standard substance (water, in the case of solids and liquids; and air, in the case of gases).

| Metals | Weight per Cu. In. Pound | Specific Gravity |
|---|---|---|
| Aluminum | .096 | 2.660 |
| Antimony | .242 | 6.712 |
| Bismuth | .352 | 9.746 |
| Brass, common | .307 | 8.500 |
| Copper, cast | .314 | 8.700 |
| Copper, rolled | .321 | 8.878 |
| Gold, pure cast | .696 | 19.258 |
| Iron, cast | .260 | 7.207 |
| Iron, wrought | .281 | 7.780 |
| Lead, pure | .409 | 11.330 |
| Mercury, at 60° F | .491 | 13.580 |
| Silver, pure | .378 | 10.474 |
| Steel, hard | .286 | 7.919 |
| Steel, soft | .283 | 7.833 |
| Tin | .256 | 7.351 |
| Zinc | .260 | 7.101 |

| Stones and Earth | Weight per Cu. In. Pound | Specific Gravity |
|---|---|---|
| Asbestos | .1110 | 3 to 3.2 |
| Brick | .0723 | 2.000 |
| Chalk | .1006 | 2.784 |
| Clay | .0686 | 1.900 |
| Coal, anthracite | .0592<br>.0519 | 1.640<br>1.436 |
| Coal, bituminous | .0488 | 1.350 |
| Earth, loose | .0491 | 1.360 |
| Emery | .1450 | 4.000 |
| Glass, flint | .1260 | 3.500 |
| Granite, Quincy | .0958 | 2.652 |
| Gypsum, opaque | .0783 | 2.168 |
| Limestone | .0980 | 2.700 |
| Marble, common | .0970 | 2.686 |
| Mica | .1012 | 2.800 |
| Quartz | .0961 | 2.660 |
| Salt, common | .0769 | 2.130 |
| Sand | .0957 | 2.650 |
| Slate | .1012 | 2.800 |
| Soil, common | .0717 | 1.984 |
| Stone, common | .0910 | 2.520 |
| Sulphur, native | .0734 | 2.033 |

| Dry Woods | Weight per Cu. In. Pound | Specific Gravity |
|---|---|---|
| Ash | .0305 | .845 |
| Beech | .0308 | .852 |
| Cedar, American | .0203 | .561 |
| Cork | .0090 | .250 |
| Ebony, American | .0441 | 1.220 |
| Elm | .0202 | .560 |
| Lignum vitæ | .0481 | 1.330 |
| Mahogany, Honduras | .0202 | .560 |
| Maple | .0285 | .790 |
| Oak | .0343 | .950 |
| Pine, Southern | .0260 | .720 |
| Pine, White | .0144 | .400 |
| Poplar | .0138 | .383 |
| Spruce | .0181 | .500 |

| Liquids | Weight per Cu. In. Pound | Specific Gravity |
|---|---|---|
| Acid, nitric | .0440 | 1.217 |
| Acid, sulphuric | .0665 | 1.841 |
| Acid, muriatic, or hydrochloric | .0434 | 1.200 |
| Alcohol, commercial | .0301 | .833 |
| Alcohol, pure | .0286 | .792 |
| Oil, linseed | .0340 | .940 |
| Oil, turpentine | .0314 | .870 |
| Water, distilled (62.425 lb. per cu. ft.) | .0361 | 1.000 |

| Gases and Vapors | Weight per Cu. Ft. Grains | Specific Gravity |
|---|---|---|
| At 32° and a tension of 1 atmosphere | | |
| Atmospheric air | 565.11 | 1.0000 |
| Ammonia gas | 333.1 | .5894 |
| Carbonic acid | 859.0 | 1.5201 |
| Carbonic oxide | 546.6 | .9673 |
| Hydrogen | 39.1 | .0692 |
| Oxygen | 624.8 | 1.1056 |
| Sulphureted hydrogen | 663.8 | 1.1747 |
| Nitrogen | 548.9 | .9713 |
| Steam at 212° F | 275.8 | .4880 |

The weight of a cubic foot of any solid or liquid is found by multiplying its specific gravity by 62.425 lb. avoir. The weight of a cubic foot of any gas at atmospheric pressure and at 32° F. is found by multiplying its specific gravity by .08073 lb. avoir.

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## CHEMISTRY AND ELECTROCHEMISTRY

**Divisions of Matter.**—Science assumes three divisions of matter—*masses*, *molecules*, and *atoms*. A **mass** is any portion of matter appreciable by the senses. A **molecule** is the smallest particle of matter into which a body can be divided; it is the smallest particle that is capable of separate existence. An **atom** is the still smaller particle produced by the division of a molecule by chemical means, and is regarded by chemists as the unit quantity of chemical combination. A molecule is a group of two or more atoms that are united by their affinity, or mutual attraction. *Elemental* molecules are formed of like atoms, *compound* molecules are formed of unlike atoms. Matter composed of elemental molecules is called *simple*, or *elementary matter;* matter composed of compound molecules is called *compound matter*.

The **atomic weight** of an element is the relative proportion, by weight, with which it enters into combinations with other elements. Hydrogen combines with other elements in the smallest proportion, by weight, of any of the elements. The weight of oxygen entering into a combination is 15.88 times the corresponding weight of hydrogen; i. e., if the atomic weight of hydrogen is 1, that of oxygen is 15.88, and if the atomic weight of oxygen is 16, that of hydrogen is about 1.008.

**Valence.**—Atoms unite in molecules always in certain fixed proportions. For example, 2 atoms of hydrogen, $H$, unite with 1 atom of oxygen, $O$, to form 1 molecule of water, $H_2O$; 1 atom of hydrogen, $H$, unites with 1 atom of chlorine, $Cl$, to form 1 molecule of hydrochloric acid, $HCl$. The **valence** of an element is the measure of its power to hold other elements in combination, and is stated on the basis that the valence of hydrogen is I. An element is mono-, di-, tri-, tetra-, etc.

## SYMBOLS, ATOMIC WEIGHTS, ETC. FOR A NUMBER OF THE MORE COMMON ELEMENTS

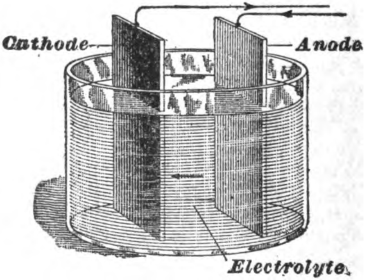
| Element and Symbol | Atomic Weight | Common Valence | Chemical Equivalent |
|---|---|---|---|
| Aluminum, *Al* | 27.1 | III | 9.03 |
| Antimony, *Sb* | 120.2 | III–V | 40.067 — 24.04 |
| *Arsenic, As* | 75.0 | III–V | 25. — 15. |
| Barium, *Ba* | 137.4 | II | 68.7 |
| Bismuth, *Bi* | 208.5 | III–V | 69.5 — 41.7 |
| *Boron, B* | 11.0 | III | 3.67 |
| *Bromine Br* | 79.96 | I | 79.96 |
| Cadmium, *Cd* | 112.4 | II | 56.2 |
| Calcium, *Ca* | 40.1 | II | 20.05 |
| *Carbon, C* | 12.0 | IV | 3. |
| *Chlorine, Cl* | 35.45 | I | 35.45 |
| Chromium, *Cr* | 52.1 | II–VI | 26.05 — 8.68 |
| Cobalt, *Co* | 59.0 | II–III | 29.5 — 19.67 |
| Copper, *Cu* | 63.6 | I–II | 63.6 — 31.8 |
| *Fluorine, F* | 19.0 | I | 19. |
| Gold, *Au* | 197.2 | III | 65.73 |
| *Hydrogen, H* | 1.008 | I | 1.008 |
| *Iodine, I* | 126.85 | I | 126.85 |
| Iron, *Fe* | 55.9 | II–III | 27.95 — 18.63 |
| Lead, *Pb* | 206.9 | II–IV | 103.45 — 51.73 |
| Lithium, *Li* | 7.03 | I | 7.03 |
| Magnesium, *Mg* | 24.36 | II | 12.18 |
| Manganese, *Mn* | 55.0 | II–VII | 27.5 — 7.86 |
| Mercury, *Hg* | 200.0 | I–II | 200.0 — 100.0 |
| Nickel, *Ni* | 58.7 | II–III | 29.35 — 19.57 |
| *Nitrogen, N* | 14.04 | III–V | 4.68 — 2.81 |
| *Oxygen, O* | 16.0 | II | 8.0 |
| Palladium, *Pd* | 106.5 | IV | 26.63 |
| *Phosphorus, P* | 31.0 | III–V | 10.33 — 6.2 |
| Platinum, *Pt* | 194.8 | IV | 48.7 |
| Potassium, *K* | 39.15 | I | 39.15 |
| *Selenium Se* | 79.2 | II | 39.6 |
| *Silicon, Si* | 28.4 | IV | 7.1 |
| Silver, *Ag* | 107.93 | I | 107.93 |
| Sodium, *Na* | 23.05 | I | 23.05 |
| Strontium, *Sr* | 87.6 | II | 43.8 |
| *Sulphur, S* | 32.06 | II | 16.03 |
| *Tellurium, Te* | 127.6 | II | 63.8 |
| Thallium, *Tl* | 204.1 | I–III | 204.1 — 68.03 |
| Thorium, *Th* | 232.5 | IV | 58.13 |
| Tin, *Sn* | 119.0 | II–IV | 59.5 — 29.75 |
| Tungsten, *W* | 184.0 | IV–VI | 46.0 — 30.67 |
| Uranium, *U* | 239.5 | IV–VI | 59.88 — 39.92 |
| Vanadium, *V* | 51.2 | III–V | 17.07 — 10.24 |
| Zinc, *Zn* | 65.4 | II | 32.7 |

**The names of non-metallic elements in the above table are printed in *Italics*. The atomic weights given are based on oxygen, *O* = 16.**

valent according to whether its atoms hold the atoms of other elements in combination in the proportion of one, two, three, four, etc. Hydrogen is monovalent and oxygen bivalent because 1 atom of oxygen holds 2 atoms of hydrogen, as represented by the symbol $H_2O$. Some elements, for instance, copper, have two or more different valencies because they unite in different proportions with certain other elements to form different compounds. Thus there is cuprous chloride, $CuCl$, and cupric chloride, $CuCl_2$.

## ELECTROLYTIC ACTION

A current of electricity in passing through an electrolyte decomposes it; e.g., an electrolyte consisting of zinc chloride (usually dissolved in water) is broken up into chlorine gas and metallic zinc. An electrolytic cell consists of a vessel




ELECTROLYTIC CELL

containing the electrolyte and the electrodes — the anode and the cathode; these are usually metal or carbon plates. The two parts into which the electrolyte is decomposed are called **ions**; those ions that appear at the anode are called *anions* and those at the cathode are called *cations*. In decomposing zinc chloride, zinc appears at the cathode and is thus a cation, and chlorine gas appears at the anode as an anion

## CHEMICAL AND ELECTROCHEMICAL EQUIVALENTS

Based on Atomic Weight of Oxygen = 16 and Electrochemical Equivalent of Silver = .001118. The Names of Non-Metallic Elements are Printed in Italics.

| Name of Element and Symbol | Common Valence | Electrochemical Equivalent Grams per Coulomb | Name of Element and Symbol | Common Valence | Electrochemical Equivalent Grams per Coulomb |
|---|---|---|---|---|---|
| Aluminum, *Al* | III | .00009354 | Mercury, *Hg* | I–II | .00207172—.00103586 |
| Antimony, *Sb* | III–V | .00041504—.00024902 | Nickel, *Ni* | II–III | .00030402—.00020272 |
| *Arsenic, As* | III–V | .00025997—.00015538 | *Nitrogen, N* | III–V | .00004848—.00002911 |
| Barium, *Ba* | II | .00071164 | *Oxygen, O* | II | .00008287 |
| Bismuth, *Bi* | III–V | .00071992—.00043195 | Palladium, *Pd* | IV | .00027585 |
| *Boron, B* | III | .00003802 | *Phosphorus, P* | III–V | .00010700—.00006422 |
| *Bromine, Br* | I | .00082827 | Platinum, *Pt* | IV | .00050446 |
| Cadmium, *Cd* | II | .00058216 | Potassium, *K* | I | .00040554 |
| Calcium, *Ca* | II | .00020768 | *Selenium, Se* | II | .00041020 |
| *Carbon, C* | IV | .00003108 | *Silicon, Si* | IV | .00075346 |
| *Chlorine, Cl* | I | .00036721 | Silver, *Ag* | I | .00111800 |
| Chromium, *Cr* | II–VI | .00026984—.00008991 | Sodium, *Na* | I | .00023877 |
| Cobalt, *Co* | II–III | .00030558—.00020375 | Strontium, *Sr* | II | .00045371 |
| Copper, *Cu* | I–II | .00065881—.00032940 | *Sulphur, S* | II | .00016606 |
| *Fluorine, F* | I | .00019681 | *Tellurium, Te* | II | .00066088 |
| Gold, *Au* | III | .00068087 | Thallium, *Tl* | I–III | .00211419—.00070470 |
| *Hydrogen, H* | I | .00001044 | Thorium, *Th* | IV | .00060215 |
| *Iodine, I* | I | .00131399 | Tin, *Sn* | II–IV | .00061634—.00030817 |
| Iron, *Fe* | II–III | .00028952—.00019298 | Tungsten, *W* | IV–VI | .00047650—.00031769 |
| Lead, *Pb* | II–IV | .00107160—.00053585 | Uranium, *U* | IV–VI | .00062027—.00041352 |
| Lithium, *Li* | I | .00007282 | Vanadium, *V* | III–V | .00017682—.00010607 |
| Magnesium, *Mg* | II | .00012617 | Zinc, *Zn* | II | .00033873 |
| Manganese, *Mn* | II–VII | .00028486—.00008142 | | | |

The **chemical equivalent** of an element is the quotient of the atomic weight divided by the valence. The **electrochemical equivalent** of an element is the weight in grams liberated electrolytically by 1 coulomb (1 ampere-second) of electricity. The electrochemical equivalent is proportional to the chemical equivalent. The electrochemical equivalent of silver has been accurately determined by experiment as .001118 gram, and that of each of the other elements can be calculated from this. For example, the electrochemical equivalent of aluminum is $\frac{9.03}{107.93} \times .001118 = .00009354$ gram.

# HEAT

## SPECIFIC HEATS OF METALS

The specific heat of a substance is the number of heat units required to raise a unit mass of the substance one degree in temperature. The specific heat of water is very nearly constant for all temperatures, but that at its temperature of maximum density (4° C. or 39.1° F.) is considered unity. The specific heats of most substances increase with increasing temperatures.

| Substance | Specific Heat at | | |
|---|---|---|---|
| | 0° C. or 32° F. | 50° C. or 122° F. | 100° C. or 212° F. |
| Aluminum | .2070 | .2185 | .2300 |
| Copper | .0901 | .0923 | .0966 |
| German silver | .0941 | .0947 | .0952 |
| Iron | .1060 | .1130 | .1200 |
| Lead | .0300 | .0315 | .0331 |
| Platinum | .0320 | .0326 | .0333 |
| Platinum silver | .0473 | .0487 | .0581 |
| Silver | .0547 | .0569 | .0591 |
| Tin | .0523 | .0568 | .0595 |
| Zinc | .0901 | .0938 | .0976 |

## TEMPERATURE OF FUSION

| Substance | Fusing Point, Degrees F. | Fusing Point, Degrees C. |
|---|---|---|
| Aluminum | 1,160 | 627 |
| Carbon | Infusible | Infusible |
| Copper | 1,931 | 1,054 |
| Gold | 1,913 | 1,045 |
| Iridium | 3,542 | 1,950 |
| Iron, cast | 2,192 | 1,200 |
| Iron, wrought | 2,912 | 1,600 |
| Lead | 617 | 325 |
| Mercury | −37.8 | −38.8 |
| Nickel | 2,642 | 1,450 |
| Osmium | 3,900 | 2,200* |
| Platinum | 3,225 | 1,774 |
| Silver, pure | 1,749 | 954 |
| Steel | 2,520 | 1,382 |
| Sulphur | 235 | 114.5 |
| Tantalum | 3,865 | 2,150* |
| Tin | 551 | 233 |
| Tungsten | above 3,420 | above 1,900* |

*J. Swinburne, F. R. S., Proc. British Institution of Electrical Engineers, Jan. 10, 1907.

## HEAT UNITS

One *British thermal unit* (B. T. U.) is the quantity of heat required to raise the temperature of 1 lb. of pure water 1° F. at or near its maximum density, 39.1° F.

One *calorie* is the quantity of heat required to raise the temperature of 1 Kg. of water 1° C. at or near 4° C.

1 B. T. U. = .252 calorie and 1 calorie = 3.968 B. T. U.

One *small*, or *gram*, *calorie* (a heat unit also in some use) is the quantity of heat required to raise the temperature of 1 gram of water 1° C. at or near 4° C.

## COEFFICIENTS OF LINEAR EXPANSION

The coefficient of expansion of a body is its expansion per degree rise of temperature. The coefficient of surface expansion is double, and that of cubical expansion three times, the coefficient of linear expansion.

| Substance | | Coefficient of Linear Expansion in Inches per Degree F. |
|---|---|---|
| Aluminum | | .00001140 |
| Brass | | .00001040 |
| Brick | | .00000306 |
| Cement and Concrete | from | .00000550 |
| | to | .00000780 |
| Copper | | .00000961 |
| Glass | from | .00000399 |
| | to | .00000521 |
| Gold | | .00000841 |
| Granite | | .00000460 |
| Iron, cast | | .00000587 |
| Iron, wrought | | .00000677 |
| Lead | | .00001580 |
| Marble | | .00000400 |
| Masonry | from | .00000206 |
| | to | .00000490 |
| Mercury | | .00003334 |
| Platinum | | .00000494 |
| Porcelain | | .00000200 |
| Sandstone | from | .00000400 |
| | to | .00000670 |
| Steel, untempered | | .00000599 |
| Steel, tempered | | .00000702 |
| Tin | | .00001160 |
| Wood (pine) | | .00000276 |
| Zinc | | .00001634 |

For example, a 60-ft. steel rail in warming from 20° F. below zero to 100° F. will expand 120 × .00000599 × 60 × 12 = .5175 in.

## THE MECHANICAL EQUIVALENT OF HEAT

1 B. T. U. = 778 ft.-lb.

1 ft.-lb. = $\frac{1}{778}$ = .001285 B. T. U.

1 H. P. = 33,000 ft.-lb. per min. = 42.416 B. T. U. per min.

## CENTIGRADE AND FAHRENHEIT DEGREES

| Deg. C. | Deg. F. | Deg. C. | Deg. F. | Deg. C. | Deg. F. | Deg. C. | Deg. F. |
|---|---|---|---|---|---|---|---|
| 0 | 32.0 | 26 | 78.8 | 51 | 123.8 | 76 | 168.8 |
| 1 | 33.8 | 27 | 80.6 | 52 | 125.6 | 77 | 170.6 |
| 2 | 35.6 | 28 | 82.4 | 53 | 127.4 | 78 | 172.4 |
| 3 | 37.4 | 29 | 84.2 | 54 | 129.2 | 79 | 174.2 |
| 4 | 39.2 | 30 | 86.0 | 55 | 131.0 | 80 | 176.0 |
| 5 | 41.0 | 31 | 87.8 | 56 | 132.8 | 81 | 177.8 |
| 6 | 42.8 | 32 | 89.6 | 57 | 134.6 | 82 | 179.6 |
| 7 | 44.6 | 33 | 91.4 | 58 | 136.4 | 83 | 181.4 |
| 8 | 46.4 | 34 | 93.2 | 59 | 138.2 | 84 | 183.2 |
| 9 | 48.2 | 35 | 95.0 | 60 | 140.0 | 85 | 185.0 |
| 10 | 50.0 | 36 | 96.8 | 61 | 141.8 | 86 | 186.8 |
| 11 | 51.8 | 37 | 98.6 | 62 | 143.6 | 87 | 188.6 |
| 12 | 53.6 | 38 | 100.4 | 63 | 145.4 | 88 | 190.4 |
| 13 | 55.4 | 39 | 102.2 | 64 | 147.2 | 89 | 192.2 |
| 14 | 57.2 | 40 | 104.0 | 65 | 149.0 | 90 | 194.0 |
| 15 | 59.0 | 41 | 105.8 | 66 | 150.8 | 91 | 195.8 |
| 16 | 60.8 | 42 | 107.6 | 67 | 152.6 | 92 | 197.6 |
| 17 | 62.6 | 43 | 109.4 | 68 | 154.4 | 93 | 199.4 |
| 18 | 64.4 | 44 | 111.2 | 69 | 156.2 | 94 | 201.2 |
| 19 | 66.2 | 45 | 113.0 | 70 | 158.0 | 95 | 203.0 |
| 20 | 68.0 | 46 | 114.8 | 71 | 159.8 | 96 | 204.8 |
| 21 | 69.8 | 47 | 116.6 | 72 | 161.6 | 97 | 206.6 |
| 22 | 71.6 | 48 | 118.4 | 73 | 163.4 | 98 | 208.4 |
| 23 | 73.4 | 49 | 120.2 | 74 | 165.2 | 99 | 210.2 |
| 24 | 75.2 | 50 | 122.0 | 75 | 167.0 | 100 | 212.0 |
| 25 | 77.0 | | | | | | |

## TEMPERATURE

The temperature of a body is its degree of sensible heat. For the measurement of temperatures there are three kinds of thermometers: the Fahrenheit, abbreviated F. or Fahr., commonly used in America; the Centigrade, abbreviated C. or Cent., used in France and by scientists everywhere; and the Réaumur, abbreviated R. or Réau., used in Germany.

| *Standard Points* | *Degrees F.* | *Degrees C.* | *Degrees R.* |
|---|---|---|---|
| Boiling point of water at sea level; i. e., pressure = 1 atmosphere..... | 212 | 100 | 80 |
| Melting point of ice....... | 32 | 0 | 0 |
| Absolute zero, i. e., the total absence of heat; theoretical only...... | −460 | −273 | −219 |

Between boiling point and freezing point = 180° F. = 100° C. = 80° R.

$$\text{Temp. F.} = \frac{9}{5}\ \text{Temp. C.} + 32° = \frac{9}{4}\ \text{Temp. R.} + 32°.$$

$$\text{Temp. C.} = \frac{5}{9}\ (\text{Temp. F.} - 32°) = \frac{5}{4}\ \text{Temp. R.}$$

$$\text{Temp. R.} = \frac{4}{9}\ (\text{Temp. F.} - 32°) = \frac{4}{5}\ \text{Temp. C.}$$

# MATHEMATICAL TABLES

## CIRCUMFERENCES AND AREAS OF CIRCLES FROM 1-64 TO 100

| Diam. | Circum. | Area | Diam. | Circum. | Area |
|---|---|---|---|---|---|
| 1/64 | .0491 | .0002 | 4 | 12.5664 | 12.5664 |
| 1/32 | .0982 | .0008 | 4 1/8 | 12.9591 | 13.3641 |
| 1/16 | .1963 | .0031 | 4 1/4 | 13.3518 | 14.1863 |
| 1/8 | .3927 | .0123 | 4 3/8 | 13.7445 | 15.0330 |
| 3/16 | .5890 | .0276 | 4 1/2 | 14.1372 | 15.9043 |
| 1/4 | .7854 | .0491 | 4 5/8 | 14.5299 | 16.8002 |
| 5/16 | .9817 | .0767 | 4 3/4 | 14.9226 | 17.7206 |
| 3/8 | 1.1781 | .1104 | 4 7/8 | 15.3153 | 18.6555 |
| 7/16 | 1.3744 | .1503 | 5 | 15.7080 | 19.6350 |
| 1/2 | 1.5708 | .1963 | 5 1/8 | 16.1007 | 20.6290 |
| 9/16 | 1.7671 | .2485 | 5 1/4 | 16.4934 | 21.6476 |
| 5/8 | 1.9635 | .3068 | 5 3/8 | 16.8861 | 22.6907 |
| 11/16 | 2.1598 | .3712 | 5 1/2 | 17.2788 | 23.7583 |
| 3/4 | 2.3562 | .4418 | 5 5/8 | 17.6715 | 24.8505 |
| 13/16 | 2.5525 | .5185 | 5 3/4 | 18.0642 | 25.9673 |
| 7/8 | 2.7489 | .6013 | 5 7/8 | 18.4569 | 27.1086 |
| 15/16 | 2.9452 | .6903 | 6 | 18.8496 | 28.2744 |
| 1 | 3.1416 | .7854 | 6 1/8 | 19.2423 | 29.4648 |
| 1 1/8 | 3.5343 | .9940 | 6 1/4 | 19.6350 | 30.6797 |
| 1 1/4 | 3.9270 | 1.2272 | 6 3/8 | 20.0277 | 31.9191 |
| 1 3/8 | 4.3197 | 1.4849 | 6 1/2 | 20.4204 | 33.1831 |
| 1 1/2 | 4.7124 | 1.7671 | 6 5/8 | 20.8131 | 34.4717 |
| 1 5/8 | 5.1051 | 2.0739 | 6 3/4 | 21.2058 | 35.7848 |
| 1 3/4 | 5.4978 | 2.4053 | 6 7/8 | 21.5985 | 37.1224 |
| 1 7/8 | 5.8905 | 2.7612 | 7 | 21.9912 | 38.4846 |
| 2 | 6.2832 | 3.1416 | 7 1/8 | 22.3839 | 39.8713 |
| 2 1/8 | 6.6759 | 3.5466 | 7 1/4 | 22.7766 | 41.2826 |
| 2 1/4 | 7.0686 | 3.9761 | 7 3/8 | 23.1693 | 42.7184 |
| 2 3/8 | 7.4613 | 4.4301 | 7 1/2 | 23.5620 | 44.1787 |
| 2 1/2 | 7.8540 | 4.9087 | 7 5/8 | 23.9547 | 45.6636 |
| 2 5/8 | 8.2467 | 5.4119 | 7 3/4 | 24.3474 | 47.1731 |
| 2 3/4 | 8.6394 | 5.9396 | 7 7/8 | 24.7401 | 48.7071 |
| 2 7/8 | 9.0321 | 6.4918 | 8 | 25.1328 | 50.2656 |
| 3 | 9.4248 | 7.0686 | 8 1/8 | 25.5255 | 51.8487 |
| 3 1/8 | 9.8175 | 7.6699 | 8 1/4 | 25.9182 | 53.4563 |
| 3 1/4 | 10.2102 | 8.2958 | 8 3/8 | 26.3109 | 55.0884 |
| 3 3/8 | 10.6029 | 8.9462 | 8 1/2 | 26.7036 | 56.7451 |
| 3 1/2 | 10.9956 | 9.6211 | 8 5/8 | 27.0963 | 58.4264 |
| 3 5/8 | 11.3883 | 10.3206 | 8 3/4 | 27.4890 | 60.1322 |
| 3 3/4 | 11.7810 | 11.0447 | 8 7/8 | 27.8817 | 61.8625 |
| 3 7/8 | 12.1737 | 11.7933 | 9 | 28.2744 | 63.6174 |

TABLE—(*Continued*)

| Diam. | Circum. | Area | Diam. | Circum. | Area |
|---|---|---|---|---|---|
| 9⅛ | 28.6671 | 65.3968 | 19½ | 61.2612 | 298.648 |
| 9¼ | 29.0598 | 67.2008 | 19¾ | 62.0466 | 306.355 |
| 9⅜ | 29.4525 | 69.0293 | 20 | 62.8320 | 314.160 |
| 9½ | 29.8452 | 70.8823 | 20¼ | 63.6174 | 322.063 |
| 9⅝ | 30.2379 | 72.7599 | 20½ | 64.4028 | 330.064 |
| 9¾ | 30.6306 | 74.6621 | 20¾ | 65.1882 | 338.164 |
| 9⅞ | 31.0233 | 76.589 | 21 | 65.9736 | 346.361 |
| 10 | 31.4160 | 78.540 | 21¼ | 66.7590 | 354.657 |
| 10¼ | 32.2014 | 82.516 | 21½ | 67.5444 | 363.051 |
| 10½ | 32.9868 | 86.590 | 21¾ | 68.3298 | 371.543 |
| 10¾ | 33.7722 | 90.763 | 22 | 69.1152 | 380.134 |
| 11 | 34.5576 | 95.033 | 22¼ | 69.9006 | 388.822 |
| 11¼ | 35.3430 | 99.402 | 22½ | 70.6860 | 397.609 |
| 11½ | 36.1284 | 103.869 | 22¾ | 71.4714 | 406.494 |
| 11¾ | 36.9138 | 108.434 | 23 | 72.2568 | 415.477 |
| 12 | 37.6992 | 113.098 | 23¼ | 73.0422 | 424.558 |
| 12¼ | 38.4846 | 117.859 | 23½ | 73.8276 | 433.737 |
| 12½ | 39.2700 | 122.719 | 23¾ | 74.6130 | 443.015 |
| 12¾ | 40.0554 | 127.677 | 24 | 75.3984 | 452.390 |
| 13 | 40.8408 | 132.733 | 24¼ | 76.1838 | 461.864 |
| 13¼ | 41.6262 | 137.887 | 24½ | 76.9692 | 471.436 |
| 13½ | 42.4116 | 143.139 | 24¾ | 77.7546 | 481.107 |
| 13¾ | 43.1970 | 148.490 | 25 | 78.5400 | 490.875 |
| 14 | 43.9824 | 153.938 | 25¼ | 79.3254 | 500.742 |
| 14¼ | 44.7678 | 159.485 | 25½ | 80.1108 | 510.706 |
| 14½ | 45.5532 | 165.130 | 25¾ | 80.8962 | 520.769 |
| 14¾ | 46.3386 | 170.874 | 26 | 81.6816 | 530.930 |
| 15 | 47.1240 | 176.715 | 26¼ | 82.4670 | 541.190 |
| 15¼ | 47.9094 | 182.655 | 26½ | 83.2524 | 551.547 |
| 15½ | 48.6948 | 188.692 | 26¾ | 84.0378 | 562.003 |
| 15¾ | 49.4802 | 194.828 | 27 | 84.8232 | 572.557 |
| 16 | 50.2656 | 201.062 | 27¼ | 85.6086 | 583.209 |
| 16¼ | 51.0510 | 207.395 | 27½ | 86.3940 | 593.959 |
| 16½ | 51.8364 | 213.825 | 27¾ | 87.1794 | 604.807 |
| 16¾ | 52.6218 | 220.354 | 28 | 87.9648 | 615.754 |
| 17 | 53.4072 | 226.981 | 28¼ | 88.7502 | 626.798 |
| 17¼ | 54.1926 | 233.706 | 28½ | 89.5356 | 637.941 |
| 17½ | 54.9780 | 240.529 | 28¾ | 90.3210 | 649.182 |
| 17¾ | 55.7634 | 247.450 | 29 | 91.1064 | 660.521 |
| 18 | 56.5488 | 254.470 | 29¼ | 91.8918 | 671.959 |
| 18¼ | 57.3342 | 261.587 | 29½ | 92.6772 | 683.494 |
| 18½ | 58.1196 | 268.803 | 29¾ | 93.4626 | 695.128 |
| 18¾ | 58.9050 | 276.117 | 30 | 94.2480 | 706.860 |
| 19 | 59.6904 | 283.529 | 30¼ | 95.0334 | 718.690 |
| 19¼ | 60.4758 | 291.040 | 30½ | 95.8188 | 730.618 |

TABLE—(*Continued*)

| Diam. | Circum. | Area | Diam. | Circum. | Area |
|---|---|---|---|---|---|
| 30¾ | 96.6042 | 742.645 | 42 | 131.947 | 1,385.450 |
| 31 | 97.3896 | 754.769 | 42¼ | 132.733 | 1,401.990 |
| 31¼ | 98.1750 | 766.992 | 42½ | 133.518 | 1,418.630 |
| 31½ | 98.9604 | 779.313 | 42¾ | 134.303 | 1,435.370 |
| 31¾ | 99.7458 | 791.732 | 43 | 135.089 | 1,452.200 |
| 32 | 100.5312 | 804.250 | 43¼ | 135.874 | 1,469.140 |
| 32¼ | 101.3166 | 816.865 | 43½ | 136.660 | 1,486.170 |
| 32½ | 102.1020 | 829.579 | 43¾ | 137.445 | 1,503.300 |
| 32¾ | 102.8874 | 842.391 | 44 | 138.230 | 1,520.530 |
| 33 | 103.673 | 855.301 | 44¼ | 139.016 | 1,537.860 |
| 33¼ | 104.458 | 868.309 | 44½ | 139.801 | 1,555.29 |
| 33½ | 105.244 | 881.415 | 44¾ | 140.587 | 1,572.81 |
| 33¾ | 106.029 | 894.620 | 45 | 141.372 | 1,590.43 |
| 34 | 106.814 | 907.922 | 45¼ | 142.157 | 1,608.16 |
| 34¼ | 107.600 | 921.323 | 45½ | 142.943 | 1,625.97 |
| 34½ | 108.385 | 934.822 | 45¾ | 143.728 | 1,643.89 |
| 34¾ | 109.171 | 948.420 | 46 | 144.514 | 1,661.91 |
| 35 | 109.956 | 962.115 | 46¼ | 145.299 | 1,680.02 |
| 35¼ | 110.741 | 975.909 | 46½ | 146.084 | 1,698.23 |
| 35½ | 111.527 | 989.800 | 46¾ | 146.870 | 1,716.54 |
| 35¾ | 112.312 | 1,003.790 | 47 | 147.655 | 1,734.95 |
| 36 | 113.098 | 1,017.878 | 47¼ | 148.441 | 1,753.45 |
| 36¼ | 113.883 | 1,032.065 | 47½ | 149.226 | 1,772.06 |
| 36½ | 114.668 | 1,046.349 | 47¾ | 150.011 | 1,790.76 |
| 36¾ | 115.454 | 1,060.732 | 48 | 150.797 | 1,809.56 |
| 37 | 116.239 | 1,075.213 | 48¼ | 151.582 | 1,828.46 |
| 37¼ | 117.025 | 1,089.792 | 48½ | 152.368 | 1,847.46 |
| 37½ | 117.810 | 1,104.469 | 48¾ | 153.153 | 1,866.55 |
| 37¾ | 118.595 | 1,119.244 | 49 | 153.938 | 1,885.75 |
| 38 | 119.381 | 1,134.118 | 49¼ | 154.724 | 1,905.04 |
| 38¼ | 120.166 | 1,149.089 | 49½ | 155.509 | 1,924.43 |
| 38½ | 120.952 | 1,164.159 | 49¾ | 156.295 | 1,943.91 |
| 38¾ | 121.737 | 1,179.327 | 50 | 157.080 | 1,963.50 |
| 39 | 122.522 | 1,194.593 | 50½ | 158.651 | 2,002.97 |
| 39¼ | 123.308 | 1,209.958 | 51 | 160.222 | 2,042.83 |
| 39½ | 124.093 | 1,225.420 | 51½ | 161.792 | 2,083.08 |
| 39¾ | 124.879 | 1,240.981 | 52 | 163.363 | 2,123.72 |
| 40 | 125.664 | 1,256.640 | 52½ | 164.934 | 2,164.76 |
| 40¼ | 126.449 | 1,272.400 | 53 | 166.505 | 2,206.19 |
| 40½ | 127.235 | 1,288.250 | 53½ | 168.076 | 2,248.01 |
| 40¾ | 128.020 | 1,304.210 | 54 | 169.646 | 2,290.23 |
| 41 | 128.806 | 1,320.260 | 54½ | 171.217 | 2,332.83 |
| 41¼ | 129.591 | 1,336.410 | 55 | 172.788 | 2,375.83 |
| 41½ | 130.376 | 1,352.660 | 55½ | 174.359 | 2,419.23 |
| 41¾ | 131.162 | 1,369.000 | 56 | 175.930 | 2,463.01 |

TABLE—(*Continued*)

| Diam | Circum. | Area | Diam. | Circum. | Area |
|---|---|---|---|---|---|
| 56½ | 177.500 | 2,507.19 | 78½ | 246.616 | 4,839.83 |
| 57 | 179.071 | 2,551.76 | 79 | 248.186 | 4,901.68 |
| 57½ | 180.642 | 2,596.73 | 79½ | 249.757 | 4,963.92 |
| 58 | 182.213 | 2,642.09 | 80 | 251.328 | 5,026.56 |
| 58½ | 183.784 | 2,687.84 | 80½ | 252.899 | 5,089.59 |
| 59 | 185.354 | 2,733.98 | 81 | 254.470 | 5,153.01 |
| 59½ | 186.925 | 2,780.51 | 81½ | 256.040 | 5,216.82 |
| 60 | 188.496 | 2,827.44 | 82 | 257.611 | 5,281.03 |
| 60½ | 190.067 | 2,874.76 | 82½ | 259.182 | 5,345.63 |
| 61 | 191.638 | 2;922.47 | 83 | 260.753 | 5,410.62 |
| 61½ | 193.208 | 2,9.0.58 | 83½ | 262.324 | 5,476.01 |
| 62 | 194.779 | 3,019.08 | 84 | 263.894 | 5,541.78 |
| 62½ | 196.350 | 3,067.97 | 84½ | 265.465 | 5,607.95 |
| 63 | 197.921 | 3,117.25 | 85 | 267.036 | 5,674.51 |
| 63½ | 199.492 | 3,166.93 | 85½ | 268.607 | 5,741.47 |
| 64 | 201.062 | 3,217.00 | 86 | 270.178 | 5,808.82 |
| 64½ | 202.633 | 3,267.46 | 86½ | 271.748 | 5,876.56 |
| 65 | 204.204 | 3,318.31 | 87 | 273.319 | 5,944.69 |
| 65½ | 205.775 | 3,369.56 | 87½ | 274.890 | 6,013.22 |
| 66 | 207.346 | 3,421.20 | 88 | 276.461 | 6,082.14 |
| 66½ | 208.916 | 3,473 24 | 88½ | 278.032 | 6,151.45 |
| 67 | 210.487 | 3,525.66 | 89 | 279.602 | 6,221.15 |
| 67½ | 212.058 | 3,578.48 | 89½ | 281.173 | 6,291.25 |
| 68 | 213.629 | 3,631.69 | 90 | 282.744 | 6,361.74 |
| 68½ | 215.200 | 3,685.29 | 90½ | 284.315 | 6,432.62 |
| 69 | 216.770 | 3,739.29 | 91 | 285.886 | 6,503.90 |
| 69½ | 218.341 | 3,793.68 | 91½ | 287.456 | 6,575.56 |
| 70 | 219.912 | 3,848.46 | 92 | 289.027 | 6,647.63 |
| 70½ | 221.483 | 3,903.63 | 92½ | 290.598 | 6,720.08 |
| 71 | 223.054 | 3,959.20 | 93 | 292.169 | 6,792.92 |
| 71½ | 224.624 | 4,015.16 | 93½ | 293.740 | 6,866.16 |
| 72 | 226.195 | 4,071.51 | 94 | 295.310 | 6,939.79 |
| 72½ | 227.766 | 4,128.26 | 94½ | 296.881 | 7,013.82 |
| 73 | 229.337 | 4,185.40 | 95 | 298.452 | 7,088.24 |
| 73½ | 230.908 | 4,242.93 | 95½ | 300.023 | 7,163.04 |
| 74 | 232.478 | 4,300.85 | 96 | 301.594 | 7,238.25 |
| 74½ | 234.049 | 4,359.17 | 96½ | 303.164 | 7,313.84 |
| 75 | 235.620 | 4,417.87 | 97 | 304.735 | 7,389.83 |
| 75½ | 237.191 | 4,476.98 | 97½ | 306.306 | 7,466.21 |
| 76 | 238.762 | 4,536.47 | 98 | 307.877 | 7,542.98 |
| 76½ | 240.332 | 4,596.36 | 98½ | 309.448 | 7,620.15 |
| 77 | 241.903 | 4,656.64 | 99 | 311.018 | 7,697.71 |
| 77½ | 243.474 | 4,717.31 | 99½ | 312.589 | 7,775.66 |
| 78 | 245.045 | 4,778.37 | 100 | 314.160 | 7,854.00 |

## DECIMAL EQUIVALENTS OF PARTS OF ONE INCH

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1–64 | .015625 | 17–64 | .265625 | 33–64 | .515625 | 49–64 | .765625 |
| 1–32 | .031250 | 9–32 | .281250 | 17–32 | .531250 | 25–32 | .781250 |
| 3–64 | .046875 | 19–64 | .296875 | 35–64 | .546875 | 51–64 | .796875 |
| 1–16 | .062500 | 5–16 | .312500 | 9–16 | .562500 | 13–16 | .812500 |
| 5–64 | .078125 | 21–64 | .328125 | 37–64 | .578125 | 53–64 | .828125 |
| 3–32 | 093750 | 11–32 | .343750 | 19–32 | .593750 | 27–32 | .843750 |
| 7–64 | .109375 | 23–64 | .359375 | 39–64 | .609375 | 55–64 | .859375 |
| 1–8 | .125000 | 3–8 | .375000 | 5–8 | .625000 | 7–8 | .875000 |
| 9–64 | .140625 | 25–64 | .390625 | 41–64 | .640625 | 57–64 | .890625 |
| 5–32 | .156250 | 13–32 | .406250 | 21–32 | .656250 | 29–32 | .906250 |
| 11–64 | .171875 | 27–64 | .421875 | 43–64 | .671875 | 59–64 | .921875 |
| 3–16 | .187500 | 7–16 | .437500 | 11–16 | .687500 | 15–16 | .937500 |
| 13–64 | .203125 | 29–64 | .453125 | 45–64 | .703125 | 61–64 | .953125 |
| 7–32 | .218750 | 15–32 | .468750 | 23–32 | .718750 | 31–32 | .968750 |
| 15–64 | .234375 | 31–64 | .484375 | 47–64 | .734375 | 63–64 | .984375 |
| 1–4 | .250000 | 1–2 | .500000 | 3–4 | .750000 | 1 | 1 |

## TRIGONOMETRIC FUNCTIONS

The table given on pages 26–27 contains the natural sines, cosines, tangents, and cotangents of angles from 0° to 90°. Angles less than 45° are given in the first column at the left-hand side of the page, and the names of the functions are given at the top of the page; angles greater than 45° appear at the right-hand side of the page, and the names of the functions are given at the bottom. Thus, the second column contains the sines of angles less than 45° and the cosines of angles greater than 45°; the sixth column contains the cotangents of angles less than 45° and the tangents of angles greater than 45°. To find the function of an angle less than 45°, look in the column of angles at the left of the page for the angle, and at the top of the page for the name of the function; to find a function of an angle greater than 45°, look in the column at the right of the page for the angle and at the bottom of the page for the name of the function. The successive angles differ by an interval of 10′; they increase downwards in the left-hand column and upwards in the right-hand column. Thus, for angles less than 45° read down from top of page, and for angles greater than 45° read up from bottom of page.

The third, fifth, seventh, and ninth columns, headed *d*, contain the differences between the successive functions; for

example, the sine of 32° 10′ is .5324 and the sine of 32° 20′ is .5348, as given in the second column, page 26; the difference is .5348 − .5324 = .0024, and the 24 is written in the third column, just opposite the space between .5324 and .5348. In like manner, the differences between the successive tabular values of the tangents are given in the fifth column, those between the cotangents in the seventh column, and those for the cosines in the ninth column. These differences in the functions correspond to a difference of 10′ in the angle; thus, when the angle 32° 10′ is increased by 10′, that is, to 32° 20′, the increase of the sine is .0024, or, as given in the table, 24. In the tabular difference, no attention is paid to the decimal point, it being understood that the difference is merely the number obtained by subtracting the last two or three figures of the smaller function from those of the larger. These differences are used to obtain the sines, cosines, etc. of angles not given in the table; for example, to find the tangent of 27° 34′ find in the table the tangent of 27° 30′, .5206, and (in column 5) the difference for 10′, 37. Difference for 1′ is 37 ÷ 10 = 3.7, and difference for 4′ is 3.7 × 4 = 14.8. Adding this difference to the value of the tan 27° 30′, gives

$$\begin{array}{rl} \tan 27^\circ 30' = & .5206 \\ \text{difference for } 4' = & \phantom{.52}14.8 \\ \hline \tan 27^\circ 34' = & .52208, \text{ or } .5221, \text{ to four places.} \end{array}$$

Since only four decimal places are retained, the 8 in the fifth place is dropped and the figure in the fourth place is increased by 1, because 8 is greater than 5.

To avoid multiplication, the column of proportional parts, headed P. P., at the extreme right of the page, is used. At the head of each table in this column is the difference for 10′, and below are the differences for any intermediate number of minutes from 1′ to 9′. In the above example, the difference at 27° 30′ for 10′ was 37; looking in the table with 37 at the head, the difference opposite 4 is 14.8; that opposite 7 is 25.9; and so on. For want of space, the differences for the cotangents for angles less than 45° (or the tangents of angles greater than 45°) have been omitted from the tables of proportional parts. The use of these functions should be

| ° | ′ | Sin. | d. | Tan. | d. | Cot. | d. | Cos. | d. | ′ | ° |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0.0000 | 29 | 0.0000 | 29 | infinit. | | 1.0000 | 0 | 0 | 90 |
| | 10 | 0.0029 | 29 | 0.0029 | 29 | 343.7737 | | 1.0000 | 0 | 50 | |
| | 20 | 0.0058 | 29 | 0.0058 | 29 | 171.8854 | | 1.0000 | 0 | 40 | |
| | 30 | 0.0087 | 29 | 0.0087 | 29 | 114.5887 | | 1.0000 | 1 | 30 | |
| | 40 | 0.0116 | 29 | 0.0116 | 29 | 85.9398 | | 0.9999 | 0 | 20 | |
| | 50 | 0.0145 | 30 | 0.0145 | 30 | 68.7501 | | 0.9999 | 1 | 10 | |
| 1 | 0 | 0.0175 | 29 | 0.0175 | 29 | 57.2900 | 81861 | 0.9998 | 0 | 0 | 89 |
| | 10 | 0.0204 | 29 | 0.0204 | 29 | 49.1039 | 61398 | 0.9998 | 1 | 50 | |
| | 20 | 0.0233 | 29 | 0.0233 | 29 | 42.9641 | 47756 | 0.9997 | 0 | 40 | |
| | 30 | 0.0262 | 29 | 0.0262 | 29 | 38.1885 | 38207 | 0.9997 | 1 | 30 | |
| | 40 | 0.0291 | 29 | 0.0291 | 29 | 34.3678 | 31262 | 0.9996 | 1 | 20 | |
| | 50 | 0.0320 | 29 | 0.0320 | 29 | 31.2416 | 26053 | 0.9995 | 1 | 10 | |
| 2 | 0 | 0.0349 | 29 | 0.0349 | 29 | 28.6363 | 22047 | 0.9994 | 1 | 0 | 88 |
| | 10 | 0.0378 | 29 | 0.0378 | 29 | 26.4316 | 18898 | 0.9993 | 1 | 50 | |
| | 20 | 0.0407 | 29 | 0.0407 | 30 | 24.5418 | 16380 | 0.9992 | 2 | 40 | |
| | 30 | 0.0436 | 29 | 0.0437 | 29 | 22.9038 | 14334 | 0.9990 | 1 | 30 | |
| | 40 | 0.0465 | 29 | 0.0466 | 29 | 21.4704 | 12648 | 0.9989 | 1 | 20 | |
| | 50 | 0.0494 | 29 | 0.0495 | 29 | 20.2056 | 11245 | 0.9988 | 2 | 10 | |
| 3 | 0 | 0.0523 | 29 | 0.0524 | 29 | 19.0811 | 10061 | 0.9986 | 1 | 0 | 87 |
| | 10 | 0.0552 | 29 | 0.0553 | 29 | 18.0750 | 9057 | 0.9985 | 2 | 50 | |
| | 20 | 0.0581 | 29 | 0.0582 | 30 | 17.1693 | 8194 | 0.9983 | 2 | 40 | |
| | 30 | 0.0610 | 30 | 0.0612 | 29 | 16.3499 | 7451 | 0.9981 | 1 | 30 | |
| | 40 | 0.0640 | 29 | 0.0641 | 29 | 15.6048 | 6804 | 0.9980 | 2 | 20 | |
| | 50 | 0.0669 | 29 | 0.0670 | 29 | 14.9244 | 6237 | 0.9978 | 2 | 10 | |
| 4 | 0 | 0.0698 | 29 | 0.0699 | 30 | 14.3007 | 5740 | 0.9976 | 2 | 0 | 86 |
| | 10 | 0.0727 | 29 | 0.0729 | 29 | 13.7267 | 5298 | 0.9974 | 3 | 50 | |
| | 20 | 0.0756 | 29 | 0.0758 | 29 | 13.1969 | 4907 | 0.9971 | 2 | 40 | |
| | 30 | 0.0785 | 29 | 0.0787 | 29 | 12.7062 | 4557 | 0.9969 | 2 | 30 | |
| | 40 | 0.0814 | 29 | 0.0816 | 30 | 12.2505 | 4243 | 0.9967 | 3 | 20 | |
| | 50 | 0.0843 | 29 | 0.0846 | 29 | 11.8262 | 3961 | 0.9964 | 2 | 10 | |
| 5 | 0 | 0.0872 | 29 | 0.0875 | 29 | 11.4301 | 3707 | 0.9962 | 3 | 0 | 85 |
| | 10 | 0.0901 | 28 | 0.0904 | 30 | 11.0594 | 3475 | 0.9959 | 2 | 50 | |
| | 20 | 0.0929 | 29 | 0.0934 | 29 | 10.7119 | 3265 | 0.9957 | 3 | 40 | |
| | 30 | 0.0958 | 29 | 0.0963 | 29 | 10.3854 | 3074 | 0.9954 | 3 | 30 | |
| | 40 | 0.0987 | 29 | 0.0992 | 30 | 10.0780 | 2898 | 0.9951 | 3 | 20 | |
| | 50 | 0.1016 | 29 | 0.1022 | 29 | 9.7882 | 2738 | 0.9948 | 3 | 10 | |
| 6 | 0 | 0.1045 | 29 | 0.1051 | 29 | 9.5144 | 2591 | 0.9945 | 3 | 0 | 84 |
| | 10 | 0.1074 | 29 | 0.1080 | 30 | 9.2553 | 2455 | 0.9942 | 3 | 50 | |
| | 20 | 0.1103 | 29 | 0.1110 | 29 | 9.0098 | 2329 | 0.9939 | 3 | 40 | |
| | 30 | 0.1132 | 29 | 0.1139 | 30 | 8.7769 | 2214 | 0.9936 | 4 | 30 | |
| | 40 | 0.1161 | 29 | 0.1169 | 29 | 8.5555 | 2105 | 0.9932 | 3 | 20 | |
| | 50 | 0.1190 | 29 | 0.1198 | 30 | 8.3450 | 2007 | 0.9929 | 4 | 10 | |
| 7 | 0 | 0.1219 | 29 | 0.1228 | 29 | 8.1443 | 1913 | 0.9925 | 3 | 0 | 83 |
| | 10 | 0.1248 | 28 | 0.1257 | 30 | 7.9530 | 1826 | 0.9922 | 4 | 50 | |
| | 20 | 0.1276 | 29 | 0.1287 | 30 | 7.7704 | 1746 | 0.9918 | 4 | 40 | |
| | 30 | 0.1305 | 29 | 0.1317 | 29 | 7.5958 | 1671 | 0.9914 | 3 | 30 | |
| | 40 | 0.1334 | 29 | 0.1346 | 30 | 7.4287 | 1600 | 0.9911 | 4 | 20 | |
| | 50 | 0.1363 | 29 | 0.1376 | 29 | 7.2687 | 1533 | 0.9907 | 4 | 10 | |
| 8 | 0 | 0.1392 | 29 | 0.1405 | 30 | 7.1154 | 1472 | 0.9903 | 4 | 0 | 82 |
| | 10 | 0.1421 | 28 | 0.1435 | 30 | 6.9682 | 1413 | 0.9899 | 5 | 50 | |
| | 20 | 0.1449 | 29 | 0.1465 | 30 | 6.8269 | 1357 | 0.9894 | 4 | 40 | |
| | 30 | 0.1478 | 29 | 0.1495 | 29 | 6.6912 | 1306 | 0.9890 | 4 | 30 | |
| | 40 | 0.1507 | 29 | 0.1524 | 30 | 6.5606 | 1258 | 0.9886 | 5 | 20 | |
| | 50 | 0.1536 | 28 | 0.1554 | 30 | 6.4348 | 1210 | 0.9881 | 4 | 10 | |
| 9 | 0 | 0.1564 | | 0.1584 | | 6.3138 | | 0.9877 | | 0 | 81 |
| | | Cos. | d. | Cot. | d. | Tan. | d. | Sin. | d. | ′ | ° |

P. P.

| | 30 | 29 | 28 | 5 | 4 |
|---|---|---|---|---|---|
| 1 | 3.0 | 2.9 | 2.8 | 0.5 | 0.4 |
| 2 | 6.0 | 5.8 | 5.6 | 1.0 | 0.8 |
| 3 | 9.0 | 8.7 | 8.4 | 1.5 | 1.2 |
| 4 | 12.0 | 11.6 | 11.2 | 2.0 | 1.6 |
| 5 | 15.0 | 14.5 | 14.0 | 2.5 | 2.0 |
| 6 | 18.0 | 17.4 | 16.8 | 3.0 | 2.4 |
| 7 | 21.0 | 20.3 | 19.6 | 3.5 | 2.8 |
| 8 | 24.0 | 23.2 | 22.4 | 4.0 | 3.2 |
| 9 | 27.0 | 26.1 | 25.2 | 4.5 | 3.6 |

| ° | ′ | Sin. | d. | Tan. | d. | Cot. | d. | Cos. | d. | ′ | ° |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 9 | 0 | 0.1564 | 29 | 0.1584 | 30 | 6.3138 | 1168 | 0.9877 | 5 | 0 | 81 |
| | 10 | 0.1593 | 29 | 0.1614 | 30 | 6.1970 | 1126 | 0.9872 | 4 | 50 | |
| | 20 | 0.1622 | 28 | 0.1644 | 29 | 6.0844 | 1086 | 0.9868 | 5 | 40 | |
| | 30 | 0.1650 | 29 | 0.1673 | 30 | 5.9758 | 1050 | 0.9863 | 5 | 30 | |
| | 40 | 0.1679 | 29 | 0.1703 | 30 | 5.8708 | 1014 | 0.9858 | 5 | 20 | |
| | 50 | 0.1708 | 28 | 0.1733 | 30 | 5.7694 | 981 | 0.9853 | 5 | 10 | |
| 10 | 0 | 0.1736 | 29 | 0.1763 | 30 | 5.6713 | 949 | 0.9848 | 5 | 0 | 80 |
| | 10 | 0.1765 | 29 | 0.1793 | 30 | 5.5764 | 919 | 0.9843 | 5 | 50 | |
| | 20 | 0.1794 | 28 | 0.1823 | 30 | 5.4845 | 890 | 0.9838 | 5 | 40 | |
| | 30 | 0.1822 | 29 | 0.1853 | 30 | 5.3955 | 862 | 0.9833 | 6 | 30 | |
| | 40 | 0.1851 | 29 | 0.1883 | 31 | 5.3093 | 836 | 0.9827 | 5 | 20 | |
| | 50 | 0.1880 | 28 | 0.1914 | 30 | 5.2257 | 811 | 0.9822 | 6 | 10 | |
| 11 | 0 | 0.1908 | 29 | 0.1944 | 30 | 5.1446 | 788 | 0.9816 | 5 | 0 | 79 |
| | 10 | 0.1937 | 28 | 0.1974 | 30 | 5.0658 | 764 | 0.9811 | 6 | 50 | |
| | 20 | 0.1965 | 29 | 0.2004 | 31 | 4.9894 | 742 | 0.9805 | 6 | 40 | |
| | 30 | 0.1994 | 28 | 0.2035 | 30 | 4.9152 | 722 | 0.9799 | 6 | 30 | |
| | 40 | 0.2022 | 29 | 0.2065 | 30 | 4.8430 | 701 | 0.9793 | 6 | 20 | |
| | 50 | 0.2051 | 28 | 0.2095 | 31 | 4.7729 | 683 | 0.9787 | 6 | 10 | |
| 12 | 0 | 0.2079 | 29 | 0.2126 | 30 | 4.7046 | 664 | 0.9781 | 6 | 0 | 78 |
| | 10 | 0.2108 | 28 | 0.2156 | 30 | 4.6382 | 646 | 0.9775 | 6 | 50 | |
| | 20 | 0.2136 | 28 | 0.2186 | 31 | 4.5736 | 629 | 0.9769 | 6 | 40 | |
| | 30 | 0.2164 | 29 | 0.2217 | 30 | 4.5107 | 613 | 0.9763 | 6 | 30 | |
| | 40 | 0.2193 | 28 | 0.2247 | 31 | 4.4494 | 597 | 0.9757 | 7 | 20 | |
| | 50 | 0.2221 | 29 | 0.2278 | 31 | 4.3897 | 582 | 0.9750 | 6 | 10 | |
| 13 | 0 | 0.2250 | 28 | 0.2309 | 30 | 4.3315 | 568 | 0.9744 | 7 | 0 | 77 |
| | 10 | 0.2278 | 28 | 0.2339 | 31 | 4.2747 | 554 | 0.9737 | 7 | 50 | |
| | 20 | 0.2306 | 28 | 0.2370 | 31 | 4.2193 | 540 | 0.9730 | 6 | 40 | |
| | 30 | 0.2334 | 29 | 0.2401 | 31 | 4.1653 | 527 | 0.9724 | 7 | 30 | |
| | 40 | 0.2363 | 28 | 0.2432 | 30 | 4.1126 | 515 | 0.9717 | 7 | 20 | |
| | 50 | 0.2391 | 28 | 0.2462 | 31 | 4.0611 | 503 | 0.9710 | 7 | 10 | |
| 14 | 0 | 0.2419 | 28 | 0.2493 | 31 | 4.0108 | 491 | 0.9703 | 7 | 0 | 76 |
| | 10 | 0.2447 | 29 | 0.2524 | 31 | 3.9617 | 481 | 0.9696 | 7 | 50 | |
| | 20 | 0.2476 | 28 | 0.2555 | 31 | 3.9136 | 469 | 0.9689 | 8 | 40 | |
| | 30 | 0.2504 | 28 | 0.2586 | 31 | 3.8667 | 459 | 0.9681 | 7 | 30 | |
| | 40 | 0.2532 | 28 | 0.2617 | 31 | 3.8208 | 448 | 0.9674 | 7 | 20 | |
| | 50 | 0.2560 | 28 | 0.2648 | 31 | 3.7760 | 439 | 0.9667 | 8 | 10 | |
| 15 | 0 | 0.2588 | 28 | 0.2679 | 32 | 3.7321 | 430 | 0.9659 | 7 | 0 | 75 |
| | 10 | 0.2616 | 28 | 0.2711 | 31 | 3.6891 | 421 | 0.9652 | 8 | 50 | |
| | 20 | 0.2644 | 28 | 0.2742 | 31 | 3.6470 | 411 | 0.9644 | 8 | 40 | |
| | 30 | 0.2672 | 28 | 0.2773 | 32 | 3.6059 | 403 | 0.9636 | 8 | 30 | |
| | 40 | 0.2700 | 28 | 0.2805 | 31 | 3.5656 | 395 | 0.9628 | 7 | 20 | |
| | 50 | 0.2728 | 28 | 0.2836 | 31 | 3.5261 | 387 | 0.9621 | 8 | 10 | |
| 16 | 0 | 0.2756 | 28 | 0.2867 | 32 | 3.4874 | 379 | 0.9613 | 8 | 0 | 74 |
| | 10 | 0.2784 | 28 | 0.2899 | 32 | 3.4495 | 371 | 0.9605 | 9 | 50 | |
| | 20 | 0.2812 | 28 | 0.2931 | 31 | 3.4124 | 365 | 0.9596 | 8 | 40 | |
| | 30 | 0.2840 | 28 | 0.2962 | 32 | 3.3759 | 357 | 0.9588 | 8 | 30 | |
| | 40 | 0.2868 | 28 | 0.2994 | 32 | 3.3402 | 350 | 0.9580 | 8 | 20 | |
| | 50 | 0.2896 | 28 | 0.3026 | 31 | 3.3052 | 343 | 0.9572 | 9 | 10 | |
| 17 | 0 | 0.2924 | 28 | 0.3057 | 32 | 3.2709 | 338 | 0.9563 | 8 | 0 | 73 |
| | 10 | 0.2952 | 27 | 0.3089 | 32 | 3.2371 | 330 | 0.9555 | 9 | 50 | |
| | 20 | 0.2979 | 28 | 0.3121 | 32 | 3.2041 | 325 | 0.9546 | 9 | 40 | |
| | 30 | 0.3007 | 28 | 0.3153 | 32 | 3.1716 | 319 | 0.9537 | 9 | 30 | |
| | 40 | 0.3035 | 27 | 0.3185 | 32 | 3.1397 | 313 | 0.9528 | 8 | 20 | |
| | 50 | 0.3062 | 28 | 0.3217 | 32 | 3.1084 | 307 | 0.9520 | 9 | 10 | |
| 18 | 0 | 0.3090 | | 0.3249 | | 3.0777 | | 0.9511 | | 0 | 72 |
| | | Cos. | d. | Cot. | d. | Tan. | d. | Sin. | d. | ′ | ° |

P. P.

| | 32 | 31 | 30 |
|---|---|---|---|
| 1 | 3.2 | 3.1 | 3.0 |
| 2 | 6.4 | 6.2 | 6.0 |
| 3 | 9.6 | 9.3 | 9.0 |
| 4 | 12 8 | 12.4 | 12.0 |
| 5 | 16.0 | 15.5 | 15.0 |
| 6 | 19.2 | 18.6 | 18.0 |
| 7 | 22.4 | 21.7 | 21.0 |
| 8 | 25.6 | 24.8 | 24.0 |
| 9 | 28.8 | 27.9 | 27.0 |

| | 29 | 28 | 27 |
|---|---|---|---|
| 1 | 2.9 | 2.8 | 2.7 |
| 2 | 5.8 | 5.6 | 5.4 |
| 3 | 8.7 | 8.4 | 8.1 |
| 4 | 11.6 | 11.2 | 10.8 |
| 5 | 14.5 | 14.0 | 13.5 |
| 6 | 17.4 | 16.8 | 16.2 |
| 7 | 20.3 | 19.6 | 18.9 |
| 8 | 23.2 | 22.4 | 21.6 |
| 9 | 26.1 | 25.2 | 24.3 |

| | 9 | 8 |
|---|---|---|
| 1 | 0.9 | 0.8 |
| 2 | 1.8 | 1.6 |
| 3 | 2.7 | 2.4 |
| 4 | 3.6 | 3.2 |
| 5 | 4.5 | 4.0 |
| 6 | 5.4 | 4.8 |
| 7 | 6.3 | 5.6 |
| 8 | 7.2 | 6.4 |
| 9 | 8.1 | 7.2 |

| | 7 | 6 |
|---|---|---|
| 1 | 0.7 | 0.6 |
| 2 | 1.4 | 1.2 |
| 3 | 2.1 | 1.8 |
| 4 | 2.8 | 2.4 |
| 5 | 3.5 | 3.0 |
| 6 | 4.2 | 3.6 |
| 7 | 4.9 | 4.2 |
| 8 | 5.6 | 4.8 |
| 9 | 6.3 | 5.4 |

| | 5 | 4 |
|---|---|---|
| 1 | 0.5 | 0.4 |
| 2 | 1.0 | 0.8 |
| 3 | 1.5 | 1.2 |
| 4 | 2.0 | 1.6 |
| 5 | 2.5 | 2.0 |
| 6 | 3.0 | 2.4 |
| 7 | 3.5 | 2.8 |
| 8 | 4.0 | 3.2 |
| 9 | 4.5 | 3.6 |

P. P.

| ° | ′ | Sin. | d. | Tan. | d. | Cot. | d. | Cos. | d. | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 18 | 0 | 0.3090 | | 0.3249 | | 3.0777 | | 0.9511 | | 0 | 72 |
| | 10 | 0.3118 | 28 | 0.3281 | 32 | 3.0475 | 302 | 0.9502 | 9 | 50 | |
| | 20 | 0.3145 | 27 | 0.3314 | 33 | 3.0178 | 297 | 0.9492 | 10 | 40 | |
| | 30 | 0.3173 | 28 | 0.3346 | 32 | 2.9887 | 291 | 0.9483 | 9 | 30 | |
| | 40 | 0.3201 | 28 | 0.3378 | 32 | 2.9600 | 287 | 0.9474 | 9 | 20 | |
| | 50 | 0.3228 | 27 | 0.3411 | 33 | 2.9319 | 281 | 0.9465 | 9 | 10 | |
| 19 | 0 | 0.3256 | 28 | 0.3443 | 32 | 2.9042 | 277 | 0.9455 | 10 | 0 | 71 |
| | 10 | 0.3283 | 27 | 0.3476 | 33 | 2.8770 | 272 | 0.9446 | 9 | 50 | |
| | 20 | 0.3311 | 28 | 0.3508 | 32 | 2.8502 | 268 | 0.9436 | 10 | 40 | |
| | 30 | 0.3338 | 27 | 0.3541 | 33 | 2.8239 | 263 | 0.9426 | 10 | 30 | |
| | 40 | 0.3365 | 27 | 0.3574 | 33 | 2.7980 | 259 | 0.9417 | 9 | 20 | |
| | 50 | 0.3393 | 28 | 0.3607 | 33 | 2.7725 | 255 | 0.9407 | 10 | 10 | |
| 20 | 0 | 0.3420 | 27 | 0.3640 | 33 | 2.7475 | 250 | 0.9397 | 10 | 0 | 70 |
| | 10 | 0.3448 | 28 | 0.3673 | 33 | 2.7228 | 247 | 0.9387 | 10 | 50 | |
| | 20 | 0.3475 | 27 | 0.3706 | 33 | 2.6985 | 243 | 0.9377 | 10 | 40 | |
| | 30 | 0.3502 | 27 | 0.3739 | 33 | 2.6746 | 239 | 0.9367 | 10 | 30 | |
| | 40 | 0.3529 | 27 | 0.3772 | 33 | 2.6511 | 235 | 0.9356 | 11 | 20 | |
| | 50 | 0.3557 | 28 | 0.3805 | 33 | 2.6279 | 232 | 0.9346 | 10 | 10 | |
| 21 | 0 | 0.3584 | 27 | 0.3839 | 34 | 2.6051 | 228 | 0.9336 | 10 | 0 | 69 |
| | 10 | 0.3611 | 27 | 0.3872 | 33 | 2.5826 | 225 | 0.9325 | 11 | 50 | |
| | 20 | 0.3638 | 27 | 0.3906 | 34 | 2.5605 | 221 | 0.9315 | 10 | 40 | |
| | 30 | 0.3665 | 27 | 0.3939 | 33 | 2.5386 | 219 | 0.9304 | 11 | 30 | |
| | 40 | 0.3692 | 27 | 0.3973 | 34 | 2.5172 | 214 | 0.9293 | 11 | 20 | |
| | 50 | 0.3719 | 27 | 0.4006 | 33 | 2.4960 | 212 | 0.9283 | 10 | 10 | |
| 22 | 0 | 0.3746 | 27 | 0.4040 | 34 | 2.4751 | 209 | 0.9272 | 11 | 0 | 68 |
| | 10 | 0.3773 | 27 | 0.4074 | 34 | 2.4545 | 206 | 0.9261 | 11 | 50 | |
| | 20 | 0.3800 | 27 | 0.4108 | 34 | 2.4342 | 203 | 0.9250 | 11 | 40 | |
| | 30 | 0.3827 | 27 | 0.4142 | 34 | 2.4142 | 200 | 0.9239 | 11 | 30 | |
| | 40 | 0.3854 | 27 | 0.4176 | 34 | 2.3945 | 197 | 0.9228 | 11 | 20 | |
| | 50 | 0.3881 | 27 | 0.4210 | 34 | 2.3750 | 195 | 0.9216 | 12 | 10 | |
| 23 | 0 | 0.3907 | 26 | 0.4245 | 35 | 2.3559 | 191 | 0.9205 | 11 | 0 | 67 |
| | 10 | 0.3934 | 27 | 0.4279 | 34 | 2.3369 | 190 | 0.9194 | 11 | 50 | |
| | 20 | 0.3961 | 27 | 0.4314 | 35 | 2.3183 | 186 | 0.9182 | 12 | 40 | |
| | 30 | 0.3987 | 26 | 0.4348 | 34 | 2.2998 | 185 | 0.9171 | 11 | 30 | |
| | 40 | 0.4014 | 27 | 0.4383 | 35 | 2.2817 | 181 | 0.9159 | 12 | 20 | |
| | 50 | 0.4041 | 27 | 0.4417 | 34 | 2.2637 | 180 | 0.9147 | 12 | 10 | |
| 24 | 0 | 0.4067 | 26 | 0.4452 | 35 | 2.2460 | 177 | 0.9135 | 12 | 0 | 66 |
| | 10 | 0.4094 | 27 | 0.4487 | 35 | 2.2286 | 174 | 0.9124 | 11 | 50 | |
| | 20 | 0.4120 | 26 | 0.4522 | 35 | 2.2113 | 173 | 0.9112 | 12 | 40 | |
| | 30 | 0.4147 | 27 | 0.4557 | 35 | 2.1943 | 170 | 0.9100 | 12 | 30 | |
| | 40 | 0.4173 | 26 | 0.4592 | 35 | 2.1775 | 168 | 0.9088 | 12 | 20 | |
| | 50 | 0.4200 | 27 | 0.4628 | 36 | 2.1609 | 166 | 0.9075 | 13 | 10 | |
| 25 | 0 | 0.4226 | 26 | 0.4663 | 35 | 2.1445 | 164 | 0.9063 | 12 | 0 | 65 |
| | 10 | 0.4253 | 27 | 0.4699 | 36 | 2.1283 | 162 | 0.9051 | 12 | 50 | |
| | 20 | 0.4279 | 26 | 0.4734 | 35 | 2.1123 | 160 | 0.9038 | 13 | 40 | |
| | 30 | 0.4305 | 26 | 0.4770 | 36 | 2.0965 | 158 | 0.9026 | 12 | 30 | |
| | 40 | 0.4331 | 26 | 0.4806 | 36 | 2.0809 | 156 | 0.9013 | 13 | 20 | |
| | 50 | 0.4358 | 27 | 0.4841 | 35 | 2.0655 | 154 | 0.9001 | 12 | 10 | |
| 26 | 0 | 0.4384 | 26 | 0.4877 | 36 | 2.0503 | 152 | 0.8988 | 13 | 0 | 64 |
| | 10 | 0.4410 | 26 | 0.4913 | 36 | 2.0353 | 150 | 0.8975 | 13 | 50 | |
| | 20 | 0.4436 | 26 | 0.49 0 | 37 | 2.0204 | 149 | 0.8962 | 13 | 40 | |
| | 30 | 0.4462 | 26 | 0.4986 | 36 | 2.0057 | 147 | 0.8949 | 13 | 30 | |
| | 40 | 0.4488 | 26 | 0.5022 | 36 | 1.9912 | 145 | 0.8936 | 13 | 20 | |
| | 50 | 0.4514 | 26 | 0.5059 | 37 | 1.9768 | 144 | 0.8923 | 13 | 10 | |
| 27 | 0 | 0.4540 | 26 | 0.5095 | 36 | 1.9626 | 142 | 0.8910 | 13 | 0 | 63 |
| | | Cos. | d. | Cot. | d. | Tan. | d. | Sin. | d. | ′ | ° |

P. P.

| | 37 | 36 | 35 |
|---|---|---|---|
| 1 | 3.7 | 3.6 | 3.5 |
| 2 | 7.4 | 7.2 | 7.0 |
| 3 | 11.1 | 10.8 | 10.5 |
| 4 | 14.8 | 14.4 | 14.0 |
| 5 | 18.5 | 18.0 | 17.5 |
| 6 | 22.2 | 21.6 | 21.0 |
| 7 | 25.9 | 25.2 | 24.5 |
| 8 | 29.6 | 28.8 | 28.0 |
| 9 | 33.3 | 32.4 | 31.5 |

| | 34 | 33 | 32 |
|---|---|---|---|
| 1 | 3.4 | 3.3 | 3.2 |
| 2 | 6.8 | 6.6 | 6.4 |
| 3 | 10.2 | 9.9 | 9.6 |
| 4 | 13.6 | 13.2 | 12.8 |
| 5 | 17.0 | 16.5 | 16.0 |
| 6 | 20.4 | 19.8 | 19.2 |
| 7 | 23.8 | 23.1 | 22.4 |
| 8 | 27.2 | 26.4 | 25.6 |
| 9 | 30.6 | 29.7 | 28.8 |

| | 28 | 27 | 26 |
|---|---|---|---|
| 1 | 2.8 | 2.7 | 2.6 |
| 2 | 5.6 | 5.4 | 5.2 |
| 3 | 8.4 | 8.1 | 7.8 |
| 4 | 11.2 | 10.8 | 10.4 |
| 5 | 14.0 | 13.5 | 13.0 |
| 6 | 16.8 | 16.2 | 15.6 |
| 7 | 19.6 | 18.9 | 18.2 |
| 8 | 22.4 | 21.6 | 20.8 |
| 9 | 25.2 | 24.3 | 23.4 |

| | 13 | 12 |
|---|---|---|
| 1 | 1.3 | 1.2 |
| 2 | 2.6 | 2.4 |
| 3 | 3.9 | 3.6 |
| 4 | 5.2 | 4.8 |
| 5 | 6.5 | 6.0 |
| 6 | 7.8 | 7.2 |
| 7 | 9.1 | 8.4 |
| 8 | 10.4 | 9.6 |
| 9 | 11.7 | 10.8 |

| | 11 | 10 | 9 |
|---|---|---|---|
| 1 | 1.1 | 1.0 | 0.9 |
| 2 | 2.2 | 2.0 | 1.8 |
| 3 | 3.3 | 3.0 | 2.7 |
| 4 | 4.4 | 4.0 | 3.6 |
| 5 | 5.5 | 5.0 | 4.5 |
| 6 | 6.6 | 6.0 | 5.4 |
| 7 | 7.7 | 7.0 | 6.3 |
| 8 | 8.8 | 8.0 | 7.2 |
| 9 | 9.9 | 9.0 | 8.1 |

P. P.

| ° | ′ | Sin. | d. | Tan. | d. | Cot. | d. | Cos. | d. | ′ | ° |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 27 | 0 | 0.4540 | | 0.5095 | | 1.9626 | | 0.8910 | | 0 | 63 |
| | | | 26 | | 37 | | 140 | | 13 | | |
| | 10 | 0.4566 | | 0.5132 | | 1.9486 | | 0.8897 | | 50 | |
| | | | 26 | | 37 | | 139 | | 13 | | |
| | 20 | 0.4592 | | 0.5169 | | 1.9347 | | 0.8884 | | 40 | |
| | | | 25 | | 37 | | 137 | | 14 | | |
| | 30 | 0.4617 | | 0.5206 | | 1.9210 | | 0.8870 | | 30 | |
| | | | 26 | | 37 | | 136 | | 13 | | |
| | 40 | 0.4643 | | 0.5243 | | 1.9074 | | 0.8857 | | 20 | |
| | | | 26 | | 37 | | 134 | | 14 | | |
| | 50 | 0.4669 | | 0.5280 | | 1.8940 | | 0.8843 | | 10 | |
| | | | 26 | | 37 | | 133 | | 14 | | |
| 28 | 0 | 0.4695 | | 0.5317 | | 1.8807 | | 0.8829 | | 0 | 62 |
| | | | 25 | | 37 | | 131 | | 13 | | |
| | 10 | 0.4720 | | 0.5354 | | 1.8676 | | 0.8816 | | 50 | |
| | | | 26 | | 38 | | 130 | | 14 | | |
| | 20 | 0.4746 | | 0.5392 | | 1.8546 | | 0.8802 | | 40 | |
| | | | 26 | | 38 | | 128 | | 14 | | |
| | 30 | 0.4772 | | 0.5430 | | 1.8418 | | 0.8788 | | 30 | |
| | | | 25 | | 37 | | 127 | | 14 | | |
| | 40 | 0.4797 | | 0.5467 | | 1.8291 | | 0.8774 | | 20 | |
| | | | 26 | | 38 | | 126 | | 14 | | |
| | 50 | 0.4823 | | 0.5505 | | 1.8165 | | 0.8760 | | 10 | |
| | | | 25 | | 38 | | 125 | | 14 | | |
| 29 | 0 | 0.4848 | | 0.5543 | | 1.8040 | | 0.8746 | | 0 | 61 |
| | | | 26 | | 38 | | 123 | | 14 | | |
| | 10 | 0.4874 | | 0.5581 | | 1.7917 | | 0.8732 | | 50 | |
| | | | 25 | | 38 | | 121 | | 14 | | |
| | 20 | 0.4899 | | 0.5619 | | 1.7796 | | 0.8718 | | 40 | |
| | | | 25 | | 39 | | 121 | | 14 | | |
| | 30 | 0.4924 | | 0.5658 | | 1.7675 | | 0.8704 | | 30 | |
| | | | 26 | | 38 | | 119 | | 15 | | |
| | 40 | 0.4950 | | 0.5696 | | 1.7556 | | 0.8689 | | 20 | |
| | | | 25 | | 39 | | 119 | | 14 | | |
| | 50 | 0.4975 | | 0.5735 | | 1.7437 | | 0.8675 | | 10 | |
| | | | 25 | | 39 | | 116 | | 15 | | |
| 30 | 0 | 0.5000 | | 0.5774 | | 1.7321 | | 0.8660 | | 0 | 60 |
| | | | 25 | | 38 | | 116 | | 14 | | |
| | 10 | 0.5025 | | 0.5812 | | 1.7205 | | 0.8646 | | 50 | |
| | | | 25 | | 39 | | 115 | | 15 | | |
| | 20 | 0.5050 | | 0.5851 | | 1.7090 | | 0.8631 | | 40 | |
| | | | 25 | | 39 | | 113 | | 15 | | |
| | 30 | 0.5075 | | 0.5890 | | 1.6977 | | 0.8616 | | 30 | |
| | | | 25 | | 40 | | 113 | | 15 | | |
| | 40 | 0.5100 | | 0.5930 | | 1.6864 | | 0.8601 | | 20 | |
| | | | 25 | | 39 | | 111 | | 14 | | |
| | 50 | 0.5125 | | 0.5969 | | 1.6753 | | 0.8587 | | 10 | |
| | | | 25 | | 40 | | 110 | | 15 | | |
| 31 | 0 | 0.5150 | | 0.6009 | | 1.6643 | | 0.8572 | | 0 | 59 |
| | | | 25 | | 39 | | 109 | | 15 | | |
| | 10 | 0.5175 | | 0.6048 | | 1.6534 | | 0.8557 | | 50 | |
| | | | 25 | | 40 | | 108 | | 15 | | |
| | 20 | 0.5200 | | 0.6088 | | 1.6426 | | 0.8542 | | 40 | |
| | | | 25 | | 40 | | 107 | | 16 | | |
| | 30 | 0.5225 | | 0.6128 | | 1.6319 | | 0.8526 | | 30 | |
| | | | 25 | | 40 | | 107 | | 15 | | |
| | 40 | 0.5250 | | 0.6168 | | 1.6212 | | 0.8511 | | 20 | |
| | | | 25 | | 40 | | 105 | | 15 | | |
| | 50 | 0.5275 | | 0.6208 | | 1.6107 | | 0.8496 | | 10 | |
| | | | 24 | | 41 | | 104 | | 16 | | |
| 32 | 0 | 0.5299 | | 0.6249 | | 1.6003 | | 0.8480 | | 0 | 58 |
| | | | 25 | | 40 | | 103 | | 15 | | |
| | 10 | 0.5324 | | 0.6289 | | 1.5900 | | 0.8465 | | 50 | |
| | | | 24 | | 41 | | 102 | | 15 | | |
| | 20 | 0.5348 | | 0.6330 | | 1.5798 | | 0.8450 | | 40 | |
| | | | 25 | | 41 | | 101 | | 16 | | |
| | 30 | 0.5373 | | 0.6371 | | 1.5697 | | 0.8434 | | 30 | |
| | | | 25 | | 41 | | 100 | | 16 | | |
| | 40 | 0.5398 | | 0.6412 | | 1.5597 | | 0.8418 | | 20 | |
| | | | 24 | | 41 | | 100 | | 15 | | |
| | 50 | 0.5422 | | 0.6453 | | 1.5497 | | 0.8403 | | 10 | |
| | | | 24 | | 41 | | 98 | | 16 | | |
| 33 | 0 | 0.5446 | | 0.6494 | | 1.5399 | | 0.8387 | | 0 | 57 |
| | | | 25 | | 42 | | 98 | | 16 | | |
| | 10 | 0.5471 | | 0.6536 | | 1.5301 | | 0.8371 | | 50 | |
| | | | 24 | | 41 | | 97 | | 16 | | |
| | 20 | 0.5495 | | 0.6577 | | 1.5204 | | 0.8355 | | 40 | |
| | | | 24 | | 42 | | 96 | | 16 | | |
| | 30 | 0.5519 | | 0.6619 | | 1.5108 | | 0.8339 | | 30 | |
| | | | 25 | | 42 | | 95 | | 16 | | |
| | 40 | 0.5544 | | 0.6661 | | 1.5013 | | 0.8323 | | 20 | |
| | | | 24 | | 42 | | 94 | | 16 | | |
| | 50 | 0.5568 | | 0.6703 | | 1.4919 | | 0.8307 | | 10 | |
| | | | 24 | | 42 | | 93 | | 17 | | |
| 34 | 0 | 0.5592 | | 0.6745 | | 1.4826 | | 0.8290 | | 0 | 56 |
| | | | 24 | | 42 | | 93 | | 16 | | |
| | 10 | 0.5616 | | 0.6787 | | 1.4733 | | 0.8274 | | 50 | |
| | | | 24 | | 43 | | 92 | | 16 | | |
| | 20 | 0.5640 | | 0.6830 | | 1.4641 | | 0.8258 | | 40 | |
| | | | 24 | | 43 | | 91 | | 17 | | |
| | 30 | 0.5664 | | 0.6873 | | 1.4550 | | 0.8241 | | 30 | |
| | | | 24 | | 43 | | 90 | | 16 | | |
| | 40 | 0.5688 | | 0.6916 | | 1.4460 | | 0.8225 | | 20 | |
| | | | 24 | | 43 | | 90 | | 17 | | |
| | 50 | 0.5712 | | 0.6959 | | 1.4370 | | 0.8208 | | 10 | |
| | | | 24 | | 43 | | 89 | | 16 | | |
| 35 | 0 | 0.5736 | | 0.7002 | | 1.4281 | | 0.8192 | | 0 | 55 |
| | | | 24 | | 44 | | 88 | | 17 | | |
| | 10 | 0.5760 | | 0.7046 | | 1.4193 | | 0.8175 | | 50 | |
| | | | 23 | | 43 | | 87 | | 17 | | |
| | 20 | 0.5783 | | 0.7089 | | 1.4106 | | 0.8158 | | 40 | |
| | | | 24 | | 44 | | 87 | | 17 | | |
| | 30 | 0.5807 | | 0.7133 | | 1.4019 | | 0.8141 | | 30 | |
| | | | 24 | | 44 | | 85 | | 17 | | |
| | 40 | 0.5831 | | 0.7177 | | 1.3934 | | 0.8124 | | 20 | |
| | | | 23 | | 44 | | 86 | | 17 | | |
| | 50 | 0.5854 | | 0.7221 | | 1.3848 | | 0.8107 | | 10 | |
| | | | 24 | | 44 | | 84 | | 17 | | |
| 36 | 0 | 0.5878 | | 0.7265 | | 1.3764 | | 0.8090 | | 0 | 54 |
| | | Cos. | d. | Cot. | d. | Tan. | d. | Sin. | d. | ′ | ° |

P. P.

| | 44 | 43 | 42 |
|---|---|---|---|
| 1 | 4.4 | 4.3 | 4.2 |
| 2 | 8.8 | 8.6 | 8.4 |
| 3 | 13.2 | 12.9 | 12.6 |
| 4 | 17.6 | 17.2 | 16.8 |
| 5 | 22.0 | 21.5 | 21.0 |
| 6 | 26.4 | 25.8 | 25.2 |
| 7 | 30.8 | 30.1 | 29.4 |
| 8 | 35.2 | 34.4 | 33.6 |
| 9 | 39.6 | 38.7 | 37.8 |

| | 41 | 40 | 39 |
|---|---|---|---|
| 1 | 4.1 | 4.0 | 3.9 |
| 2 | 8.2 | 8.0 | 7.8 |
| 3 | 12.3 | 12.0 | 11.7 |
| 4 | 16.4 | 16.0 | 15.6 |
| 5 | 20.5 | 20.0 | 19.5 |
| 6 | 24.6 | 24.0 | 23.4 |
| 7 | 28.7 | 28.0 | 27.3 |
| 8 | 32.8 | 32.0 | 31.2 |
| 9 | 36.9 | 36.0 | 35.1 |

| | 38 | 37 |
|---|---|---|
| 1 | 3.8 | 3.7 |
| 2 | 7.6 | 7.4 |
| 3 | 11.4 | 11.1 |
| 4 | 15.2 | 14.8 |
| 5 | 19.0 | 18.5 |
| 6 | 22.8 | 22.2 |
| 7 | 26.6 | 25.9 |
| 8 | 30.4 | 29.6 |
| 9 | 34.2 | 33.3 |

| | 26 | 25 | 24 |
|---|---|---|---|
| 1 | 2.6 | 2.5 | 2.4 |
| 2 | 5.2 | 5.0 | 4.8 |
| 3 | 7.8 | 7.5 | 7.2 |
| 4 | 10.4 | 10.0 | 9.6 |
| 4 | 13.0 | 12.5 | 12.0 |
| 6 | 15.6 | 15.0 | 14.4 |
| 7 | 18.2 | 17.5 | 16.8 |
| 8 | 20.8 | 20.0 | 19.2 |
| 9 | 23.4 | 22.5 | 21.6 |

| | 23 | 17 | 16 |
|---|---|---|---|
| 1 | 2.3 | 1.7 | 1.6 |
| 2 | 4.6 | 3.4 | 3.2 |
| 3 | 6.9 | 5.1 | 4.8 |
| 4 | 9.2 | 6.8 | 6.4 |
| 5 | 11.5 | 8.5 | 8.0 |
| 6 | 13.8 | 10.2 | 9.6 |
| 7 | 16.1 | 11.9 | 11.2 |
| 8 | 18.4 | 13.6 | 12.8 |
| 9 | 20.7 | 15.3 | 14.4 |

| | 15 | 14 | 13 |
|---|---|---|---|
| 1 | 1.5 | 1.4 | 1.3 |
| 2 | 3.0 | 2.8 | 2.6 |
| 3 | 4.5 | 4.2 | 3.9 |
| 4 | 6.0 | 5.6 | 5.2 |
| 5 | 7.5 | 7.0 | 6.5 |
| 6 | 9.0 | 8.4 | 7.8 |
| 7 | 10.5 | 9.8 | 9.1 |
| 8 | 12.0 | 11.2 | 10.4 |
| 9 | 13.5 | 12.6 | 11.7 |

P. P.

| ° | ′ | Sin. | d. | Tan. | d. | Cot. | d. | Cos. | d. | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 36 | 0 | 0.5878 | 23 | 0.7265 | 45 | 1.3764 | 84 | 0.8090 | 17 | 0 | 54 |
| | 10 | 0.5901 | 24 | 0.7310 | 45 | 1.3680 | 83 | 0.8073 | 17 | 50 | |
| | 20 | 0.5925 | 23 | 0.7355 | 45 | 1.3597 | 83 | 0.8056 | 17 | 40 | |
| | 30 | 0.5948 | 24 | 0.7400 | 45 | 1.3514 | 82 | 0.8039 | 18 | 30 | |
| | 40 | 0.5972 | 23 | 0.7445 | 45 | 1.3432 | 81 | 0.8021 | 17 | 20 | |
| | 50 | 0.5995 | 23 | 0.7490 | 46 | 1.3351 | 81 | 0.8004 | 18 | 10 | |
| 37 | 0 | 0.6018 | 23 | 0.7536 | 45 | 1.3270 | 80 | 0.7986 | 17 | 0 | 53 |
| | 10 | 0.6041 | 24 | 0.7581 | 46 | 1.3190 | 79 | 0.7969 | 18 | 50 | |
| | 20 | 0.6065 | 23 | 0.7627 | 46 | 1.3111 | 79 | 0.7951 | 17 | 40 | |
| | 30 | 0.6088 | 23 | 0.7673 | 47 | 1.3032 | 78 | 0.7934 | 18 | 30 | |
| | 40 | 0.6111 | 23 | 0.7720 | 46 | 1.2954 | 78 | 0.7916 | 18 | 20 | |
| | 50 | 0.6134 | 23 | 0.7766 | 47 | 1.2876 | 77 | 0.7898 | 18 | 10 | |
| 38 | 0 | 0.6157 | 23 | 0.7813 | 47 | 1.2799 | 76 | 0.7880 | 18 | 0 | 52 |
| | 10 | 0.6180 | 22 | 0.7860 | 47 | 1.2723 | 76 | 0.7862 | 18 | 50 | |
| | 20 | 0.6202 | 23 | 0.7907 | 47 | 1.2647 | 75 | 0.7844 | 18 | 40 | |
| | 30 | 0.6225 | 23 | 0.7954 | 48 | 1.2572 | 75 | 0.7826 | 18 | 30 | |
| | 40 | 0.6248 | 23 | 0.8002 | 48 | 1.2497 | 74 | 0.7808 | 18 | 20 | |
| | 50 | 0.6271 | 22 | 0.8050 | 48 | 1.2423 | 74 | 0.7790 | 19 | 10 | |
| 39 | 0 | 0.6293 | 23 | 0.8098 | 48 | 1.2349 | 73 | 0.7771 | 18 | 0 | 51 |
| | 10 | 0.6316 | 22 | 0.8146 | 49 | 1.2276 | 73 | 0.7753 | 18 | 50 | |
| | 20 | 0.6338 | 23 | 0.8195 | 48 | 1.2203 | 72 | 0.7735 | 19 | 40 | |
| | 30 | 0.6361 | 22 | 0.8243 | 49 | 1.2131 | 72 | 0.7716 | 18 | 30 | |
| | 40 | 0.6383 | 23 | 0.8292 | 50 | 1.2059 | 71 | 0.7698 | 19 | 20 | |
| | 50 | 0.6406 | 22 | 0.8342 | 49 | 1.1988 | 70 | 0.7679 | 19 | 10 | |
| 40 | 0 | 0.6428 | 22 | 0.8391 | 50 | 1.1918 | 71 | 0.7660 | 18 | 0 | 50 |
| | 10 | 0.6450 | 22 | 0.8441 | 50 | 1.1847 | 69 | 0.7642 | 19 | 50 | |
| | 20 | 0.6472 | 22 | 0.8491 | 50 | 1.1778 | 70 | 0.7623 | 19 | 40 | |
| | 30 | 0.6494 | 23 | 0.8541 | 50 | 1.1708 | 68 | 0.7604 | 19 | 30 | |
| | 40 | 0.6517 | 22 | 0.8591 | 51 | 1.1640 | 69 | 0.7585 | 19 | 20 | |
| | 50 | 0.6539 | 22 | 0.8642 | 51 | 1.1571 | 67 | 0.7566 | 19 | 10 | |
| 41 | 0 | 0.6561 | 22 | 0.8693 | 51 | 1.1504 | 68 | 0.7547 | 19 | 0 | 49 |
| | 10 | 0.6583 | 21 | 0.8744 | 52 | 1.1436 | 67 | 0.7528 | 19 | 50 | |
| | 20 | 0.6604 | 22 | 0.8796 | 51 | 1.1369 | 66 | 0.7509 | 19 | 40 | |
| | 30 | 0.6626 | 22 | 0.8847 | 52 | 1.1303 | 66 | 0.7490 | 20 | 30 | |
| | 40 | 0.6648 | 22 | 0.8899 | 53 | 1.1237 | 66 | 0.7470 | 19 | 20 | |
| | 50 | 0.6670 | 21 | 0.8952 | 52 | 1.1171 | 65 | 0.7451 | 20 | 10 | |
| 42 | 0 | 0.6691 | 22 | 0.9004 | 53 | 1.1106 | 65 | 0.7431 | 19 | 0 | 48 |
| | 10 | 0.6713 | 21 | 0.9057 | 53 | 1.1041 | 64 | 0.7412 | 20 | 50 | |
| | 20 | 0.6734 | 22 | 0.9110 | 53 | 1.0977 | 64 | 0.7392 | 19 | 40 | |
| | 30 | 0.6756 | 21 | 0.9163 | 54 | 1.0913 | 63 | 0.7373 | 20 | 30 | |
| | 40 | 0.6777 | 22 | 0.9217 | 54 | 1.0850 | 64 | 0.7353 | 20 | 20 | |
| | 50 | 0.6799 | 21 | 0.9271 | 54 | 1.0786 | 62 | 0.7333 | 19 | 10 | |
| 43 | 0 | 0.6820 | 21 | 0.9325 | 55 | 1.0724 | 63 | 0.7314 | 20 | 0 | 47 |
| | 10 | 0.6841 | 21 | 0.9380 | 55 | 1.0661 | 62 | 0.7294 | 20 | 50 | |
| | 20 | 0.6862 | 22 | 0.9435 | 55 | 1.0599 | 61 | 0.7274 | 20 | 40 | |
| | 30 | 0.6884 | 21 | 0.9490 | 55 | 1.0538 | 61 | 0.7254 | 20 | 30 | |
| | 40 | 0.6905 | 21 | 0.9545 | 56 | 1.0477 | 61 | 0.7234 | 20 | 20 | |
| | 50 | 0.6926 | 21 | 0.9601 | 56 | 1.0416 | 61 | 0.7214 | 21 | 10 | |
| 44 | 0 | 0.6947 | 20 | 0.9657 | 56 | 1.0355 | 60 | 0.7193 | 20 | 0 | 46 |
| | 10 | 0.6967 | 21 | 0.9713 | 57 | 1.0295 | 60 | 0.7173 | 20 | 50 | |
| | 20 | 0.6988 | 21 | 0.9770 | 57 | 1.0235 | 59 | 0.7153 | 20 | 40 | |
| | 30 | 0.7009 | 21 | 0.9827 | 57 | 1.0176 | 59 | 0.7133 | 21 | 30 | |
| | 40 | 0.7030 | 20 | 0.9884 | 58 | 1.0117 | 59 | 0.7112 | 20 | 20 | |
| | 50 | 0.7050 | 21 | 0.9942 | 58 | 1.0058 | 58 | 0.7092 | 21 | 10 | |
| 45 | 0 | 0.7071 | | 1.0000 | | 1.0000 | | 0.7071 | | 0 | 45 |
| | | Cos. | d. | Cot. | d. | Tan. | d. | Sin. | d. | ′ | ° |

P. P.

| | 58 | 57 | 56 | 55 |
|---|---|---|---|---|
| 1 | 5.8 | 5.7 | 5.6 | 5.5 |
| 2 | 11.6 | 11.4 | 11.2 | 11.0 |
| 3 | 17.4 | 17.1 | 16.8 | 16.5 |
| 4 | 23.2 | 22.8 | 22.4 | 22.0 |
| 5 | 29.0 | 28.5 | 28.0 | 27.5 |
| 6 | 34.8 | 34.2 | 33.6 | 33.0 |
| 7 | 40.6 | 39.9 | 39.2 | 38.5 |
| 8 | 46.4 | 45.6 | 44.8 | 44.0 |
| 9 | 52.2 | 51.3 | 50.4 | 49.5 |

| | 54 | 53 | 52 | 51 |
|---|---|---|---|---|
| 1 | 5.4 | 5.3 | 5.2 | 5.1 |
| 2 | 10.8 | 10.6 | 10.4 | 10.2 |
| 3 | 16.2 | 15.9 | 15.6 | 15.3 |
| 4 | 21.6 | 21.2 | 20.8 | 20.4 |
| 5 | 27.0 | 26.5 | 26.0 | 25.5 |
| 6 | 32.4 | 31.8 | 31.2 | 30.6 |
| 7 | 37.8 | 37.1 | 36.4 | 35.7 |
| 8 | 43.2 | 42.4 | 41.6 | 40.8 |
| 9 | 48.6 | 47.7 | 46.8 | 45.9 |

| | 50 | 49 | 48 |
|---|---|---|---|
| 1 | 5.0 | 4.9 | 4.8 |
| 2 | 10.0 | 9.8 | 9.6 |
| 3 | 15.0 | 14.7 | 14.4 |
| 4 | 20.0 | 19.6 | 19.2 |
| 5 | 25.0 | 24.5 | 24.0 |
| 6 | 30.0 | 29.4 | 28.8 |
| 7 | 35.0 | 34.3 | 33.6 |
| 8 | 40.0 | 39.2 | 38.4 |
| 9 | 45.0 | 44.1 | 43.2 |

| | 47 | 46 | 45 |
|---|---|---|---|
| 1 | 4.7 | 4.6 | 4.5 |
| 2 | 9.4 | 9.2 | 9.0 |
| 3 | 14.1 | 13.8 | 13.5 |
| 4 | 18.8 | 18.4 | 18.0 |
| 5 | 23.5 | 23.0 | 22.5 |
| 6 | 28.2 | 27.6 | 27.0 |
| 7 | 32.9 | 32.2 | 31.5 |
| 8 | 37.6 | 36.8 | 36.0 |
| 9 | 42.3 | 41.4 | 40.5 |

| | 24 | 23 | 22 | 21 |
|---|---|---|---|---|
| 1 | 2.4 | 2.3 | 2.2 | 2.1 |
| 2 | 4.8 | 4.6 | 4.4 | 4.2 |
| 3 | 7.2 | 6.9 | 6.6 | 6.3 |
| 4 | 9.6 | 9.2 | 8.8 | 8.4 |
| 5 | 12.0 | 11.5 | 11.0 | 10.5 |
| 6 | 14.4 | 13.8 | 13.2 | 12.6 |
| 7 | 16.8 | 16.1 | 15.4 | 14.7 |
| 8 | 19.2 | 18.4 | 17.6 | 16.8 |
| 9 | 21.6 | 20.7 | 19.8 | 18.9 |

| | 20 | 19 | 18 | 17 |
|---|---|---|---|---|
| 1 | 2.0 | 1.9 | 1.8 | 1.7 |
| 2 | 4.0 | 3.8 | 3.6 | 3.4 |
| 3 | 6.0 | 5.7 | 5.4 | 5.1 |
| 4 | 8.0 | 7.6 | 7.2 | 6.8 |
| 5 | 10.0 | 9.5 | 9.0 | 8.5 |
| 6 | 12.0 | 11.4 | 10.8 | 10.2 |
| 7 | 14.0 | 13.3 | 12.6 | 11.9 |
| 8 | 16.0 | 15.2 | 14.4 | 13.6 |
| 9 | 18.0 | 17.1 | 16.2 | 15.3 |

P. P.

avoided, if possible, since the differences change very rapidly, and the computation is therefore likely to be inexact.

In finding the functions of an angle, note carefully whether the difference obtained from the table of proportional parts is to be added or subtracted, by observing whether the function is increasing or decreasing as the angle increases. For example, the sine of 21° is .3584, and the following sines, reading downwards, are .3611, .3638, etc. The sine of 21° 6′ is greater than that of 21°, and the difference for 6′ must be added. On the other hand, the cosine of 21° is .9336, and the following cosines, reading downwards, are .9325, .9315, etc.; that is, as the angle grows larger the cosine decreases, and the difference obtained for any angle between 21° and 21° 10′, say 21° 6′, must be subtracted from the cosine of 21°.

Suppose the function, i. e., the sine, cosine, tangent, or cotangent is given and the corresponding angle is to be found; for example, find the angle whose sine is .4943. First find in the second column the sine next *smaller* than .4943, which is .4924, and the difference for 10′ is 26. The angle corresponding to .4924 is 29° 30′. Subtracting the .4924 from .4943, the first remainder is 19; in the table of proportional parts under 26, the part next lower than this difference, is 18.2, opposite which is 7′. Subtracting 18.2 from 19 leaves .8 as the second remainder. In the table under 26 is found 7.8, which with its decimal point moved one place to the left is nearest to the second remainder, and opposite 7.8 is 3, which indicates .3′ or 18″. Hence, the angle is 29° 30′ + 7′ + 18″ = 29° 37′ 18″.

## INVOLUTION AND EVOLUTION

By means of the following table, the square, cube, square root, cube root, and reciprocal of any number may be obtained correct always to five significant figures, and in the majority of cases correct to six significant figures.

In any number, the figures beginning with the first digit* at the left and ending with the last digit at the right, are

*Ciphers (used merely to locate the decimal point) are not digits.

called the *significant figures* of the number. Thus, the number 405,800 has the four significant figures 4, 0, 5, 8; and the *significant part* of the number is 4058. The number .000090067 has five significant figures, 9, 0, 0, 6, 7, and the significant part is 90067. *All numbers that differ only in the position of the decimal point have the same significant figures and the same significant part.* For example, .002103, 21.03, 21,030, and 210,300 have the same significant figures 2, 1, 0, and 3, and the same significant part 2103.

The *integral part* of a number is the part to the left of the decimal point.

**Square and Cube Roots.**—If the given number contains less than four significant figures, the required root can be found in the table, the square root under $\sqrt{n}$, or $\sqrt{10n}$, and the cube root under $\sqrt[3]{n}$, $\sqrt[3]{10n}$, or $\sqrt[3]{100n}$, according to the number of significant figures in the integral part of the number. Thus, $\sqrt{3.14} = 1.772$; $\sqrt{31.4} = \sqrt{10 \times 3.14} = 5.60357$; $\sqrt[3]{3.14} = 1.46434$; $\sqrt[3]{31.4} = \sqrt[3]{10 \times 3.14} = 3.15484$; $\sqrt[3]{314} = \sqrt[3]{100 \times 3.14} = 6.79688$.

In order to locate the decimal point, the given number must be pointed off into periods of two figures each for square root and three figures each for cube root, beginning always at the decimal point. Thus, for square root: 12703, 1'27'03; 12.703, 12.70'30; 220000, 22'00'00; .000442, .00'04'42; and for cube root: 3141.6, 3'141.6; 67296428, 67'296'428; .0000000217, .000'000'021'700, etc.

*There are as many figures in the root preceding the decimal point as there are periods preceding the decimal point in the given number; if the number is entirely decimal, the root is entirely decimal, and there are as many ciphers following the decimal point in the root as there are cipher periods following the decimal point in the given number.*

Applying this rule, $\sqrt{220000} = 469.04$, $\sqrt{.000442} = .021024$, $\sqrt[3]{518000} = 80.3113$, and $\sqrt[3]{.000073} = .0418$.

If the number has more than three significant figures, point off the number into periods, place a decimal point between the first and second periods of the significant part of the number, and proceed as in the following examples:

| $n$ | $n^2$ | $n^3$ | $\sqrt{n}$ | $\sqrt{10n}$ | $\sqrt[3]{n}$ | $\sqrt[3]{10n}$ | $\sqrt[3]{100n}$ | $\frac{1}{n}$ |
|---|---|---|---|---|---|---|---|---|
| 1.01 | 1.0201 | 1.03030 | 1.00499 | 3.17805 | 1.00332 | 2.16159 | 4.65701 | .990099 |
| 1.02 | 1.0404 | 1.06121 | 1.00995 | 3.19374 | 1.00662 | 2.16870 | 4.67233 | .980392 |
| 1.03 | 1.0609 | 1.09273 | 1.01489 | 3.20936 | 1.00990 | 2.17577 | 4.68755 | .970874 |
| 1.04 | 1.0816 | 1.12486 | 1.01980 | 3.22490 | 1.01316 | 2.18278 | 4.70267 | .961539 |
| 1.05 | 1.1025 | 1.15763 | 1.02470 | 3.24037 | 1.01640 | 2.18976 | 4.71769 | .952381 |
| 1.06 | 1.1236 | 1.19102 | 1.02956 | 3.25576 | 1.01961 | 2.19669 | 4.73262 | .943396 |
| 1.07 | 1.1449 | 1.22504 | 1.03441 | 3.27109 | 1.02281 | 2.20358 | 4.74746 | .934579 |
| 1.08 | 1.1664 | 1.25971 | 1.03923 | 3.28634 | 1.02599 | 2.21042 | 4.76220 | .925926 |
| 1.09 | 1.1881 | 1.29503 | 1.04403 | 3.30151 | 1.02914 | 2.21722 | 4.77686 | .917431 |
| 1.10 | 1.2100 | 1.33100 | 1.04881 | 3.31662 | 1.03228 | 2.22398 | 4.79142 | .909091 |
| 1.11 | 1.2321 | 1 36763 | 1.05357 | 3.33167 | 1.03540 | 2.23070 | 4.80590 | .900901 |
| 1.12 | 1.2544 | 1.40493 | 1.05830 | 3.34664 | 1.03850 | 2.23738 | 4.82028 | .892857 |
| 1.13 | 1.2769 | 1.44290 | 1.06301 | 3.36155 | 1.04158 | 2.24402 | 4.83459 | .884956 |
| 1.14 | 1.2996 | 1.48154 | 1.06771 | 3.37639 | 1.04464 | 2.25062 | 4.84881 | .877193 |
| 1.15 | 1.3225 | 1.52088 | 1.07238 | 3.39116 | 1.04769 | 2.25718 | 4.86294 | .869565 |
| 1.16 | 1.3456 | 1.56090 | 1.07703 | 3.40588 | 1.05072 | 2.26370 | 4.87700 | .862069 |
| 1.17 | 1.3689 | 1.60161 | 1.08167 | 3.42053 | 1.05373 | 2.27019 | 4.89097 | .854701 |
| 1.18 | 1.3924 | 1.64303 | 1.08628 | 3.43511 | 1.05672 | 2.27664 | 4.90487 | .847458 |
| 1.19 | 1.4161 | 1.68516 | 1.09087 | 3.44964 | 1.05970 | 2.28305 | 4.91868 | .840336 |
| 1.20 | 1.4400 | 1.72800 | 1.09545 | 3.46410 | 1.06266 | 2.28943 | 4.93242 | .833333 |
| 1.21 | 1.4641 | 1.77156 | 1.10000 | 3.47851 | 1.06560 | 2.29577 | 4.94609 | .826446 |
| 1.22 | 1.4884 | 1.81585 | 1.10454 | 3.49285 | 1.06853 | 2.30208 | 4.95968 | .819672 |
| 1.23 | 1.5129 | 1.86087 | 1.10905 | 3.50714 | 1.07144 | 2.30835 | 4.97319 | .813008 |
| 1.24 | 1.5376 | 1.90662 | 1.11355 | 3.52136 | 1.07434 | 2.31459 | 4.98663 | .806452 |
| 1.25 | 1.5625 | 1.95313 | 1.11803 | 3.53553 | 1.07722 | 2.32080 | 5.00000 | .800000 |
| 1.26 | 1.5876 | 2.00038 | 1.12250 | 3.54965 | 1.08008 | 2.32697 | 5.01330 | .793651 |
| 1.27 | 1.6129 | 2.04838 | 1.12694 | 3.56371 | 1.08293 | 2.33310 | 5.02653 | .787402 |
| 1.28 | 1.6384 | 2.09715 | 1.13137 | 3.57771 | 1.08577 | 2.33921 | 5.03968 | .781250 |
| 1.29 | 1.6641 | 2.14669 | 1.13578 | 3.59166 | 1.08859 | 2.34529 | 5.05277 | .775194 |
| 1.30 | 1.6900 | 2.19700 | 1.14018 | 3.60555 | 1.09139 | 2.35134 | 5.06580 | .769231 |
| 1.31 | 1.7161 | 2.24809 | 1.14455 | 3.61939 | 1.09418 | 2.35735 | 5.07875 | .763359 |
| 1.32 | 1.7424 | 2.29997 | 1.14891 | 3.63318 | 1.09696 | 2.36333 | 5.09164 | .757576 |
| 1.33 | 1.7689 | 2.35264 | 1.15326 | 3.64692 | 1.09972 | 2.36928 | 5.10447 | .751880 |
| 1.34 | 1.7956 | 2.40610 | 1.15758 | 3.66060 | 1.10247 | 2.37521 | 5.11723 | .746269 |
| 1.35 | 1.8225 | 2.46038 | 1.16190 | 3.67423 | 1.10521 | 2.38110 | 5.12993 | .740741 |
| 1.36 | 1.8496 | 2.51546 | 1.16619 | 3.68782 | 1.10793 | 2.38696 | 5.14256 | .735294 |
| 1.37 | 1.8769 | 2.57135 | 1.17047 | 3.70135 | 1.11064 | 2.39280 | 5.15514 | .729927 |
| 1.38 | 1.9044 | 2.62807 | 1.17473 | 3.71484 | 1.11334 | 2.39861 | 5.16765 | .724638 |
| 1.39 | 1.9321 | 2.68562 | 1.17898 | 3.72827 | 1.11602 | 2.40439 | 5.18010 | .719425 |
| 1.40 | 1.9600 | 2.74400 | 1.18322 | 3.74166 | 1.11869 | 2.41014 | 5.19249 | .714286 |
| 1.41 | 1.9881 | 2.80322 | 1.18743 | 3.75500 | 1.12135 | 2.41587 | 5.20483 | .709220 |
| 1.42 | 2.0164 | 2.86329 | 1.19164 | 3.76829 | 1.12399 | 2.42156 | 5.21710 | .704225 |
| 1.43 | 2.0449 | 2.92421 | 1.19583 | 3.78153 | 1.12662 | 2.42724 | 5.22932 | .699301 |
| 1.44 | 2.0736 | 2.98598 | 1.20000 | 3.79473 | 1.12924 | 2.43288 | 5.24148 | .694444 |
| 1.45 | 2.1025 | 3.04863 | 1.20416 | 3.80789 | 1.13185 | 2.43850 | 5.25359 | .689655 |
| 1.46 | 2.1316 | 3.11214 | 1.20830 | 3.82099 | 1.13445 | 2.44409 | 5.26564 | .684932 |
| 1.47 | 2.1609 | 3.17652 | 1.21244 | 3.83406 | 1.13703 | 2.44966 | 5.27763 | .680272 |
| 1.48 | 2.1904 | 3.24179 | 1.21655 | 3.84708 | 1.13960 | 2.45520 | 5.28957 | .675676 |
| 1.49 | 2.2201 | 3.30795 | 1.22066 | 3.86005 | 1.14216 | 2.46072 | 5.30146 | .671141 |
| 1.50 | 2.2500 | 3.37500 | 1.22474 | 3.87298 | 1.14471 | 2.46621 | 5.31329 | .666667 |

| $n$ | $n^2$ | $n^3$ | $\sqrt{n}$ | $\sqrt{10\,n}$ | $\sqrt[3]{n}$ | $\sqrt[3]{10\,n}$ | $\sqrt[3]{100\,n}$ | $\frac{1}{n}$ |
|---|---|---|---|---|---|---|---|---|
| 1.51 | 2.2801 | 3.44295 | 1.22882 | 3.88587 | 1.14725 | 2.47168 | 5.32507 | .662252 |
| 1.52 | 2.3104 | 3.51181 | 1.23288 | 3.89872 | 1.14978 | 2.47713 | 5.33680 | .657895 |
| 1.53 | 2.3409 | 3.58158 | 1.23693 | 3.91152 | 1.15230 | 2.48255 | 5.34848 | .653595 |
| 1.54 | 2.3716 | 3.65226 | 1.24097 | 3.92428 | 1.15480 | 2.48794 | 5.36011 | .649351 |
| 1.55 | 2.4025 | 3.72388 | 1.24499 | 3.93700 | 1.15729 | 2.49332 | 5.37169 | .645161 |
| 1.56 | 2.4336 | 3.79642 | 1.24900 | 3.94968 | 1.15978 | 2.49866 | 5.38321 | .641026 |
| 1.57 | 2.4649 | 3.86989 | 1.25300 | 3.96232 | 1.16225 | 2.50399 | 5.39469 | .636943 |
| 1.58 | 2.4964 | 3.94431 | 1.25698 | 3.97492 | 1.16471 | 2.50930 | 5.40612 | .632911 |
| 1.59 | 2.5281 | 4.01968 | 1.26095 | 3.98748 | 1.16717 | 2.51458 | 5.41750 | .628931 |
| 1.60 | 2.5600 | 4.09600 | 1.26491 | 4.00000 | 1.16961 | 2.51984 | 5.42884 | .625000 |
| 1.61 | 2.5921 | 4.17328 | 1.26886 | 4.01248 | 1.17204 | 2.52508 | 5.44012 | .621118 |
| 1.62 | 2.6244 | 4.25153 | 1.27279 | 4.02492 | 1.17446 | 2.53030 | 5.45136 | .617284 |
| 1.63 | 2.6569 | 4.33075 | 1.27671 | 4.03733 | 1.17687 | 2.53549 | 5.46256 | .613497 |
| 1.64 | 2.6896 | 4.41094 | 1.28062 | 4.04969 | 1.17927 | 2.54067 | 5.47370 | .609756 |
| 1.65 | 2.7225 | 4.49213 | 1.28452 | 4.06202 | 1.18167 | 2.54582 | 5.48481 | .606061 |
| 1.66 | 2.7556 | 4.57430 | 1.28841 | 4.07431 | 1.18405 | 2.55095 | 5.49586 | .602410 |
| 1.67 | 2.7889 | 4.65746 | 1.29228 | 4.08656 | 1.18642 | 2.55607 | 5.50688 | .598802 |
| 1.68 | 2.8224 | 4.74163 | 1.29615 | 4.09878 | 1.18878 | 2.56116 | 5.51785 | .595238 |
| 1.69 | 2.8561 | 4.82681 | 1.30000 | 4.11096 | 1.19114 | 2.56623 | 5.52877 | .591716 |
| 1.70 | 2.8900 | 4.91300 | 1.30384 | 4.12311 | 1.19348 | 2.57128 | 5.53966 | .588235 |
| 1.71 | 2.9241 | 5.00021 | 1.30767 | 4.13521 | 1.19582 | 2.57631 | 5.55050 | .584795 |
| 1.72 | 2.9584 | 5.08845 | 1.31149 | 4.14729 | 1.19815 | 2.58133 | 5.56130 | .581395 |
| 1.73 | 2.9929 | 5.17772 | 1.31529 | 4.15933 | 1.20046 | 2.58632 | 5.57205 | .578035 |
| 1.74 | 3.0276 | 5.26802 | 1.31909 | 4.17133 | 1.20277 | 2.59129 | 5.58277 | .574713 |
| 1.75 | 3.0625 | 5.35938 | 1.32288 | 4.18330 | 1.20507 | 2.59625 | 5.59344 | .571429 |
| 1.76 | 3.0976 | 5.45178 | 1.32665 | 4.19524 | 1.20736 | 2.60118 | 5.60408 | .568182 |
| 1.77 | 3.1329 | 5.54523 | 1.33041 | 4.20714 | 1.20964 | 2.60610 | 5.61467 | .564972 |
| 1.78 | 3.1684 | 5.63975 | 1.33417 | 4.21900 | 1.21192 | 2.61100 | 5.62523 | .561798 |
| 1.79 | 3.2041 | 5.73534 | 1.33791 | 4.23084 | 1.21418 | 2.61588 | 5.63574 | .558659 |
| 1.80 | 3.2400 | 5.83200 | 1.34164 | 4.24264 | 1.21644 | 2.62074 | 5.64622 | .555556 |
| 1.81 | 3.2761 | 5.92974 | 1.34536 | 4.25441 | 1.21869 | 2.62558 | 5.65665 | .552486 |
| 1.82 | 3.3124 | 6.02857 | 1.34907 | 4.26615 | 1.22093 | 2.63041 | 5.66705 | .549451 |
| 1.83 | 3.3489 | 6.12849 | 1.35277 | 4.27785 | 1.22316 | 2.63522 | 5.67741 | .546448 |
| 1.84 | 3.3856 | 6.22950 | 1.35647 | 4.28952 | 1.22539 | 2.64001 | 5.68773 | .543478 |
| 1.85 | 3.4225 | 6.33163 | 1.36015 | 4.30116 | 1.22760 | 2.64479 | 5.69802 | .540541 |
| 1.86 | 3.4596 | 6.43486 | 1.36382 | 4.31277 | 1.22981 | 2.64954 | 5.70827 | .537634 |
| 1.87 | 3.4969 | 6.53920 | 1.36748 | 4.32435 | 1.23201 | 2.65428 | 5.71848 | .534759 |
| 1.88 | 3.5344 | 6.64467 | 1.37113 | 4.33590 | 1.23420 | 2.65900 | 5.72865 | .531915 |
| 1.89 | 3.5721 | 6.75127 | 1.37477 | 4.34741 | 1.23639 | 2.66371 | 5.73879 | .529101 |
| 1.90 | 3.6100 | 6.85900 | 1.37840 | 4.35890 | 1.23856 | 2.66840 | 5.74890 | .526316 |
| 1.91 | 3.6481 | 6.96787 | 1.38203 | 4.37035 | 1.24073 | 2.67307 | 5.75897 | .523560 |
| 1.92 | 3.6864 | 7.07789 | 1.38564 | 4.38178 | 1.24289 | 2.67773 | 5.76900 | .520833 |
| 1.93 | 3.7249 | 7.18906 | 1.38924 | 4.39318 | 1.24505 | 2.68237 | 5.77900 | .518135 |
| 1.94 | 3.7636 | 7.30138 | 1.39284 | 4.40454 | 1.24719 | 2.68700 | 5.78896 | .515464 |
| 1.95 | 3.8025 | 7.41488 | 1.39642 | 4.41588 | 1.24933 | 2.69161 | 5.79889 | .512821 |
| 1.96 | 3.8416 | 7.52954 | 1.40000 | 4.42719 | 1.25146 | 2.69620 | 5.80879 | .510204 |
| 1.97 | 3.8809 | 7.64537 | 1.40357 | 4.43847 | 1.25359 | 2.70078 | 5.81865 | .507614 |
| 1.98 | 3.9204 | 7.76239 | 1.40712 | 4.44972 | 1.25571 | 2.70534 | 5.82848 | .505051 |
| 1.99 | 3.9601 | 7.88060 | 1.41067 | 4.46094 | 1.25782 | 2.70989 | 5.83827 | .502513 |
| 2.00 | 4.0000 | 8.00000 | 1.41421 | 4.47214 | 1.25992 | 2.71442 | 5.84804 | .500000 |

| $n$ | $n^2$ | $n^3$ | $\sqrt{n}$ | $\sqrt{10\,n}$ | $\sqrt[3]{n}$ | $\sqrt[3]{10\,n}$ | $\sqrt[3]{100\,n}$ | $\frac{1}{n}$ |
|---|---|---|---|---|---|---|---|---|
| 2.01 | 4.0401 | 8.12060 | 1.41774 | 4.48330 | 1.26202 | 2.71893 | 5.85777 | .497512 |
| 2.02 | 4.0804 | 8.24241 | 1.42127 | 4.49444 | 1.26411 | 2.72348 | 5.86746 | .495050 |
| 2.03 | 4.1209 | 8.36543 | 1.42478 | 4.50555 | 1.26619 | 2.72792 | 5.87713 | .492611 |
| 2.04 | 4.1616 | 8.48966 | 1.42829 | 4.51664 | 1.26827 | 2.73239 | 5.88677 | .490196 |
| 2.05 | 4.2025 | 8.61513 | 1.43178 | 4.52769 | 1.27033 | 2.73685 | 5.89637 | .487805 |
| 2.06 | 4.2436 | 8.74182 | 1.43527 | 4.53872 | 1.27240 | 2.74129 | 5.90594 | .485437 |
| 2.07 | 4.2849 | 8.86974 | 1.43875 | 4.54973 | 1.27445 | 2.74572 | 5.91548 | .483092 |
| 2.08 | 4.3264 | 8.99891 | 1.44222 | 4.56070 | 1.27650 | 2.75014 | 5.92499 | .480769 |
| 2.09 | 4.3681 | 9.12933 | 1.44568 | 4.57165 | 1.27854 | 2.75454 | 5.93447 | .478469 |
| 2.10 | 4.4100 | 9.26100 | 1.44914 | 4.58258 | 1.28058 | 2.75893 | 5.94392 | .476191 |
| 2.11 | 4.4521 | 9.39393 | 1.45258 | 4.59347 | 1.28261 | 2.76330 | 5.95334 | .473934 |
| 2.12 | 4.4944 | 9.52813 | 1.45602 | 4.60435 | 1.28463 | 2.76766 | 5.96273 | .471698 |
| 2.13 | 4.5369 | 9.66360 | 1.45945 | 4.61519 | 1.28665 | 2.77200 | 5.97209 | .469484 |
| 2.14 | 4.5796 | 9.80034 | 1.46287 | 4.62601 | 1.28866 | 2.77633 | 5.98142 | .467290 |
| 2.15 | 4.6225 | 9.93838 | 1.46629 | 4.63681 | 1.29066 | 2.78065 | 5.99073 | .465116 |
| 2.16 | 4.6656 | 10.0777 | 1.46969 | 4.64758 | 1.29266 | 2.78495 | 6.00000 | .462963 |
| 2.17 | 4.7089 | 10.2183 | 1.47309 | 4.65833 | 1.29465 | 2.78924 | 6.00925 | .460830 |
| 2.18 | 4.7524 | 10.3602 | 1.47648 | 4.66905 | 1.29664 | 2.79352 | 6.01846 | .458716 |
| 2.19 | 4.7961 | 10.5035 | 1.47986 | 4.67974 | 1.29862 | 2.79779 | 6.02765 | .456621 |
| 2.20 | 4.8400 | 10.6480 | 1.48324 | 4.69042 | 1.30059 | 2.80204 | 6.03681 | .454546 |
| 2.21 | 4.8841 | 10.7939 | 1.48661 | 4.70106 | 1.30256 | 2.80628 | 6.04594 | .452489 |
| 2.22 | 4.9284 | 10.9410 | 1.48997 | 4.71169 | 1.30452 | 2.81051 | 6.05505 | .450451 |
| 2.23 | 4.9729 | 11.0896 | 1.49332 | 4.72229 | 1.30648 | 2.81472 | 6.06413 | .448431 |
| 2.24 | 5.0176 | 11.2394 | 1.49666 | 4.73286 | 1.30843 | 2.81892 | 6.07318 | .446429 |
| 2.25 | 5.0625 | 11.3906 | 1.50000 | 4.74342 | 1.31037 | 2.82311 | 6.08220 | .444444 |
| 2.26 | 5.1076 | 11.5432 | 1.50333 | 4.75395 | 1.31231 | 2.82728 | 6.09120 | .442478 |
| 2.27 | 5.1529 | 11.6971 | 1.50665 | 4.76445 | 1.31424 | 2.83145 | 6.10017 | .440529 |
| 2.28 | 5.1984 | 11.8524 | 1.50997 | 4.77493 | 1.31617 | 2.83560 | 6.10911 | .438597 |
| 2.29 | 5.2441 | 12.0090 | 1.51327 | 4.78539 | 1.31809 | 2.83974 | 6.11803 | .436681 |
| 2.30 | 5.2900 | 12.1670 | 1.51658 | 4.79583 | 1.32001 | 2.84387 | 6.12693 | .434783 |
| 2.31 | 5.3361 | 12.3264 | 1.51987 | 4.80625 | 1.32192 | 2.84798 | 6.13579 | .432900 |
| 2.32 | 5.3824 | 12.4872 | 1.52315 | 4.81664 | 1.32382 | 2.85209 | 6.14463 | .431035 |
| 2.33 | 5.4289 | 12.6493 | 1.52643 | 4.82701 | 1.32572 | 2.85618 | 6.15345 | .429185 |
| 2.34 | 5.4756 | 12.8129 | 1.52971 | 4.83735 | 1.32761 | 2.86026 | 6.16224 | .427350 |
| 2.35 | 5.5225 | 12.9779 | 1.53297 | 4.84768 | 1.32950 | 2.86433 | 6.17101 | .425532 |
| 2.36 | 5.5696 | 13.1443 | 1.53623 | 4.85798 | 1.33139 | 2.86838 | 6.17975 | .423729 |
| 2.37 | 5.6169 | 13.3121 | 1.53948 | 4.86826 | 1.33326 | 2.87243 | 6.18846 | .421941 |
| 2.38 | 5.6644 | 13.4813 | 1.54272 | 4.87852 | 1.33514 | 2.87646 | 6.19715 | .420168 |
| 2.39 | 5.7121 | 13.6519 | 1.54596 | 4.88876 | 1.33700 | 2.88049 | 6.20582 | .418410 |
| 2.40 | 5.7600 | 13.8240 | 1.54919 | 4.89898 | 1.33887 | 2.88450 | 6.21447 | .416667 |
| 2.41 | 5.8081 | 13.9975 | 1.55242 | 4.90918 | 1.34072 | 2.88850 | 6.22308 | .414938 |
| 2.42 | 5.8564 | 14.1725 | 1.55563 | 4.91935 | 1.34257 | 2.89249 | 6.23168 | .413223 |
| 2.43 | 5.9049 | 14.3489 | 1.55885 | 4.92950 | 1.34442 | 2.89647 | 6.24025 | .411523 |
| 2.44 | 5.9536 | 14.5268 | 1.56205 | 4.93964 | 1.34626 | 2.90044 | 6.24880 | .409836 |
| 2.45 | 6.0025 | 14.7061 | 1.56525 | 4.94975 | 1.34810 | 2.90439 | 6.25732 | .408163 |
| 2.46 | 6.0516 | 14.8869 | 1.56844 | 4.95984 | 1.34993 | 2.90834 | 6.26583 | .406504 |
| 2.47 | 6.1009 | 15.0692 | 1.57162 | 4.96991 | 1.35176 | 2.91227 | 6.27431 | .404858 |
| 2.48 | 6.1504 | 15.2530 | 1.57480 | 4.97996 | 1.35358 | 2.91620 | 6.28276 | .403226 |
| 2.49 | 6.2001 | 15.4382 | 1.57797 | 4.98999 | 1.35540 | 2.92011 | 6.29119 | .401606 |
| 2.50 | 6.2500 | 15.6250 | 1.58114 | 5.00000 | 1.35721 | 2.92402 | 6.29961 | .400000 |

| $n$ | $n^2$ | $n^3$ | $\sqrt{n}$ | $\sqrt{10n}$ | $\sqrt[3]{n}$ | $\sqrt[3]{10n}$ | $\sqrt[3]{100n}$ | $\frac{1}{n}$ |
|---|---|---|---|---|---|---|---|---|
| 2.51 | 6.3001 | 15.8133 | 1.58430 | 5.00999 | 1.35902 | 2.92791 | 6.30799 | .398406 |
| 2.52 | 6.3504 | 16.0030 | 1.58745 | 5.01996 | 1.36082 | 2.93179 | 6.31636 | .396825 |
| 2.53 | 6.4009 | 16.1943 | 1.59060 | 5.02991 | 1.36262 | 2.93567 | 6.32470 | .395257 |
| 2.54 | 6.4516 | 16.3871 | 1.59374 | 5.03984 | 1.36441 | 2.93953 | 6.33303 | .393701 |
| 2.55 | 6.5025 | 16.5814 | 1.59687 | 5.04975 | 1.36620 | 2.94338 | 6.34133 | .392157 |
| 2.56 | 6.5536 | 16.7772 | 1.60000 | 5.05964 | 1.36798 | 2.94723 | 6.34960 | .390625 |
| 2.57 | 6.6049 | 16.9746 | 1.60312 | 5.06952 | 1.36976 | 2.95106 | 6.35786 | .389105 |
| 2.58 | 6.6564 | 17.1735 | 1.60624 | 5.07937 | 1.37153 | 2.95488 | 6.36610 | .387597 |
| 2.59 | 6.7081 | 17.3740 | 1.60935 | 5.08920 | 1.37330 | 2.95869 | 6.37431 | .386100 |
| 2.60 | 6.7600 | 17.5760 | 1.61245 | 5.09902 | 1.37507 | 2.96250 | 6.38250 | .384615 |
| 2.61 | 6.8121 | 17.7796 | 1.61555 | 5.10882 | 1.37683 | 2.96629 | 6.39068 | .383142 |
| 2.62 | 6.8644 | 17.9847 | 1.61864 | 5.11859 | 1.37859 | 2.97007 | 6.39883 | .381679 |
| 2.63 | 6.9169 | 18.1914 | 1.62173 | 5.12835 | 1.38034 | 2.97385 | 6.40696 | .380228 |
| 2.64 | 6.9696 | 18.3997 | 1.62481 | 5.13809 | 1.38208 | 2.97761 | 6.41507 | .378788 |
| 2.65 | 7.0225 | 18.6096 | 1.62788 | 5.14782 | 1.38383 | 2.98137 | 6.42316 | .377359 |
| 2.66 | 7.0756 | 18.8211 | 1.63095 | 5.15752 | 1.38557 | 2.98511 | 6.43123 | .375940 |
| 2.67 | 7.1289 | 19.0342 | 1.63401 | 5.16720 | 1.38730 | 2.98885 | 6.43928 | .374532 |
| 2.68 | 7.1824 | 19.2488 | 1.63707 | 5.17687 | 1.38903 | 2.99257 | 6.44731 | .373134 |
| 2.69 | 7.2361 | 19.4651 | 1.64012 | 5.18652 | 1.39076 | 2.99629 | 6.45531 | .371747 |
| 2.70 | 7.2900 | 19.6830 | 1.64317 | 5.19615 | 1.39248 | 3.00000 | 6.46330 | .370370 |
| 2.71 | 7.3441 | 19.9025 | 1.64621 | 5.20577 | 1.39419 | 3.00370 | 6.47127 | .369004 |
| 2.72 | 7.3984 | 20.1236 | 1.64924 | 5.21536 | 1.39591 | 3.00739 | 6.47922 | .367647 |
| 2.73 | 7.4529 | 20.3464 | 1.65227 | 5.22494 | 1.39761 | 3.01107 | 6.48715 | .366300 |
| 2.74 | 7.5076 | 20.5708 | 1.65529 | 5.23450 | 1.39932 | 3.01474 | 6.49507 | .364964 |
| 2.75 | 7.5625 | 20.7969 | 1.65831 | 5.24404 | 1.40102 | 3.01841 | 6.50296 | .363636 |
| 2.76 | 7.6176 | 21.0246 | 1.66132 | 5.25357 | 1.40272 | 3.02206 | 6.51083 | .362319 |
| 2.77 | 7.6729 | 21.2539 | 1.66433 | 5.26308 | 1.40441 | 3.02571 | 6.51868 | .361011 |
| 2.78 | 7.7284 | 21.4850 | 1.66733 | 5.27257 | 1.40610 | 3.02934 | 6.52652 | .359712 |
| 2.79 | 7.7841 | 21.7176 | 1.67033 | 5.28205 | 1.40778 | 3.03297 | 6.53434 | .358423 |
| 2.80 | 7.8400 | 21.9520 | 1.67332 | 5.29150 | 1.40946 | 3.03659 | 6.54213 | .357142 |
| 2.81 | 7.8961 | 22.1880 | 1.67631 | 5.30094 | 1.41114 | 3.04020 | 6.54991 | .355872 |
| 2.82 | 7.9524 | 22.4258 | 1.67929 | 5.31037 | 1.41281 | 3.04380 | 6.55767 | .354610 |
| 2.83 | 8.0089 | 22.6652 | 1.68226 | 5.31977 | 1.41448 | 3.04740 | 6.56541 | .353357 |
| 2.84 | 8.0656 | 22.9063 | 1.68523 | 5.32917 | 1.41614 | 3.05098 | 6.57314 | .352113 |
| 2.85 | 8.1225 | 23.1491 | 1.68819 | 5.33854 | 1.41780 | 3.05456 | 6.58084 | .350877 |
| 2.86 | 8.1796 | 23.3937 | 1.69115 | 5.34790 | 1.41946 | 3.05813 | 6.58853 | .349650 |
| 2.87 | 8.2369 | 23.6399 | 1.69411 | 5.35724 | 1.42111 | 3.06169 | 6.59620 | .348432 |
| 2.88 | 8.2944 | 23.8879 | 1.69706 | 5.36656 | 1.42276 | 3.06524 | 6.60385 | .347222 |
| 2.89 | 8.3521 | 24.1376 | 1.70000 | 5.37587 | 1.42440 | 3.06878 | 6.61149 | .346021 |
| 2.90 | 8.4100 | 24.3890 | 1.70294 | 5.38516 | 1.42604 | 3.07232 | 6.61911 | .344828 |
| 2.91 | 8.4681 | 24.6422 | 1.70587 | 5.39444 | 1.42768 | 3.07585 | 6.62671 | .343643 |
| 2.92 | 8.5264 | 24.8971 | 1.70880 | 5.40370 | 1.42931 | 3.07936 | 6.63429 | .342466 |
| 2.93 | 8.5849 | 25.1538 | 1.71172 | 5.41295 | 1.43094 | 3.08287 | 6.64185 | .341297 |
| 2.94 | 8.6436 | 25.4122 | 1.71464 | 5.42218 | 1.43257 | 3.08638 | 6.64940 | .340136 |
| 2.95 | 8.7025 | 25.6724 | 1.71756 | 5.43139 | 1.43419 | 3.08987 | 6.65693 | .338983 |
| 2.96 | 8.7616 | 25.9343 | 1.72047 | 5.44059 | 1.43581 | 3.09336 | 6.66444 | .337838 |
| 2.97 | 8.8209 | 26.1981 | 1.72337 | 5.44977 | 1.43743 | 3.09684 | 6.67194 | .336700 |
| 2.98 | 8.8804 | 26.4636 | 1.72627 | 5.45894 | 1.43904 | 3.10031 | 6.67942 | .335571 |
| 2.99 | 8.9401 | 26.7309 | 1.72916 | 5.46809 | 1.44065 | 3.10378 | 6.68688 | .334448 |
| 3.00 | 9.0000 | 27.0000 | 1.73205 | 5.47723 | 1.44225 | 3.10723 | 6.69433 | .333333 |

| $n$ | $n^2$ | $n^3$ | $\sqrt{n}$ | $\sqrt{10\,n}$ | $\sqrt[3]{n}$ | $\sqrt[3]{10\,n}$ | $\sqrt[3]{100\,n}$ | $\frac{1}{n}$ |
|---|---|---|---|---|---|---|---|---|
| 3.01 | 9.0601 | 27.2709 | 1.73494 | 5.48635 | 1.44385 | 3.11068 | 6.70176 | .332226 |
| 3.02 | 9.1204 | 27.5436 | 1.73781 | 5.49545 | 1.44545 | 3.11412 | 6.70917 | .331126 |
| 3.03 | 9.1809 | 27.8181 | 1.74069 | 5.50454 | 1.44704 | 3.11756 | 6.71657 | .330033 |
| 3.04 | 9.2416 | 28.0945 | 1.74356 | 5.51362 | 1.44863 | 3.12098 | 6.72395 | .328947 |
| 3.05 | 9.3025 | 28.3726 | 1.74642 | 5.52268 | 1.45022 | 3.12440 | 6.73132 | .327869 |
| 3.06 | 9.3636 | 28.6526 | 1.74929 | 5.53173 | 1.45180 | 3.12781 | 6.73866 | .326797 |
| 3.07 | 9.4249 | 28.9344 | 1.75214 | 5.54076 | 1.45338 | 3.13121 | 6.74600 | .325733 |
| 3.08 | 9.4864 | 29.2181 | 1.75499 | 5.54977 | 1.45496 | 3.13461 | 6.75331 | .324675 |
| 3.09 | 9.5481 | 29.5036 | 1.75784 | 5.55878 | 1.45653 | 3.13800 | 6.76061 | .323625 |
| 3.10 | 9.6100 | 29.7910 | 1.76068 | 5.56776 | 1.45810 | 3.14138 | 6.76790 | .322581 |
| 3.11 | 9.6721 | 30.0802 | 1.76352 | 5.57674 | 1.45967 | 3.14475 | 6.77517 | .321543 |
| 3.12 | 9.7344 | 30.3713 | 1.76635 | 5.58570 | 1.46123 | 3.14812 | 6.78242 | .320513 |
| 3.13 | 9.7969 | 30.6643 | 1.76918 | 5.59464 | 1.46279 | 3.15148 | 6.78966 | .319489 |
| 3.14 | 9.8596 | 30.9591 | 1.77200 | 5.60357 | 1.46434 | 3.15484 | 6.79688 | .318471 |
| 3.15 | 9.9225 | 31.2559 | 1.77482 | 5.61249 | 1.46590 | 3.15818 | 6.80409 | .317460 |
| 3.16 | 9.9856 | 31.5545 | 1.77764 | 5.62139 | 1.46745 | 3.16152 | 6.81128 | .316456 |
| 3.17 | 10.0489 | 31.8550 | 1.78045 | 5.63028 | 1.46899 | 3.16485 | 6.81846 | .315457 |
| 3.18 | 10.1124 | 32.1574 | 1.78326 | 5.63915 | 1.47054 | 3.16817 | 6.82562 | .314465 |
| 3.19 | 10.1761 | 32.4618 | 1.78606 | 5.64801 | 1.47208 | 3.17149 | 6.83277 | .313480 |
| 3.20 | 10.2400 | 32.7680 | 1.78885 | 5.65685 | 1.47361 | 3.17480 | 6.83990 | .312500 |
| 3.21 | 10.3041 | 33.0762 | 1.79165 | 5.66569 | 1.47515 | 3.17811 | 6.84702 | .311527 |
| 3.22 | 10.3684 | 33.3862 | 1.79444 | 5.67450 | 1.47668 | 3.18140 | 6.85412 | .310559 |
| 3.23 | 10.4329 | 33.6983 | 1.79722 | 5.68331 | 1.47820 | 3.18469 | 6.86121 | .309598 |
| 3.24 | 10.4976 | 34.0122 | 1.80000 | 5.69210 | 1.47973 | 3.18798 | 6.86829 | .308642 |
| 3.25 | 10.5625 | 34.3281 | 1.80278 | 5.70088 | 1.48125 | 3.19125 | 6.87534 | .307692 |
| 3.26 | 10.6276 | 34.6460 | 1.80555 | 5.70964 | 1.48277 | 3.19452 | 6.88239 | .306749 |
| 3.27 | 10.6929 | 34.9658 | 1.80831 | 5.71839 | 1.48428 | 3.19779 | 6.88942 | .305810 |
| 3.28 | 10.7584 | 35.2876 | 1.81108 | 5.72713 | 1.48579 | 3.20104 | 6.89643 | .304878 |
| 3.29 | 10.8241 | 35.6129 | 1.81384 | 5.73585 | 1.48730 | 3.20429 | 6.90344 | .303951 |
| 3.30 | 10.8900 | 35.9370 | 1.81659 | 5.74456 | 1.48881 | 3.20753 | 6.91042 | .303030 |
| 3.31 | 10.9561 | 36.2647 | 1.81934 | 5.75326 | 1.49031 | 3.21077 | 6.91740 | .302115 |
| 3.32 | 11.0224 | 36.5944 | 1.82209 | 5.76194 | 1.49181 | 3.21400 | 6.92436 | .301205 |
| 3.33 | 11.0889 | 36.9260 | 1.82483 | 5.77062 | 1.49330 | 3.21723 | 6.93130 | .300300 |
| 3.34 | 11.1556 | 37.2597 | 1.82757 | 5.77927 | 1.49480 | 3.22044 | 6.93823 | .299401 |
| 3.35 | 11.2225 | 37.5954 | 1.83030 | 5.78792 | 1.49629 | 3.22365 | 6.94515 | .298508 |
| 3.36 | 11.2896 | 37.9331 | 1.83303 | 5.79655 | 1.49777 | 3.22686 | 6.95205 | .297619 |
| 3.37 | 11.3569 | 38.2728 | 1.83576 | 5.80517 | 1.49926 | 3.23005 | 6.95894 | .296736 |
| 3.38 | 11.4244 | 38.6145 | 1.83848 | 5.81378 | 1.50074 | 3.23325 | 6.96582 | .295858 |
| 3.39 | 11.4921 | 38.9582 | 1.84120 | 5.82237 | 1.50222 | 3.23643 | 6.97268 | .294985 |
| 3.40 | 11.5600 | 39.3040 | 1.84391 | 5.83095 | 1.50369 | 3.23961 | 6.97953 | .294118 |
| 3.41 | 11.6281 | 39.6518 | 1.84662 | 5.83952 | 1.50517 | 3.24278 | 6.98637 | .293255 |
| 3.42 | 11.6964 | 40.0017 | 1.84932 | 5.84808 | 1.50664 | 3.24595 | 6.99319 | .292398 |
| 3.43 | 11.7649 | 40.3536 | 1.85203 | 5.85662 | 1.50810 | 3.24911 | 7.00000 | .291545 |
| 3.44 | 11.8336 | 40.7076 | 1.85472 | 5.86515 | 1.50957 | 3.25227 | 7.00680 | .290698 |
| 3.45 | 11.9025 | 41.0636 | 1.85742 | 5.87367 | 1.51103 | 3.25542 | 7.01358 | .289855 |
| 3.46 | 11.9716 | 41.4217 | 1.86011 | 5.88218 | 1.51249 | 3.25856 | 7.02035 | .289017 |
| 3.47 | 12.0409 | 41.7819 | 1.86279 | 5.89067 | 1.51394 | 3.26169 | 7.02711 | .288184 |
| 3.48 | 12.1104 | 42.1442 | 1.86548 | 5.89915 | 1.51540 | 3.26482 | 7.03385 | .287356 |
| 3.49 | 12.1801 | 42.5085 | 1.86815 | 5.90762 | 1.51685 | 3.26795 | 7.04058 | .286533 |
| 3.50 | 12.2500 | 42.8750 | 1.87083 | 5.91608 | 1.51829 | 3.27107 | 7.04730 | .285714 |

| $n$ | $n^2$ | $n^3$ | $\sqrt{n}$ | $\sqrt{10n}$ | $\sqrt[3]{n}$ | $\sqrt[3]{10n}$ | $\sqrt[3]{100n}$ | $\frac{1}{n}$ |
|---|---|---|---|---|---|---|---|---|
| 3.51 | 12.3201 | 43.2436 | 1.87350 | 5.92453 | 1.51974 | 3.27418 | 7.05400 | .284900 |
| 3.52 | 12.3904 | 43.6142 | 1.87617 | 5.93296 | 1.52118 | 3.27729 | 7.06070 | .284091 |
| 3.53 | 12.4609 | 43.9870 | 1.87883 | 5.94138 | 1.52262 | 3.28039 | 7.06738 | .283286 |
| 3.54 | 12.5316 | 44.3619 | 1.88149 | 5.94979 | 1.52406 | 3.28348 | 7.07404 | .282486 |
| 3.55 | 12.6025 | 44.7389 | 1.88414 | 5.95819 | 1.52549 | 3.28657 | 7.08070 | .281690 |
| 3.56 | 12.6736 | 45.1180 | 1.88680 | 5.96657 | 1.52692 | 3.28965 | 7.08734 | .280899 |
| 3.57 | 12.7449 | 45.4993 | 1.88944 | 5.97495 | 1.52835 | 3.29273 | 7.09397 | .280112 |
| 3.58 | 12.8164 | 45.8827 | 1.89209 | 5.98331 | 1.52978 | 3.29580 | 7.10059 | .279330 |
| 3.59 | 12.8881 | 46.2683 | 1.89473 | 5.99166 | 1.53120 | 3.29887 | 7.10719 | .278552 |
| 3.60 | 12.9600 | 46.6560 | 1.89737 | 6.00000 | 1.53262 | 3.30193 | 7.11379 | .277778 |
| 3.61 | 13.0321 | 47.0459 | 1.90000 | 6.00833 | 1.53404 | 3.30498 | 7.12037 | .277008 |
| 3.62 | 13.1044 | 47.4379 | 1.90263 | 6.01664 | 1.53545 | 3.30803 | 7.12694 | .276243 |
| 3.63 | 13.1769 | 47.8321 | 1.90526 | 6.02495 | 1.53686 | 3.31107 | 7.13349 | .275482 |
| 3.64 | 13.2496 | 48.2285 | 1.90788 | 6.03324 | 1.53827 | 3.31411 | 7.14004 | .274725 |
| 3.65 | 13.3225 | 48.6271 | 1.91050 | 6.04152 | 1.53968 | 3.31714 | 7.14657 | .273973 |
| 3.66 | 13.3956 | 49.0279 | 1.91311 | 6.04979 | 1.54109 | 3.32017 | 7.15309 | .273224 |
| 3.67 | 13.4689 | 49.4309 | 1.91572 | 6.05805 | 1.54249 | 3.32319 | 7.15960 | .272480 |
| 3.68 | 13.5424 | 49.8360 | 1.91833 | 6.06630 | 1.54389 | 3.32621 | 7.16610 | .271739 |
| 3.69 | 13.6161 | 50.2434 | 1.92094 | 6.07454 | 1.54529 | 3.32922 | 7.17258 | .271003 |
| 3.70 | 13.6900 | 50.6530 | 1.92354 | 6.08276 | 1.54668 | 3.33222 | 7.17905 | .270270 |
| 3.71 | 13.7641 | 51.0648 | 1.92614 | 6.09098 | 1.54807 | 3.33522 | 7.18552 | .269542 |
| 3.72 | 13.8384 | 51.4788 | 1.92873 | 6.09918 | 1.54946 | 3.33822 | 7.19197 | .268817 |
| 3.73 | 13.9129 | 51.8951 | 1.93132 | 6.10737 | 1.55085 | 3.34120 | 7.19841 | .268097 |
| 3.74 | 13.9876 | 52.3136 | 1.93391 | 6.11555 | 1.55223 | 3.34419 | 7.20483 | .267380 |
| 3.75 | 14.0625 | 52.7344 | 1.93649 | 6.12372 | 1.55362 | 3.34716 | 7.21125 | .266667 |
| 3.76 | 14.1376 | 53.1574 | 1.93907 | 6.13188 | 1.55500 | 3.35014 | 7.21765 | .265957 |
| 3.77 | 14.2129 | 53.5826 | 1.94165 | 6.14003 | 1.55637 | 3.35310 | 7.22405 | .265252 |
| 3.78 | 14.2884 | 54.0102 | 1.94422 | 6.14817 | 1.55775 | 3.35607 | 7.23043 | .264550 |
| 3.79 | 14.3641 | 54.4399 | 1.94679 | 6.15630 | 1.55912 | 3.35902 | 7.23680 | .263852 |
| 3.80 | 14.4400 | 54.8720 | 1.94936 | 6.16441 | 1.56049 | 3.36198 | 7.24316 | .263158 |
| 3.81 | 14.5161 | 55.3063 | 1.95192 | 6.17252 | 1.56186 | 3.36492 | 7.24950 | .262467 |
| 3.82 | 14.5924 | 55.7430 | 1.95448 | 6.18061 | 1.56322 | 3.36786 | 7.25584 | .261780 |
| 3.83 | 14.6689 | 56.1819 | 1.95704 | 6.18870 | 1.56459 | 3.37080 | 7.26217 | .261097 |
| 3.84 | 14.7456 | 56.6231 | 1.95959 | 6.19677 | 1.56595 | 3.37373 | 7.26848 | .260417 |
| 3.85 | 14.8225 | 57.0666 | 1.96214 | 6.20484 | 1.56731 | 3.37666 | 7.27479 | .259740 |
| 3.86 | 14.8996 | 57.5125 | 1.96469 | 6.21289 | 1.56866 | 3.37958 | 7.28108 | .259067 |
| 3.87 | 14.9769 | 57.9606 | 1.96723 | 6.22093 | 1.57001 | 3.38249 | 7.28736 | .258398 |
| 3.88 | 15.0544 | 58.4111 | 1.96977 | 6.22896 | 1.57137 | 3.38540 | 7.29363 | .257732 |
| 3.89 | 15.1321 | 58.8639 | 1.97231 | 6.23699 | 1.57271 | 3.38831 | 7.29989 | .257069 |
| 3.90 | 15.2100 | 59.3190 | 1.97484 | 6.24500 | 1.57406 | 3.39121 | 7.30614 | .256410 |
| 3.91 | 15.2881 | 59.7765 | 1.97737 | 6.25300 | 1.57541 | 3.39411 | 7.31238 | .255755 |
| 3.92 | 15.3664 | 60.2363 | 1.97990 | 6.26099 | 1.57675 | 3.39700 | 7.31861 | .255102 |
| 3.93 | 15.4449 | 60.6985 | 1.98242 | 6.26897 | 1.57809 | 3.39988 | 7.32483 | .254453 |
| 3.94 | 15.5236 | 61.1630 | 1.98494 | 6.27694 | 1.57942 | 3.40277 | 7.33104 | .253807 |
| 3.95 | 15.6025 | 61.6299 | 1.98746 | 6.28490 | 1.58076 | 3.40564 | 7.33723 | .253165 |
| 3.96 | 15.6816 | 62.0991 | 1.98997 | 6.29285 | 1.58209 | 3.40851 | 7.34342 | .252525 |
| 3.97 | 15.7609 | 62.5708 | 1.99249 | 6.30079 | 1.58342 | 3.41138 | 7.34960 | .251889 |
| 3.98 | 15.8404 | 63.0448 | 1.99499 | 6.30872 | 1.58475 | 3.41424 | 7.35576 | .251256 |
| 3.99 | 15.9201 | 63.5212 | 1.99750 | 6.31664 | 1.58608 | 3.41710 | 7.36192 | .250627 |
| 4.00 | 16.0000 | 64.0000 | 2.00000 | 6.32456 | 1.58740 | 3.41995 | 7.36806 | .250000 |

| $n$ | $n^2$ | $n^3$ | $\sqrt{n}$ | $\sqrt{10n}$ | $\sqrt[3]{n}$ | $\sqrt[3]{10n}$ | $\sqrt[3]{100n}$ | $\frac{1}{n}$ |
|---|---|---|---|---|---|---|---|---|
| 4.01 | 16.0801 | 64.4812 | 2.00250 | 6.33246 | 1.58872 | 3.42280 | 7.37420 | .249377 |
| 4.02 | 16.1604 | 64.9648 | 2.00499 | 6.34035 | 1.59004 | 3.42564 | 7.38032 | .248756 |
| 4.03 | 16.2409 | 65.4508 | 2.00749 | 6.34823 | 1.59136 | 3.42848 | 7.38644 | .248139 |
| 4.04 | 16.3216 | 65.9393 | 2.00998 | 6.35610 | 1.59267 | 3.43131 | 7.39254 | .247525 |
| 4.05 | 16.4025 | 66.4301 | 2.01246 | 6.36396 | 1.59399 | 3.43414 | 7.39864 | .246914 |
| 4.06 | 16.4836 | 66.9234 | 2.01494 | 6.37181 | 1.59530 | 3.43697 | 7.40472 | .246305 |
| 4.07 | 16.5649 | 67.4191 | 2.01742 | 6.37966 | 1.59661 | 3.43979 | 7.41080 | .245700 |
| 4.08 | 16.6464 | 67.9173 | 2.01990 | 6.38749 | 1.59791 | 3.44260 | 7.41686 | .245098 |
| 4.09 | 16.7281 | 68.4179 | 2.02237 | 6.39531 | 1.59922 | 3.44541 | 7.42291 | .244499 |
| 4.10 | 16.8100 | 68.9210 | 2.02485 | 6.40312 | 1.60052 | 3.44822 | 7.42896 | .243902 |
| 4.11 | 16.8921 | 69.4265 | 2.02731 | 6.41093 | 1.60182 | 3.45102 | 7.43499 | .243309 |
| 4.12 | 16.9744 | 69.9345 | 2.02978 | 6.41872 | 1.60312 | 3.45382 | 7.44102 | .242718 |
| 4.13 | 17.0569 | 70.4450 | 2.03224 | 6.42651 | 1.60441 | 3.45661 | 7.44703 | .242131 |
| 4.14 | 17.1396 | 70.9579 | 2.03470 | 6.43428 | 1.60571 | 3.45939 | 7.45304 | .241546 |
| 4.15 | 17.2225 | 71.4734 | 2.03715 | 6.44205 | 1.60700 | 3.46218 | 7.45904 | .240964 |
| 4.16 | 17.3056 | 71.9913 | 2.03961 | 6.44981 | 1.60829 | 3.46496 | 7.46502 | .240385 |
| 4.17 | 17.3889 | 72.5117 | 2.04206 | 6.45755 | 1.60958 | 3.46773 | 7.47100 | .239808 |
| 4.18 | 17.4724 | 73.0346 | 2.04450 | 6.46529 | 1.61086 | 3.47050 | 7.47697 | .239234 |
| 4.19 | 17.5561 | 73.5601 | 2.04695 | 6.47302 | 1.61215 | 3.47327 | 7.48292 | .238664 |
| 4.20 | 17.6400 | 74.0880 | 2.04939 | 6.48074 | 1.61343 | 3.47603 | 7.48887 | .238095 |
| 4.21 | 17.7241 | 74.6185 | 2.05183 | 6.48845 | 1.61471 | 3.47878 | 7.49481 | .237530 |
| 4.22 | 17.8084 | 75.1514 | 2.05426 | 6.49615 | 1.61599 | 3.48154 | 7.50074 | .236967 |
| 4.23 | 17.8929 | 75.6870 | 2.05670 | 6.50385 | 1.61726 | 3.48428 | 7.50666 | .236407 |
| 4.24 | 17.9776 | 76.2250 | 2.05913 | 6.51153 | 1.61853 | 3.48703 | 7.51257 | .235849 |
| 4.25 | 18.0625 | 76.7656 | 2.06155 | 6.51920 | 1.61981 | 3.48977 | 7.51847 | .235294 |
| 4.26 | 18.1476 | 77.3088 | 2.06398 | 6.52687 | 1.62108 | 3.49250 | 7.52437 | .234742 |
| 4.27 | 18.2329 | 77.8545 | 2.06640 | 6.53452 | 1.62234 | 3.49523 | 7.53025 | .234192 |
| 4.28 | 18.3184 | 78.4028 | 2.06882 | 6.54217 | 1.62361 | 3.49796 | 7.53612 | .233645 |
| 4.29 | 18.4041 | 78.9536 | 2.07123 | 6.54981 | 1.62487 | 3.50068 | 7.54199 | .233100 |
| 4.30 | 18.4900 | 79.5070 | 2.07364 | 6.55744 | 1.62613 | 3.50340 | 7.54784 | .232558 |
| 4.31 | 18.5761 | 80.0630 | 2.07605 | 6.56506 | 1.62739 | 3.50611 | 7.55369 | .232019 |
| 4.32 | 18.6624 | 80.6216 | 2.07846 | 6.57267 | 1.62865 | 3.50882 | 7.55953 | .231482 |
| 4.33 | 18.7489 | 81.1827 | 2.08087 | 6.58027 | 1.62991 | 3.51153 | 7.56535 | .230947 |
| 4.34 | 18.8356 | 81.7465 | 2.08327 | 6.58787 | 1.63116 | 3.51423 | 7.57117 | .230415 |
| 4.35 | 18.9225 | 82.3129 | 2.08567 | 6.59545 | 1.63241 | 3.51692 | 7.57698 | .229885 |
| 4.36 | 19.0096 | 82.8819 | 2.08806 | 6.60303 | 1.63366 | 3.51962 | 7.58279 | .229358 |
| 4.37 | 19.0969 | 83.4535 | 2.09045 | 6.61060 | 1.63491 | 3.52231 | 7.58858 | .228833 |
| 4.38 | 19.1844 | 84.0277 | 2.09284 | 6.61816 | 1.63616 | 3.52499 | 7.59436 | .228311 |
| 4.39 | 19.2721 | 84.6045 | 2.09523 | 6.62571 | 1.63740 | 3.52767 | 7.60014 | .227790 |
| 4.40 | 19.3600 | 85.1840 | 2.09762 | 6.63325 | 1.63864 | 3.53035 | 7.60590 | .227273 |
| 4.41 | 19.4481 | 85.7661 | 2.10000 | 6.64078 | 1.63988 | 3.53302 | 7.61166 | .226757 |
| 4.42 | 19.5364 | 86.3509 | 2.10238 | 6.64831 | 1.64112 | 3.53569 | 7.61741 | .226244 |
| 4.43 | 19.6249 | 86.9383 | 2.10476 | 6.65582 | 1.64236 | 3.53835 | 7.62315 | .225734 |
| 4.44 | 19.7136 | 87.5284 | 2.10713 | 6.66333 | 1.64359 | 3.54101 | 7.62888 | .225225 |
| 4.45 | 19.8025 | 88.1211 | 2.10950 | 6.67083 | 1.64483 | 3.54367 | 7.63461 | .224719 |
| 4.46 | 19.8916 | 88.7165 | 2.11187 | 6.67832 | 1.64606 | 3.54632 | 7.64032 | .224215 |
| 4.47 | 19.9809 | 89.3146 | 2.11424 | 6.68581 | 1.64729 | 3.54897 | 7.64603 | .223714 |
| 4.48 | 20.0704 | 89.9154 | 2.11660 | 6.69328 | 1.64851 | 3.55162 | 7.65172 | .223214 |
| 4.49 | 20.1601 | 90.5188 | 2.11896 | 6.70075 | 1.64974 | 3.55426 | 7.65741 | .222717 |
| 4.50 | 20.2500 | 91.1250 | 2.12132 | 6.70820 | 1.65096 | 3.55689 | 7.66309 | .222222 |

| $n$ | $n^2$ | $n^3$ | $\sqrt{n}$ | $\sqrt{10n}$ | $\sqrt[3]{n}$ | $\sqrt[3]{10n}$ | $\sqrt[3]{100n}$ | $\frac{1}{n}$ |
|---|---|---|---|---|---|---|---|---|
| 4.51 | 20.3401 | 91.7339 | 2.12368 | 6.71565 | 1.65219 | 3.55953 | 7.66877 | .221730 |
| 4.52 | 20.4304 | 92.3454 | 2.12603 | 6.72309 | 1.65341 | 3.56215 | 7.67443 | .221239 |
| 4.53 | 20.5209 | 92.9597 | 2.12838 | 6.73053 | 1.65462 | 3.56478 | 7.68009 | .220751 |
| 4.54 | 20.6116 | 93.5767 | 2.13073 | 6.73795 | 1.65584 | 3.56740 | 7.68573 | .220264 |
| 4.55 | 20.7025 | 94.1964 | 2.13307 | 6.74537 | 1.65706 | 3.57002 | 7.69137 | .219780 |
| 4.56 | 20.7936 | 94.8188 | 2.13542 | 6.75278 | 1.65827 | 3.57263 | 7.69700 | .219298 |
| 4.57 | 20.8849 | 95.4440 | 2.13776 | 6.76018 | 1.65948 | 3.57524 | 7.70262 | .218818 |
| 4.58 | 20.9764 | 96.0719 | 2.14009 | 6.76757 | 1.66069 | 3.57785 | 7.70824 | .218341 |
| 4.59 | 21.0681 | 96.7026 | 2.14243 | 6.77495 | 1.66190 | 3.58045 | 7.71384 | .217865 |
| 4.60 | 21.1600 | 97.3360 | 2.14476 | 6.78233 | 1.66310 | 3.58305 | 7.71944 | .217391 |
| 4.61 | 21.2521 | 97.9722 | 2.14709 | 6.78970 | 1.66431 | 3.58564 | 7.72503 | .216920 |
| 4.62 | 21.3444 | 98.6111 | 2.14942 | 6.79706 | 1.66551 | 3.58823 | 7.73061 | .216450 |
| 4.63 | 21.4369 | 99.2528 | 2.15174 | 6.80441 | 1.66671 | 3.59082 | 7.73619 | .215983 |
| 4.64 | 21.5296 | 99.8973 | 2.15407 | 6.81175 | 1.66791 | 3.59340 | 7.74175 | .215517 |
| 4.65 | 21.6225 | 100.545 | 2.15639 | 6.81909 | 1.66911 | 3.59598 | 7.74731 | .215054 |
| 4.66 | 21.7156 | 101.195 | 2.15870 | 6.82642 | 1.67030 | 3.59856 | 7.75286 | .214592 |
| 4.67 | 21.8089 | 101.848 | 2.16102 | 6.83374 | 1.67150 | 3.60113 | 7.75840 | .214133 |
| 4.68 | 21.9024 | 102.503 | 2.16333 | 6.84105 | 1.67269 | 3.60370 | 7.76394 | .213675 |
| 4.69 | 21.9961 | 103.162 | 2.16564 | 6.84836 | 1.67388 | 3.60626 | 7.76946 | .213220 |
| 4.70 | 22.0900 | 103.823 | 2.16795 | 6.85565 | 1.67507 | 3.60883 | 7.77498 | .212766 |
| 4.71 | 22.1841 | 104.487 | 2.17025 | 6.86294 | 1.67626 | 3.61138 | 7.78049 | .212314 |
| 4.72 | 22.2784 | 105.154 | 2.17256 | 6.87023 | 1.67744 | 3.61394 | 7.78599 | .211864 |
| 4.73 | 22.3729 | 105.824 | 2.17486 | 6.87750 | 1.67863 | 3.61649 | 7.79149 | .211417 |
| 4.74 | 22.4676 | 106.496 | 2.17715 | 6.88477 | 1.67981 | 3.61904 | 7.79697 | .210971 |
| 4.75 | 22.5625 | 107.172 | 2.17945 | 6.89202 | 1.68099 | 3.62158 | 7.80245 | .210526 |
| 4.76 | 22.6576 | 107.850 | 2.18174 | 6.89928 | 1.68217 | 3.62412 | 7.80793 | .210084 |
| 4.77 | 22.7529 | 108.531 | 2.18403 | 6.90652 | 1.68334 | 3.62665 | 7.81339 | .209644 |
| 4.78 | 22.8484 | 109.215 | 2.18632 | 6.91375 | 1.68452 | 3.62919 | 7.81885 | .209205 |
| 4.79 | 22.9441 | 109.902 | 2.18861 | 6.92098 | 1.68569 | 3.63171 | 7.82429 | .208768 |
| 4.80 | 23.0400 | 110.592 | 2.19089 | 6.92820 | 1.68687 | 3.63424 | 7.82974 | .208333 |
| 4.81 | 23.1361 | 111.285 | 2.19317 | 6.93542 | 1.68804 | 3.63676 | 7.83517 | .207900 |
| 4.82 | 23.2324 | 111.980 | 2.19545 | 6.94262 | 1.68920 | 3.63928 | 7.84059 | .207469 |
| 4.83 | 23.3289 | 112.679 | 2.19773 | 6.94982 | 1.69037 | 3.64180 | 7.84601 | .207039 |
| 4.84 | 23.4256 | 113.380 | 2.20000 | 6.95701 | 1.69154 | 3.64431 | 7.85142 | .206612 |
| 4.85 | 23.5225 | 114.084 | 2.20227 | 6.96419 | 1.69270 | 3.64682 | 7.85683 | .206186 |
| 4.86 | 23.6196 | 114.791 | 2.20454 | 6.97137 | 1.69386 | 3.64932 | 7.86222 | .205761 |
| 4.87 | 23.7169 | 115.501 | 2.20681 | 6.97854 | 1.69503 | 3.65182 | 7.86761 | .205339 |
| 4.88 | 23.8144 | 116.214 | 2.20907 | 6.98570 | 1.69619 | 3.65432 | 7.87299 | .204918 |
| 4.89 | 23.9121 | 116.930 | 2.21133 | 6.99285 | 1.69734 | 3.65682 | 7.87837 | .204499 |
| 4.90 | 24.0100 | 117.649 | 2.21359 | 7.00000 | 1.69850 | 3.65931 | 7.88374 | .204082 |
| 4.91 | 24.1081 | 118.371 | 2.21585 | 7.00714 | 1.69965 | 3.66179 | 7.88909 | .203666 |
| 4.92 | 24.2064 | 119.095 | 2.21811 | 7.01427 | 1.70081 | 3 66428 | 7.89445 | .203252 |
| 4.93 | 24.3049 | 119.823 | 2.22036 | 7.02140 | 1.70196 | 3.66676 | 7.89979 | .202840 |
| 4.94 | 24.4036 | 120.554 | 2 22261 | 7.02851 | 1.70311 | 3.66924 | 7.90513 | .202429 |
| 4.95 | 24.5025 | 121.287 | 2.22486 | 7.03562 | 1.70426 | 3.67171 | 7.91046 | .202020 |
| 4.96 | 24.6016 | 122.024 | 2.22711 | 7.04273 | 1.70540 | 3.67418 | 7.91578 | .201613 |
| 4.97 | 24.7009 | 122.763 | 2.22935 | 7.04982 | 1.70655 | 3.67665 | 7.92110 | .201207 |
| 4.98 | 24.8004 | 123.506 | 2.23159 | 7.05691 | 1.70769 | 3.67911 | 7.92641 | .200803 |
| 4.99 | 24.9001 | 124.251 | 2.23383 | 7.06399 | 1.70884 | 3.68157 | 7.93171 | .200401 |
| 5.00 | 25.0000 | 125.000 | 2.23607 | 7.07107 | 1.70998 | 3.68403 | 7.93701 | .200000 |

| $n$ | $n^2$ | $n^3$ | $\sqrt{n}$ | $\sqrt{10\,n}$ | $\sqrt[3]{n}$ | $\sqrt[3]{10\,n}$ | $\sqrt[3]{100\,n}$ | $\frac{1}{n}$ |
|---|---|---|---|---|---|---|---|---|
| 5.01 | 25.1001 | 125.752 | 2.23830 | 7.07814 | 1.71112 | 3.68649 | 7.94229 | .199601 |
| 5.02 | 25.2004 | 126.506 | 2.24054 | 7.08520 | 1.71225 | 3.68894 | 7.94757 | .199203 |
| 5.03 | 25.3009 | 127.264 | 2.24277 | 7.09225 | 1.71339 | 3.69138 | 7.95285 | .198807 |
| 5.04 | 25.4016 | 128.024 | 2.24499 | 7.09930 | 1.71452 | 3.69383 | 7.95811 | .198413 |
| 5.05 | 25.5025 | 128.788 | 2.24722 | 7.10634 | 1.71566 | 3.69627 | 7.96337 | .198020 |
| 5.06 | 25.6036 | 129.554 | 2.24944 | 7.11337 | 1.71679 | 3.69871 | 7.96863 | .197629 |
| 5.07 | 25.7049 | 130.324 | 2.25167 | 7.12039 | 1.71792 | 3.70114 | 7.97387 | .197239 |
| 5.08 | 25.8064 | 131.097 | 2.25389 | 7.12741 | 1.71905 | 3.70358 | 7.97911 | .196850 |
| 5.09 | 25.9081 | 131.872 | 2.25610 | 7.13442 | 1.72017 | 3.70600 | 7.98434 | .196464 |
| 5.10 | 26.0100 | 132.651 | 2.25832 | 7.14143 | 1.72130 | 3.70843 | 7.98957 | .196078 |
| 5.11 | 26.1121 | 133.433 | 2.26053 | 7.14843 | 1.72242 | 3.71085 | 7.99479 | .195695 |
| 5.12 | 26.2144 | 134.218 | 2.26274 | 7.15542 | 1.72355 | 3.71327 | 8.00000 | .195313 |
| 5.13 | 26.3169 | 135.006 | 2.26495 | 7.16240 | 1.72467 | 3.71566 | 8.00520 | .194932 |
| 5.14 | 26.4196 | 135.797 | 2.26716 | 7.16938 | 1.72579 | 3.71810 | 8.01040 | .194553 |
| 5.15 | 26.5225 | 136.591 | 2.26936 | 7.17635 | 1.72691 | 3.72051 | 8.01559 | .194175 |
| 5.16 | 26.6256 | 137.388 | 2.27156 | 7.18331 | 1.72802 | 3.72292 | 8.02078 | .193798 |
| 5.17 | 26.7289 | 138.188 | 2.27376 | 7.19027 | 1.72914 | 3.72532 | 8.02596 | .193424 |
| 5.18 | 26.8324 | 138.992 | 2.27596 | 7.19722 | 1.73025 | 3.72772 | 8.03113 | .193050 |
| 5.19 | 26.9361 | 139.798 | 2.27816 | 7.20417 | 1.73137 | 3.73012 | 8.03629 | .192678 |
| 5.20 | 27.0400 | 140.608 | 2.28035 | 7.21110 | 1.73248 | 3.73251 | 8.04145 | .192308 |
| 5.21 | 27.1441 | 141.421 | 2.28254 | 7.21803 | 1.73359 | 3.73490 | 8.04660 | .191939 |
| 5.22 | 27.2484 | 142.237 | 2.28473 | 7.22496 | 1.73470 | 3.73729 | 8.05175 | .191571 |
| 5.23 | 27.3529 | 143.056 | 2.28692 | 7.23187 | 1.73580 | 3.73968 | 8.05689 | .191205 |
| 5.24 | 27.4576 | 143.878 | 2.28910 | 7.23878 | 1.73691 | 3.74206 | 8.06202 | .190840 |
| 5.25 | 27.5625 | 144.703 | 2.29129 | 7.24569 | 1.73801 | 3.74443 | 8.06714 | .190476 |
| 5.26 | 27.6676 | 145.532 | 2.29347 | 7.25259 | 1.73912 | 3.74681 | 8.07226 | .190114 |
| 5.27 | 27.7729 | 146.363 | 2.29565 | 7.25948 | 1.74022 | 3.74918 | 8.07737 | .189753 |
| 5.28 | 27.8784 | 147.198 | 2.29783 | 7.26636 | 1.74132 | 3.75158 | 8.08248 | .189394 |
| 5.29 | 27.9841 | 148.036 | 2.30000 | 7.27324 | 1.74242 | 3.75392 | 8.08758 | .189036 |
| 5.30 | 28.0900 | 148.877 | 2.30217 | 7.28011 | 1.74351 | 3.75629 | 8.09267 | .188679 |
| 5.31 | 28.1961 | 149.721 | 2.30434 | 7.28697 | 1.74461 | 3.75865 | 8.09776 | .188324 |
| 5.32 | 28.3024 | 150.569 | 2.30651 | 7.29383 | 1.74570 | 3.76100 | 8.10284 | .187970 |
| 5.33 | 28.4089 | 151.419 | 2.30868 | 7.30068 | 1.74680 | 3.76336 | 8.10791 | .187617 |
| 5.34 | 28.5156 | 152.273 | 2.31084 | 7.30753 | 1.74789 | 3.76571 | 8.11298 | .187266 |
| 5.35 | 28.6225 | 153.130 | 2.31301 | 7.31437 | 1.74898 | 3.76806 | 8.11804 | .186916 |
| 5.36 | 28.7296 | 153.991 | 2.31517 | 7.32120 | 1.75007 | 3.77041 | 8.12310 | .186567 |
| 5.37 | 28.8369 | 154.854 | 2.31733 | 7.32803 | 1.75116 | 3.77275 | 8.12814 | .186220 |
| 5.38 | 28.9444 | 155.721 | 2.31948 | 7.33485 | 1.75224 | 3.77509 | 8.13319 | .185874 |
| 5.39 | 29.0521 | 156.591 | 2.32164 | 7.34166 | 1.75333 | 3.77740 | 8.13822 | .185529 |
| 5.40 | 29.1600 | 157.464 | 2.32379 | 7.34847 | 1.75441 | 3.77976 | 8.14325 | .185185 |
| 5.41 | 29.2681 | 158.340 | 2.32594 | 7.35527 | 1.75549 | 3.78210 | 8.14828 | .184843 |
| 5.42 | 29.3764 | 159.220 | 2.32809 | 7.36206 | 1.75657 | 3.78442 | 8.15329 | .184502 |
| 5.43 | 29.4849 | 160.103 | 2.33024 | 7.36885 | 1.75765 | 3.78675 | 8.15831 | .184162 |
| 5.44 | 29.5936 | 160.989 | 2.33238 | 7.37564 | 1.75873 | 3.78907 | 8.16331 | .183824 |
| 5.45 | 29.7025 | 161.879 | 2.33452 | 7.38241 | 1.75981 | 3.79139 | 8.16831 | .183486 |
| 5.46 | 29.8116 | 162.771 | 2.33666 | 7.38918 | 1.76088 | 3.79371 | 8.17330 | .183150 |
| 5.47 | 29.9209 | 163.667 | 2.33880 | 7.39594 | 1.76196 | 3.79603 | 8.17829 | .182815 |
| 5.48 | 30.0304 | 164.567 | 2.34094 | 7.40270 | 1.76303 | 3.79834 | 8.18327 | .182482 |
| 5.49 | 30.1401 | 165.469 | 2.34307 | 7.40945 | 1.76410 | 3.80065 | 8.18824 | .182149 |
| 5.50 | 30.2500 | 166.375 | 2.34521 | 7.41620 | 1.76517 | 3.80295 | 8.19321 | .181818 |

| $n$ | $n^2$ | $n^3$ | $\sqrt{n}$ | $\sqrt{10n}$ | $\sqrt[3]{n}$ | $\sqrt[3]{10n}$ | $\sqrt[3]{100n}$ | $\frac{1}{n}$ |
|---|---|---|---|---|---|---|---|---|
| 5.51 | 30.3601 | 167.284 | 2.34734 | 7.42294 | 1.76624 | 3.80526 | 8.19818 | .181488 |
| 5.52 | 30.4704 | 168.197 | 2.34947 | 7.42967 | 1.76731 | 3.80756 | 8.20313 | .181159 |
| 5.53 | 30.5809 | 169.112 | 2.35160 | 7.43640 | 1.76838 | 3.80986 | 8.20808 | .180832 |
| 5.54 | 30.6916 | 170.031 | 2.35372 | 7.44312 | 1.76944 | 3.80115 | 8.21303 | .180505 |
| 5.55 | 30.8025 | 170.954 | 2.35584 | 7.44983 | 1.77051 | 3.81444 | 8.21797 | .180180 |
| 5.56 | 30.9136 | 171.880 | 2.35797 | 7.45654 | 1.77157 | 3.81673 | 8.22290 | .179856 |
| 5.57 | 31.0249 | 172.809 | 2.36008 | 7.46324 | 1.77263 | 3.81902 | 8.22783 | .179533 |
| 5.58 | 31.1364 | 173.741 | 2.36220 | 7.46994 | 1.77369 | 3.82130 | 8.23275 | .179212 |
| 5.59 | 31.2481 | 174.677 | 2.36432 | 7.47663 | 1.77475 | 3.82358 | 8.23766 | .178891 |
| 5.60 | 31.3600 | 175.616 | 2.36643 | 7.48331 | 1.77581 | 3.82586 | 8.24257 | .178571 |
| 5.61 | 31.4721 | 176.558 | 2.36854 | 7.48999 | 1.77686 | 3.82814 | 8.24747 | .178253 |
| 5.62 | 31.5844 | 177.504 | 2.37065 | 7.49667 | 1.77792 | 3.83041 | 8.25237 | .177936 |
| 5.63 | 31.6969 | 178.454 | 2.37276 | 7.50333 | 1.77897 | 3.83268 | 8.25726 | .177620 |
| 5.64 | 31.8096 | 179.406 | 2.37487 | 7.50999 | 1.78003 | 3.83495 | 8.26215 | .177305 |
| 5.65 | 31.9225 | 180.362 | 2.37697 | 7.51665 | 1.78108 | 3.83721 | 8.26703 | .176991 |
| 5.66 | 32.0356 | 181.321 | 2.37908 | 7.52330 | 1.78213 | 3.83948 | 8.27190 | .176678 |
| 5.67 | 32.1489 | 182.284 | 2.38118 | 7.52994 | 1.78318 | 3.84174 | 8.27677 | .176367 |
| 5.68 | 32.2624 | 183.250 | 2.38328 | 7.53658 | 1.78422 | 3.84400 | 8.28164 | .176056 |
| 5.69 | 32.3761 | 184.220 | 2.38537 | 7.54321 | 1.78527 | 3.84625 | 8.28649 | .175747 |
| 5.70 | 32.4900 | 185.193 | 2.38747 | 7.54983 | 1.78632 | 3.84850 | 8.29134 | .175439 |
| 5.71 | 32.6041 | 186.169 | 2.38956 | 7.55645 | 1.78736 | 3.85075 | 8.29619 | .175131 |
| 5.72 | 32.7184 | 187.149 | 2.39165 | 7.56307 | 1.78840 | 3.85300 | 8.30103 | .174825 |
| 5.73 | 32.8329 | 188.133 | 2.39374 | 7.56968 | 1.78944 | 3.85524 | 8.30587 | .174520 |
| 5.74 | 32.9476 | 189.119 | 2.39583 | 7.57628 | 1.79048 | 3.85748 | 8.31069 | .174216 |
| 5.75 | 33.0625 | 190.109 | 2.39792 | 7.58288 | 1.79152 | 3.85972 | 8.31552 | .173913 |
| 5.76 | 33.1776 | 191.103 | 2.40000 | 7.58947 | 1.79256 | 3.86196 | 8.32034 | .173611 |
| 5.77 | 33.2929 | 192.100 | 2.40208 | 7.59605 | 1.79360 | 3.86419 | 8.32515 | .173310 |
| 5.78 | 33.4084 | 193.101 | 2.40416 | 7.60263 | 1.79463 | 3.86642 | 8.32995 | .173010 |
| 5.79 | 33.5241 | 194.105 | 2.40624 | 7.60920 | 1.79567 | 3.86865 | 8.33476 | .172712 |
| 5.80 | 33.6400 | 195.112 | 2.40832 | 7.61577 | 1.79670 | 3.87088 | 8.33955 | .172414 |
| 5.81 | 33.7561 | 196.123 | 2.41039 | 7.62234 | 1.79773 | 3.87310 | 8.34434 | .172117 |
| 5.82 | 33.8724 | 197.137 | 2.41247 | 7.62889 | 1.79876 | 3.87532 | 8.34913 | .171821 |
| 5.83 | 33.9889 | 198.155 | 2.41454 | 7.63544 | 1.79979 | 3.87754 | 8.35390 | .171527 |
| 5.84 | 34.1056 | 199.177 | 2.41661 | 7.64199 | 1.80082 | 3.87975 | 8.35868 | .171233 |
| 5.85 | 34.2225 | 200.202 | 2.41868 | 7.64853 | 1.80185 | 3.88197 | 8.36345 | .170940 |
| 5.86 | 34.3396 | 201.230 | 2.42074 | 7.65506 | 1.80288 | 3.88418 | 8.36821 | .170649 |
| 5.87 | 34.4569 | 202.262 | 2.42281 | 7.66159 | 1.80390 | 3.88639 | 8.37297 | .170358 |
| 5.88 | 34.5744 | 203.297 | 2.42487 | 7.66812 | 1.80492 | 3.88859 | 8.37772 | .170068 |
| 5.89 | 34.6921 | 204.336 | 2.42693 | 7.67463 | 1.80595 | 3.89082 | 8.38247 | .169779 |
| 5.90 | 34.8100 | 205.379 | 2.42899 | 7.68115 | 1.80697 | 3.89300 | 8.38721 | .169492 |
| 5.91 | 34.9281 | 206.425 | 2.43105 | 7.68765 | 1.80799 | 3.89520 | 8.39194 | .169205 |
| 5.92 | 35.0464 | 207.475 | 2.43311 | 7.69415 | 1.80901 | 3.89739 | 8.39667 | .168919 |
| 5.93 | 35.1649 | 208.528 | 2.43516 | 7.70065 | 1.81003 | 3.89958 | 8.40140 | .168634 |
| 5.94 | 35.2836 | 209.585 | 2.43721 | 7.70714 | 1.81104 | 3.90177 | 8.40612 | .168350 |
| 5.95 | 35.4025 | 210.645 | 2.43926 | 7.71362 | 1.81206 | 3.90396 | 8.41083 | .168067 |
| 5.96 | 35.5216 | 211.709 | 2.44131 | 7.72010 | 1.81307 | 3.90615 | 8.41554 | .167785 |
| 5.97 | 35.6409 | 212.776 | 2.44336 | 7.72658 | 1.81409 | 3.90833 | 8.42025 | .167504 |
| 5.98 | 35.7604 | 213.847 | 2.44540 | 7.73305 | 1.81510 | 3.91051 | 8.42494 | .167224 |
| 5.99 | 35.8801 | 214.922 | 2.44745 | 7.73951 | 1.81611 | 3.91269 | 8.42964 | .166945 |
| 6.00 | 36.0000 | 216.000 | 2.44949 | 7.74597 | 1.81712 | 3.91487 | 8.43433 | .166667 |

| $n$ | $n^2$ | $n^3$ | $\sqrt{n}$ | $\sqrt{10n}$ | $\sqrt[3]{n}$ | $\sqrt[3]{10n}$ | $\sqrt[3]{100n}$ | $\frac{1}{n}$ |
|---|---|---|---|---|---|---|---|---|
| 6.01 | 36.1201 | 217.082 | 2.45153 | 7.75242 | 1.81813 | 3.91704 | 8.43901 | .166389 |
| 6.02 | 36.2404 | 218.167 | 2.45357 | 7.75887 | 1.81914 | 3.91921 | 8.44369 | .166113 |
| 6.03 | 36.3609 | 219.256 | 2.45561 | 7.76531 | 1.82014 | 3.92138 | 8.44836 | .165838 |
| 6.04 | 36.4816 | 220.349 | 2.45764 | 7.77174 | 1.82115 | 3.92355 | 8.45303 | .165563 |
| 6.05 | 36.6025 | 221.445 | 2.45967 | 7.77817 | 1.82215 | 3.92571 | 8.45769 | .165289 |
| 6.06 | 36.7236 | 222.545 | 2.46171 | 7.78460 | 1.82316 | 3.92787 | 8.46235 | .165017 |
| 6.07 | 36.8449 | 223.649 | 2.46374 | 7.79102 | 1.82416 | 3.93003 | 8.46700 | .164745 |
| 6.08 | 36.9664 | 224.756 | 2.46577 | 7.79744 | 1.82516 | 3.93219 | 8.47165 | .164474 |
| 6.09 | 37.0881 | 225.867 | 2.46779 | 7.80385 | 1.82616 | 3.93434 | 8.47629 | .164204 |
| 6.10 | 37.2100 | 226.981 | 2.46982 | 7.81025 | 1.82716 | 3.93650 | 8.48093 | .163934 |
| 6.11 | 37.3321 | 228.099 | 2.47184 | 7.81665 | 1.82816 | 3.93865 | 8.48556 | .163666 |
| 6.12 | 37.4544 | 229.221 | 2.47386 | 7.82304 | 1.82915 | 3.94079 | 8.49018 | .163399 |
| 6.13 | 37.5769 | 230.346 | 2.47588 | 7.82943 | 1.83015 | 3.94294 | 8.49481 | .163132 |
| 6.14 | 37.6996 | 231.476 | 2.47790 | 7.83582 | 1.83115 | 3.94508 | 8.49942 | .162866 |
| 6.15 | 37.8225 | 232.608 | 2.47992 | 7.84219 | 1.83214 | 3.94722 | 8.50404 | .162602 |
| 6.16 | 37.9456 | 233.745 | 2.48193 | 7.84857 | 1.83313 | 3.94936 | 8.50864 | .162338 |
| 6.17 | 38.0689 | 234.885 | 2.48395 | 7.85493 | 1.83412 | 3.95150 | 8.51324 | .162075 |
| 6.18 | 38.1924 | 236.029 | 2.48596 | 7.86130 | 1.83511 | 3.95363 | 8.51784 | .161812 |
| 6.19 | 38.3161 | 237.177 | 2.48797 | 7.86766 | 1.83610 | 3.95576 | 8.52243 | .161551 |
| 6.20 | 38.4400 | 238.328 | 2.48998 | 7.87401 | 1.83709 | 3.95789 | 8.52702 | .161290 |
| 6.21 | 38.5641 | 239.483 | 2.49199 | 7.88036 | 1.83808 | 3.96002 | 8.53160 | .161031 |
| 6.22 | 38.6884 | 240.642 | 2.49399 | 7.88670 | 1.83906 | 3.96214 | 8.53618 | .160772 |
| 6.23 | 38.8129 | 241.804 | 2.49600 | 7.89303 | 1.84005 | 3.96426 | 8.54075 | .160514 |
| 6.24 | 38.9376 | 242.971 | 2.49800 | 7.89937 | 1.84103 | 3.96639 | 8.54532 | .160256 |
| 6.25 | 39.0625 | 244.141 | 2.50000 | 7.90569 | 1.84202 | 3.96850 | 8.54988 | .160000 |
| 6.26 | 39.1876 | 245.314 | 2.50200 | 7.91202 | 1.84300 | 3.97062 | 8.55444 | .159744 |
| 6.27 | 39.3129 | 246.492 | 2.50400 | 7.91833 | 1.84398 | 3.97273 | 8.55899 | .159490 |
| 6.28 | 39.4384 | 247.673 | 2.50599 | 7.92465 | 1.84496 | 3.97484 | 8.56354 | .159236 |
| 6.29 | 39.5641 | 248.858 | 2.50799 | 7.93095 | 1.84594 | 3.97695 | 8.56808 | .158983 |
| 6.30 | 39.6900 | 250.047 | 2.50998 | 7.93725 | 1.84691 | 3.97906 | 8.57262 | .158730 |
| 6.31 | 39.8161 | 251.240 | 2.51197 | 7.94355 | 1.84789 | 3.98116 | 8.57715 | .158479 |
| 6.32 | 39.9424 | 252.436 | 2.51396 | 7.94984 | 1.84887 | 3.98326 | 8.58168 | .158228 |
| 6.33 | 40.0689 | 253.636 | 2.51595 | 7.95613 | 1.84984 | 3.98536 | 8.58620 | .157978 |
| 6.34 | 40.1956 | 254.840 | 2.51794 | 7.96241 | 1.85082 | 3.98746 | 8.59072 | .157729 |
| 6.35 | 40.3225 | 256.048 | 2.51992 | 7.96869 | 1.85179 | 3.98956 | 8.59524 | .157480 |
| 6.36 | 40.4496 | 257.259 | 2.52190 | 7.97496 | 1.85276 | 3.99165 | 8.59975 | .157233 |
| 6.37 | 40.5769 | 258.475 | 2.52389 | 7.98123 | 1.85373 | 3.99374 | 8.60425 | .156986 |
| 6.38 | 40.7044 | 259.694 | 2.52587 | 7.98749 | 1.85470 | 3.99583 | 8.60875 | .156740 |
| 6.39 | 40.8321 | 260.917 | 2.52784 | 7.99375 | 1.85567 | 3.99792 | 8.61325 | .156495 |
| 6.40 | 40.9600 | 262.144 | 2.52982 | 8.00000 | 1.85664 | 4.00000 | 8.61774 | .156250 |
| 6.41 | 41.0881 | 263.375 | 2.53180 | 8.00625 | 1.85760 | 4.00208 | 8.62222 | .156006 |
| 6.42 | 41.2164 | 264.609 | 2.53377 | 8.01249 | 1.85857 | 4.00416 | 8.62671 | .155763 |
| 6.43 | 41.3449 | 265.848 | 2.53574 | 8.01873 | 1.85953 | 4.00624 | 8.63118 | .155521 |
| 6.44 | 41.4736 | 267.090 | 2.53772 | 8.02496 | 1.86050 | 4.00832 | 8.63566 | .155280 |
| 6.45 | 41.6025 | 268.336 | 2.53969 | 8.03119 | 1.86146 | 4.01039 | 8.64012 | .155039 |
| 6.46 | 41.7316 | 269.586 | 2.54165 | 8.03741 | 1.86242 | 4.01246 | 8.64459 | .154799 |
| 6.47 | 41.8609 | 270.840 | 2.54362 | 8.04363 | 1.86338 | 4.01453 | 8.64904 | .154560 |
| 6.48 | 41.9904 | 272.098 | 2.54558 | 8.04984 | 1.86434 | 4.01660 | 8.65350 | .154321 |
| 6.49 | 42.1201 | 273.359 | 2.54755 | 8.05605 | 1.86530 | 4.01866 | 8.65795 | .154083 |
| 6.50 | 42.2500 | 274.625 | 2.54951 | 8.06226 | 1.86626 | 4.02073 | 8.66239 | .153846 |

| $n$ | $n^2$ | $n^3$ | $\sqrt{n}$ | $\sqrt{10\,n}$ | $\sqrt[3]{n}$ | $\sqrt[3]{10\,n}$ | $\sqrt[3]{100\,n}$ | $\frac{1}{n}$ |
|---|---|---|---|---|---|---|---|---|
| 6.51 | 42.3801 | 275.894 | 2.55147 | 8.06846 | 1.86721 | 4.02279 | 8.66683 | .153610 |
| 6.52 | 42.5104 | 277.168 | 2.55343 | 8.07465 | 1.86817 | 4.02485 | 8.67127 | .153374 |
| 6.53 | 42.6409 | 278.445 | 2.55539 | 8.08084 | 1.86912 | 4.02690 | 8.67570 | .153139 |
| 6.54 | 42.7716 | 279.726 | 2.55734 | 8.08703 | 1.87008 | 4.02896 | 8.68012 | .152905 |
| 6.55 | 42.9025 | 281.011 | 2.55930 | 8.09321 | 1.87103 | 4.03101 | 8.68455 | .152672 |
| 6.56 | 43.0336 | 282.300 | 2.56125 | 8.09938 | 1.87198 | 4.03306 | 8.68896 | .152439 |
| 6.57 | 43.1649 | 283.593 | 2.56320 | 8.10555 | 1.87293 | 4.03511 | 8.69338 | .152207 |
| 6.58 | 43.2964 | 284.890 | 2.56515 | 8.11172 | 1.87388 | 4.03715 | 8.69778 | .151976 |
| 6.59 | 43.4281 | 286.191 | 2.56710 | 8.11788 | 1.87483 | 4.03920 | 8.70219 | .151745 |
| 6.60 | 43.5600 | 287.496 | 2.56905 | 8.12404 | 1.87578 | 4.04124 | 8.70659 | .151515 |
| 6.61 | 43.6921 | 288.805 | 2.57099 | 8.13019 | 1.87672 | 4.04328 | 8.71098 | .151286 |
| 6.62 | 43.8244 | 290.118 | 2.57294 | 8.13634 | 1.87767 | 4.04532 | 8.71537 | .151057 |
| 6.63 | 43.9569 | 291.434 | 2.57488 | 8.14248 | 1.87862 | 4.04735 | 8.71976 | .150830 |
| 6.64 | 44.0896 | 292.755 | 2.57682 | 8.14862 | 1.87956 | 4.04939 | 8.72414 | .150602 |
| 6.65 | 44.2225 | 294.080 | 2.57876 | 8.15475 | 1.88050 | 4.05142 | 8.72852 | .150376 |
| 6.66 | 44.3556 | 295.408 | 2.58070 | 8.16088 | 1.88144 | 4.05345 | 8.73289 | .150150 |
| 6.67 | 44.4889 | 296.741 | 2.58263 | 8.16701 | 1.88239 | 4.05548 | 8.73726 | .149925 |
| 6.68 | 44.6224 | 298.078 | 2.58457 | 8.17313 | 1.88333 | 4.05750 | 8.74162 | .149701 |
| 6.69 | 44.7561 | 299.418 | 2.58650 | 8.17924 | 1.88427 | 4.05953 | 8.74598 | .149477 |
| 6.70 | 44.8900 | 300.763 | 2.58844 | 8.18535 | 1.88520 | 4.06155 | 8.75034 | .149254 |
| 6.71 | 45.0241 | 302.112 | 2.59037 | 8.19146 | 1.88614 | 4.06357 | 8.75469 | .149031 |
| 6.72 | 45.1584 | 303.464 | 2.59230 | 8.19756 | 1.88708 | 4.06558 | 8.75904 | .148810 |
| 6.73 | 45.2929 | 304.821 | 2.59422 | 8.20366 | 1.88801 | 4.06760 | 8.76338 | .148588 |
| 6.74 | 45.4276 | 306.182 | 2.59615 | 8.20975 | 1.88895 | 4.06961 | 8.76772 | .148368 |
| 6.75 | 45.5625 | 307.547 | 2.59808 | 8.21584 | 1.88988 | 4.07163 | 8.77205 | .148148 |
| 6.76 | 45.6976 | 308.916 | 2.60000 | 8.22192 | 1.89081 | 4.07364 | 8.77638 | .147929 |
| 6.77 | 45.8329 | 310.289 | 2.60192 | 8.22800 | 1.89175 | 4.07564 | 8.78071 | .147711 |
| 6.78 | 45.9684 | 311.666 | 2.60384 | 8.23408 | 1.89268 | 4.07765 | 8.78503 | .147493 |
| 6.79 | 46.1041 | 313.047 | 2.60576 | 8.24015 | 1.89361 | 4.07965 | 8.78935 | .147275 |
| 6.80 | 46.2400 | 314.432 | 2.60768 | 8.24621 | 1.89454 | 4.08166 | 8.79366 | .147059 |
| 6.81 | 46.3761 | 315.821 | 2.60960 | 8.25227 | 1.89546 | 4.08365 | 8.79797 | .146843 |
| 6.82 | 46.5124 | 317.215 | 2.61151 | 8.25833 | 1.89639 | 4.08565 | 8.80227 | .146628 |
| 6.83 | 46.6489 | 318.612 | 2.61343 | 8.26438 | 1.89732 | 4.08765 | 8.80657 | .146413 |
| 6.84 | 46.7856 | 320.014 | 2.61534 | 8.27043 | 1.89824 | 4.08964 | 8.81087 | .146199 |
| 6.85 | 46.9225 | 321.419 | 2.61725 | 8.27647 | 1.89917 | 4.09164 | 8.81516 | .145985 |
| 6.86 | 47.0596 | 322.829 | 2.61916 | 8.28251 | 1.90009 | 4.09362 | 8.81945 | .145773 |
| 6.87 | 47.1969 | 324.243 | 2.62107 | 8.28855 | 1.90102 | 4.09561 | 8.82373 | .145560 |
| 6.88 | 47.3344 | 325.661 | 2.62298 | 8.29458 | 1.90194 | 4.09760 | 8.82801 | .145349 |
| 6.89 | 47.4721 | 327.083 | 2.62488 | 8.30060 | 1.90286 | 4.09958 | 8.83229 | .145138 |
| 6.90 | 47.6100 | 328.509 | 2.62679 | 8.30662 | 1.90378 | 4.10157 | 8.83656 | .144928 |
| 6.91 | 47.7481 | 329.939 | 2.62869 | 8.31264 | 1.90470 | 4.10355 | 8.84082 | .144718 |
| 6.92 | 47.8864 | 331.374 | 2.63059 | 8.31865 | 1.90562 | 4.10552 | 8.84509 | .144509 |
| 6.93 | 48.0249 | 332.813 | 2.63249 | 8.32466 | 1.90653 | 4.10750 | 8.84934 | .144300 |
| 6.94 | 48.1636 | 334.255 | 2.63439 | 8.33067 | 1.90745 | 4.10948 | 8.85360 | .144092 |
| 6.95 | 48.3025 | 335.702 | 2.63629 | 8.33667 | 1.90837 | 4.11145 | 8.85785 | .143885 |
| 6.96 | 48.4416 | 337.154 | 2.63818 | 8.34266 | 1.90928 | 4.11342 | 8.86210 | .143678 |
| 6.97 | 48.5809 | 338.609 | 2.64008 | 8.34865 | 1.91019 | 4.11539 | 8.86634 | .143472 |
| 6.98 | 48.7204 | 340.068 | 2.64197 | 8.35464 | 1.91111 | 4.11736 | 8.87058 | .143267 |
| 6.99 | 48.8601 | 341.532 | 2.64386 | 8.36062 | 1.91202 | 4.11932 | 8.87481 | .143062 |
| 7.00 | 49.0000 | 343.000 | 2.64575 | 8.36660 | 1.91293 | 4.12129 | 8.87904 | .142857 |

| $n$ | $n^2$ | $n^3$ | $\sqrt{n}$ | $\sqrt{10\,n}$ | $\sqrt[3]{n}$ | $\sqrt[3]{10\,n}$ | $\sqrt[3]{100\,n}$ | $\frac{1}{n}$ |
|---|---|---|---|---|---|---|---|---|
| 7.01 | 49.1401 | 344.472 | 2.64764 | 8.37257 | 1.91384 | 4.12325 | 8.88327 | .142653 |
| 7.02 | 49.2804 | 345.948 | 2.64953 | 8.37854 | 1.91475 | 4.12521 | 8.88749 | .142450 |
| 7.03 | 49.4209 | 347.429 | 2.65141 | 8.38451 | 1.91566 | 4.12716 | 8.89171 | .142248 |
| 7.04 | 49.5616 | 348.914 | 2.65330 | 8.39047 | 1.91657 | 4.12912 | 8.89592 | .142046 |
| 7.05 | 49.7025 | 350.403 | 2.65518 | 8.39643 | 1.91747 | 4.13107 | 8.90013 | .141844 |
| 7.06 | 49.8436 | 351.896 | 2.65707 | 8.40238 | 1.91838 | 4.13303 | 8.90434 | .141643 |
| 7.07 | 49.9849 | 353.393 | 2.65895 | 8.40833 | 1.91929 | 4.13498 | 8.90854 | .141443 |
| 7.08 | 50.1264 | 354.895 | 2.66083 | 8.41427 | 1.92019 | 4.13693 | 8.91274 | .141243 |
| 7.09 | 50.2681 | 356.401 | 2.66271 | 8.42021 | 1.92109 | 4.13887 | 8.91693 | .141044 |
| 7.10 | 50.4100 | 357.911 | 2.66458 | 8.42615 | 1.92200 | 4.14082 | 8.92112 | .140845 |
| 7.11 | 50.5521 | 359.425 | 2.66646 | 8.43208 | 1.92290 | 4.14276 | 8.92531 | .140647 |
| 7.12 | 50.6944 | 360.944 | 2.66833 | 8.43801 | 1.92380 | 4.14470 | 8.92949 | .140449 |
| 7.13 | 50.8369 | 362.467 | 2.67021 | 8.44393 | 1.92470 | 4.14664 | 8.93367 | .140253 |
| 7.14 | 50.9796 | 363.994 | 2.67208 | 8.44985 | 1.92560 | 4.14858 | 8.93784 | .140056 |
| 7.15 | 51.1225 | 365.526 | 2.67395 | 8.45577 | 1.92650 | 4.15051 | 8.94201 | .139860 |
| 7.16 | 51.2656 | 367.062 | 2.67582 | 8.46168 | 1.92740 | 4.15245 | 8.94618 | .139665 |
| 7.17 | 51.4089 | 368.602 | 2.67769 | 8.46759 | 1.92829 | 4.15438 | 8.95034 | .139470 |
| 7.18 | 51.5524 | 370.146 | 2.67955 | 8.47349 | 1.92919 | 4.15631 | 8.95450 | .139276 |
| 7.19 | 51.6961 | 371.695 | 2.68142 | 8.47939 | 1.93008 | 4.15824 | 8.95866 | .139082 |
| 7.20 | 51.8400 | 373.248 | 2.68328 | 8.48528 | 1.93098 | 4.16017 | 8.96281 | .138889 |
| 7.21 | 51.9841 | 374.805 | 2.68514 | 8.49117 | 1.93187 | 4.16209 | 8.96696 | .138696 |
| 7.22 | 52.1284 | 376.367 | 2.68701 | 8.49706 | 1.93277 | 4.16402 | 8.97110 | .138504 |
| 7.23 | 52.2729 | 377.933 | 2.68887 | 8.50294 | 1.93366 | 4.16594 | 8.97524 | .138313 |
| 7.24 | 52.4176 | 379.503 | 2.69072 | 8.50882 | 1.93455 | 4.16786 | 8.97938 | .138122 |
| 7.25 | 52.5625 | 381.078 | 2.69258 | 8.51469 | 1.93544 | 4.16978 | 8.98351 | .137931 |
| 7.26 | 52.7076 | 382.657 | 2.69444 | 8.52056 | 1.93633 | 4.17169 | 8.98764 | .137741 |
| 7.27 | 52.8529 | 384.241 | 2.69629 | 8.52643 | 1.93722 | 4.17361 | 8.99176 | .137552 |
| 7.28 | 52.9984 | 385.828 | 2.69815 | 8.53229 | 1.93810 | 4.17552 | 8.99588 | .137363 |
| 7.29 | 53.1441 | 387.420 | 2.70000 | 8.53815 | 1.93899 | 4.17743 | 9.00000 | .137174 |
| 7.30 | 53.2900 | 389.017 | 2.70185 | 8.54400 | 1.93988 | 4.17934 | 9.00411 | .136986 |
| 7.31 | 53.4361 | 390.618 | 2.70370 | 8.54985 | 1.94076 | 4.18125 | 9.00822 | .136799 |
| 7.32 | 53.5824 | 392.223 | 2.70555 | 8.55570 | 1.94165 | 4.18315 | 9.01233 | .136612 |
| 7.33 | 53.7289 | 393.833 | 2.70740 | 8.56154 | 1.94253 | 4.18506 | 9.01643 | .136426 |
| 7.34 | 53.8756 | 395.447 | 2.70924 | 8.56738 | 1.94341 | 4.18696 | 9.02053 | .136240 |
| 7.35 | 54.0225 | 397.065 | 2.71109 | 8.57321 | 1.94430 | 4.18886 | 9.02462 | .136054 |
| 7.36 | 54.1696 | 398.688 | 2.71293 | 8.57904 | 1.94518 | 4.19076 | 9.02871 | .135870 |
| 7.37 | 54.3169 | 400.316 | 2.71477 | 8.58487 | 1.94606 | 4.19266 | 9.03280 | .135685 |
| 7.38 | 54.4644 | 401.947 | 2.71662 | 8.59069 | 1.94694 | 4.19455 | 9.03689 | .135501 |
| 7.39 | 54.6121 | 403.583 | 2.71846 | 8.59651 | 1.94782 | 4.19644 | 9.04097 | .135318 |
| 7.40 | 54.7600 | 405.224 | 2.72029 | 8.60233 | 1.94870 | 4.19834 | 9.04504 | .135135 |
| 7.41 | 54.9081 | 406.869 | 2.72213 | 8.60814 | 1.94957 | 4.20023 | 9.04911 | .134953 |
| 7.42 | 55.0564 | 408.518 | 2.72397 | 8.61394 | 1.95045 | 4.20212 | 9.05318 | .134771 |
| 7.43 | 55.2049 | 410.172 | 2.72580 | 8.61974 | 1.95132 | 4.20400 | 9.05725 | .134590 |
| 7.44 | 55.3536 | 411.831 | 2.72764 | 8.62554 | 1.95220 | 4.20589 | 9.06131 | .134409 |
| 7.45 | 55.5025 | 413.494 | 2.72947 | 8.63134 | 1.95307 | 4.20777 | 9.06537 | .134228 |
| 7.46 | 55.6516 | 415.161 | 2.73130 | 8.63713 | 1.95395 | 4.20965 | 9.06942 | .134048 |
| 7.47 | 55.8009 | 416.833 | 2.73313 | 8.64292 | 1.95482 | 4.21153 | 9.07347 | .133869 |
| 7.48 | 55.9504 | 418.509 | 2.73496 | 8.64870 | 1.95569 | 4.21341 | 9.07752 | .133690 |
| 7.49 | 56.1001 | 420.190 | 2.73679 | 8.65448 | 1.95656 | 4.21529 | 9.08156 | .133511 |
| 7.50 | 56.2500 | 421.875 | 2.73861 | 8.66025 | 1.95743 | 4.21716 | 9.08560 | .133333 |

| $n$ | $n^2$ | $n^3$ | $\sqrt{n}$ | $\sqrt{10n}$ | $\sqrt[3]{n}$ | $\sqrt[3]{10n}$ | $\sqrt[3]{100n}$ | $\frac{1}{n}$ |
|---|---|---|---|---|---|---|---|---|
| 7.51 | 56.4001 | 423.565 | 2.74044 | 8.66603 | 1.95830 | 4.21904 | 9.08964 | .133156 |
| 7.52 | 56.5504 | 425.259 | 2.74226 | 8.67179 | 1.95917 | 4.22091 | 9.09367 | .132979 |
| 7.53 | 56.7009 | 426.958 | 2.74408 | 8.67756 | 1.96004 | 4.22278 | 9.09770 | .132802 |
| 7.54 | 56.8516 | 428.661 | 2.74591 | 8.68332 | 1.96091 | 4.22465 | 9.10173 | .132626 |
| 7.55 | 57.0025 | 430.369 | 2.74773 | 8.68907 | 1.96177 | 4.22651 | 9.10575 | .132450 |
| 7.56 | 57.1536 | 432.081 | 2.74955 | 8.69483 | 1.96264 | 4.22838 | 9.10977 | .132275 |
| 7.57 | 57.3049 | 433.798 | 2.75136 | 8.70057 | 1.96350 | 4.23024 | 9.11378 | .132100 |
| 7.58 | 57.4564 | 435.520 | 2.75318 | 8.70632 | 1.96437 | 4.23210 | 9.11779 | .131926 |
| 7.59 | 57.6081 | 437.245 | 2.75500 | 8.71206 | 1.96523 | 4.23396 | 9.12180 | .131752 |
| 7.60 | 57.7600 | 438.976 | 2.75681 | 8.71780 | 1.96610 | 4.23582 | 9.12581 | .131579 |
| 7.61 | 57.9121 | 440.711 | 2.75862 | 8.72353 | 1.96696 | 4.23768 | 9.12981 | .131406 |
| 7.62 | 58.0644 | 442.451 | 2.76043 | 8.72926 | 1.96782 | 4.23954 | 9.13380 | .131234 |
| 7.63 | 58.2169 | 444.195 | 2.76225 | 8.73499 | 1.96868 | 4.24139 | 9.13780 | .131062 |
| 7.64 | 58.3696 | 445.944 | 2.76405 | 8.74071 | 1.96954 | 4.24324 | 9.14179 | .130890 |
| 7.65 | 58.5225 | 447.697 | 2.76586 | 8.74643 | 1.97040 | 4.24509 | 9.14577 | .130719 |
| 7.66 | 58.6756 | 449.455 | 2.76767 | 8.75214 | 1.97126 | 4.24694 | 9.14976 | .130548 |
| 7.67 | 58.8289 | 451.218 | 2.76948 | 8.75785 | 1.97211 | 4.24879 | 9.15374 | .130378 |
| 7.68 | 58.9824 | 452.985 | 2.77128 | 8.76356 | 1.97297 | 4.25063 | 9.15771 | .130208 |
| 7.69 | 59.1361 | 454.757 | 2.77308 | 8.76926 | 1.97383 | 4.25248 | 9.16169 | .130039 |
| 7.70 | 59.2900 | 456.533 | 2.77489 | 8.77496 | 1.97468 | 4.25432 | 9.16566 | .129870 |
| 7.71 | 59.4441 | 458.314 | 2.77669 | 8.78066 | 1.97554 | 4.25616 | 9.16962 | .129702 |
| 7.72 | 59.5984 | 460.100 | 2.77849 | 8.78635 | 1.97639 | 4.25800 | 9.17359 | .129534 |
| 7.73 | 59.7529 | 461.890 | 2.78029 | 8.79204 | 1.97724 | 4.25984 | 9.17754 | .129366 |
| 7.74 | 59.9076 | 463.685 | 2.78209 | 8.79773 | 1.97809 | 4.26168 | 9.18150 | .129199 |
| 7.75 | 60.0625 | 465.484 | 2.78388 | 8.80341 | 1.97895 | 4.26351 | 9.18545 | .129032 |
| 7.76 | 60.2176 | 467.289 | 2.78568 | 8.80909 | 1.97980 | 4.26534 | 9.18940 | .128866 |
| 7.77 | 60.3729 | 469.097 | 2.78747 | 8.81476 | 1.98065 | 4.26717 | 9.19335 | .128700 |
| 7.78 | 60.5284 | 470.911 | 2.78927 | 8.82043 | 1.98150 | 4.26900 | 9.19729 | .128535 |
| 7.79 | 60.6841 | 472.729 | 2.79106 | 8.82610 | 1.98234 | 4.27083 | 9.20123 | .128370 |
| 7.80 | 60.8400 | 474.552 | 2.79285 | 8.83176 | 1.98319 | 4.27266 | 9.20516 | .128205 |
| 7.81 | 60.9961 | 476.380 | 2.79464 | 8.83742 | 1.98404 | 4.27448 | 9.20910 | .128041 |
| 7.82 | 61.1524 | 478.212 | 2.79643 | 8.84308 | 1.98489 | 4.27631 | 9.21303 | .127877 |
| 7.83 | 61.3089 | 480.049 | 2.79821 | 8.84873 | 1.98573 | 4.27813 | 9.21695 | .127714 |
| 7.84 | 61.4656 | 481.890 | 2.80000 | 8.85438 | 1.98658 | 4.27995 | 9.22087 | .127551 |
| 7.85 | 61.6225 | 483.737 | 2.80179 | 8.86002 | 1.98742 | 4.28177 | 9.22479 | .127389 |
| 7.86 | 61.7796 | 485.588 | 2.80357 | 8.86566 | 1.98826 | 4.28359 | 9.22871 | .127227 |
| 7.87 | 61.9369 | 487.443 | 2.80535 | 8.87130 | 1.98911 | 4.28540 | 9.23262 | .127065 |
| 7.88 | 62.0944 | 489.304 | 2.80713 | 8.87694 | 1.98995 | 4.28722 | 9.23653 | .126904 |
| 7.89 | 62.2521 | 491.169 | 2.80891 | 8.88257 | 1.99079 | 4.28903 | 9.24043 | .126743 |
| 7.90 | 62.4100 | 493.039 | 2.81069 | 8.88819 | 1.99163 | 4.29084 | 9.24433 | .126582 |
| 7.91 | 62.5681 | 494.914 | 2.81247 | 8.89382 | 1.99247 | 4.29265 | 9.24823 | .126422 |
| 7.92 | 62.7264 | 496.793 | 2.81425 | 8.89944 | 1.99331 | 4.29446 | 9.25213 | .126263 |
| 7.93 | 62.8849 | 498.677 | 2.81603 | 8.90505 | 1.99415 | 4.29627 | 9.25602 | .126103 |
| 7.94 | 63.0436 | 500.566 | 2.81780 | 8.91067 | 1.99499 | 4.29807 | 9.25991 | .125945 |
| 7.95 | 63.2025 | 502.460 | 2.81957 | 8.91628 | 1.99582 | 4.29987 | 9.26380 | .125786 |
| 7.96 | 63.3616 | 504.358 | 2.82135 | 8.92188 | 1.99666 | 4.30168 | 9.26768 | .125628 |
| 7.97 | 63.5209 | 506.262 | 2.82312 | 8.92749 | 1.99750 | 4.30348 | 9.27156 | .125471 |
| 7.98 | 63.6804 | 508.170 | 2.82489 | 8.93308 | 1.99833 | 4.30528 | 9.27544 | .125313 |
| 7.99 | 63.8401 | 510.082 | 2.82666 | 8.93868 | 1.99917 | 4.30707 | 9.27931 | .125156 |
| 8.00 | 64.0000 | 512.000 | 2.82843 | 8.94427 | 2.00000 | 4.30887 | 9.28318 | .125000 |

| $n$ | $n^2$ | $n^3$ | $\sqrt{n}$ | $\sqrt{10\,n}$ | $\sqrt[3]{n}$ | $\sqrt[3]{10\,n}$ | $\sqrt[3]{100\,n}$ | $\frac{1}{n}$ |
|---|---|---|---|---|---|---|---|---|
| 8.01 | 64.1601 | 513.922 | 2.83019 | 8.94986 | 2.00083 | 4.31066 | 9.28704 | .124844 |
| 8.02 | 64.3204 | 515.850 | 2.83196 | 8.95545 | 2.00167 | 4.31246 | 9.29091 | .124688 |
| 8.03 | 64.4809 | 517.782 | 2.83373 | 8.96103 | 2.00250 | 4.31425 | 9.29477 | .124533 |
| 8.04 | 64.6416 | 519.718 | 2.83549 | 8.96660 | 2.00333 | 4.31604 | 9.29862 | .124378 |
| 8.05 | 64.8025 | 521.660 | 2.83725 | 8.97218 | 2.00416 | 4.31783 | 9.30248 | .124224 |
| 8.06 | 64.9636 | 523.607 | 2.83901 | 8.97775 | 2.00499 | 4.31961 | 9.30633 | .124070 |
| 8.07 | 65.1249 | 525.558 | 2.84077 | 8.98332 | 2.00582 | 4.32140 | 9.31018 | .123916 |
| 8.08 | 65.2864 | 527.514 | 2.84253 | 8.98888 | 2.00664 | 4.32318 | 9.31402 | .123762 |
| 8.09 | 65.4481 | 529.475 | 2.84429 | 8.99444 | 2.00747 | 4.32497 | 9.31786 | .123609 |
| 8.10 | 65.6100 | 531.441 | 2.84605 | 9.00000 | 2.00830 | 4.32675 | 9.32170 | .123457 |
| 8.11 | 65.7721 | 533.412 | 2.84781 | 9.00555 | 2.00912 | 4.32853 | 9.32553 | .123305 |
| 8.12 | 65.9344 | 535.387 | 2.84956 | 9.01110 | 2.00995 | 4.33031 | 9.32936 | .123153 |
| 8.13 | 66.0969 | 537.368 | 2.85132 | 9.01665 | 2.01078 | 4.33208 | 9.33319 | .123001 |
| 8.14 | 66.2596 | 539.353 | 2.85307 | 9.02219 | 2.01160 | 4.33386 | 9.33702 | .122850 |
| 8.15 | 66.4225 | 541.343 | 2.85482 | 9.02774 | 2.01242 | 4.33563 | 9.34084 | .122699 |
| 8.16 | 66.5856 | 543.338 | 2.85657 | 9.03327 | 2.01325 | 4.33741 | 9.34466 | .122549 |
| 8.17 | 66.7489 | 545.339 | 2.85832 | 9.03881 | 2.01407 | 4.33918 | 9.34847 | .122399 |
| 8.18 | 66.9124 | 547.343 | 2.86007 | 9.04434 | 2.01489 | 4.34095 | 9.35229 | .122249 |
| 8.19 | 67.0761 | 549.353 | 2.86182 | 9.04986 | 2.01571 | 4.34272 | 9.35610 | .122100 |
| 8.20 | 67.2400 | 551.368 | 2.86356 | 9.05539 | 2.01653 | 4.34448 | 9.35990 | .121951 |
| 8.21 | 67.4041 | 553.388 | 2.86531 | 9.06091 | 2.01735 | 4.34625 | 9.36370 | .121803 |
| 8.22 | 67.5684 | 555.412 | 2.86705 | 9.06642 | 2.01817 | 4.34801 | 9.36751 | .121655 |
| 8.23 | 67.7329 | 557.442 | 2.86880 | 9.07193 | 2.01899 | 4.34977 | 9.37130 | .121507 |
| 8.24 | 67.8976 | 559.476 | 2.87054 | 9.07744 | 2.01980 | 4.35153 | 9.37510 | .121359 |
| 8.25 | 68.0625 | 561.516 | 2.87228 | 9.08295 | 2.02062 | 4.35329 | 9.37889 | .121212 |
| 8.26 | 68.2276 | 563.560 | 2.87402 | 9.08845 | 2.02144 | 4.35505 | 9.38268 | .121065 |
| 8.27 | 68.3929 | 565.609 | 2.87576 | 9.09395 | 2.02225 | 4.35681 | 9.38646 | .120919 |
| 8.28 | 68.5584 | 567.664 | 2.87750 | 9.09945 | 2.02307 | 4.35856 | 9.39024 | .120773 |
| 8.29 | 68.7241 | 569.723 | 2.87924 | 9.10494 | 2.02388 | 4.36032 | 9.39402 | .120627 |
| 8.30 | 68.8900 | 571.787 | 2.88097 | 9.11043 | 2.02469 | 4.36207 | 9.39780 | .120482 |
| 8.31 | 69.0561 | 573.856 | 2.88271 | 9.11592 | 2.02551 | 4.36382 | 9.40157 | .120337 |
| 8.32 | 69.2224 | 575.930 | 2.88444 | 9.12140 | 2.02632 | 4.36557 | 9.40534 | .120192 |
| 8.33 | 69.3889 | 578.010 | 2.88617 | 9.12688 | 2.02713 | 4.36732 | 9.40911 | .120048 |
| 8.34 | 69.5556 | 580.094 | 2.88791 | 9.13236 | 2.02794 | 4.36907 | 9.41287 | .119904 |
| 8.35 | 69.7225 | 582.183 | 2.88964 | 9.13783 | 2.02875 | 4.37081 | 9.41663 | .119761 |
| 8.36 | 69.8896 | 584.277 | 2.89137 | 9.14330 | 2.02956 | 4.37255 | 9.42039 | .119617 |
| 8.37 | 70.0569 | 586.376 | 2.89310 | 9.14877 | 2.03037 | 4.37430 | 9.42414 | .119474 |
| 8.38 | 70.2244 | 588.480 | 2.89482 | 9.15423 | 2.03118 | 4.37604 | 9.42789 | .119332 |
| 8.39 | 70.3921 | 590.590 | 2.89655 | 9.15969 | 2.03199 | 4.37778 | 9.43164 | .119190 |
| 8.40 | 70.5600 | 592.704 | 2.89828 | 9.16515 | 2.03279 | 4.37952 | 9.43539 | .119048 |
| 8.41 | 70.7281 | 594.823 | 2.90000 | 9.17061 | 2.03360 | 4.38126 | 9.43913 | .118906 |
| 8.42 | 70.8964 | 596.948 | 2.90172 | 9.17606 | 2.03440 | 4.38299 | 9.44287 | .118765 |
| 8.43 | 71.0649 | 599.077 | 2.90345 | 9.18150 | 2.03521 | 4.38473 | 9.44661 | .118624 |
| 8.44 | 71.2336 | 601.212 | 2.90517 | 9.18695 | 2.03601 | 4.38646 | 9.45034 | .118483 |
| 8.45 | 71.4025 | 603.351 | 2.90689 | 9.19239 | 2.03682 | 4.38819 | 9.45407 | .118343 |
| 8.46 | 71.5716 | 605.496 | 2.90861 | 9.19783 | 2.03762 | 4.38992 | 9.45780 | .118203 |
| 8.47 | 71.7409 | 607.645 | 2.91033 | 9.20326 | 2.03842 | 4.39165 | 9.46152 | .118064 |
| 8.48 | 71.9104 | 609.800 | 2.91204 | 9.20869 | 2.03923 | 4.39338 | 9.46525 | .117925 |
| 8.49 | 72.0801 | 611.960 | 2.91376 | 9.21412 | 2.04003 | 4.39511 | 9.46897 | .117786 |
| 8.50 | 72.2500 | 614.125 | 2.91548 | 9.21954 | 2.04083 | 4.39683 | 9.47268 | .117647 |

| $n$ | $n^2$ | $n^3$ | $\sqrt{n}$ | $\sqrt{10\,n}$ | $\sqrt[3]{n}$ | $\sqrt[3]{10\,n}$ | $\sqrt[3]{100\,n}$ | $\frac{1}{n}$ |
|---|---|---|---|---|---|---|---|---|
| 8.51 | 72.4201 | 616.295 | 2.91719 | 9.22497 | 2.04163 | 4.39855 | 9.47640 | .117509 |
| 8.52 | 72.5904 | 618.470 | 2.91890 | 9.23038 | 2.04243 | 4.40028 | 9.48011 | .117371 |
| 8.53 | 72.7609 | 620.650 | 2.92062 | 9.23580 | 2.04323 | 4.40200 | 9.48381 | .117233 |
| 8.54 | 72.9316 | 622.836 | 2.92233 | 9.24121 | 2.04402 | 4.40372 | 9.48752 | .117096 |
| 8.55 | 73.1025 | 625.026 | 2.92404 | 9.24662 | 2.04482 | 4.40543 | 9.49122 | .116959 |
| 8.56 | 73.2736 | 627.222 | 2.92575 | 9.25203 | 2.04562 | 4.40715 | 9.49492 | .116822 |
| 8.57 | 73.4449 | 629.423 | 2.92746 | 9.25743 | 2.04641 | 4.40887 | 9.49861 | .116686 |
| 8.58 | 73.6164 | 631.629 | 2.92916 | 9.26283 | 2.04721 | 4.41058 | 9.50231 | .116550 |
| 8.59 | 73.7881 | 633.840 | 2.93087 | 9.26823 | 2.04801 | 4.41229 | 9.50600 | .116414 |
| 8.60 | 73.9600 | 636.056 | 2.93258 | 9.27362 | 2.04880 | 4.41400 | 9.50969 | .116279 |
| 8.61 | 74.1321 | 638.277 | 2.93428 | 9.27901 | 2.04959 | 4.41571 | 9.51337 | .116144 |
| 8.62 | 74.3044 | 640.504 | 2.93598 | 9.28440 | 2.05039 | 4.41742 | 9.51705 | .116009 |
| 8.63 | 74.4769 | 642.736 | 2.93769 | 9.28978 | 2.05118 | 4.41913 | 9.52073 | .115875 |
| 8.64 | 74.6496 | 644.973 | 2.93939 | 9.29516 | 2.05197 | 4.42084 | 9.52441 | .115741 |
| 8.65 | 74.8225 | 647.215 | 2.94109 | 9.30054 | 2.05276 | 4.42254 | 9.52808 | .115607 |
| 8.66 | 74.9956 | 649.462 | 2.94279 | 9.30591 | 2.05355 | 4.42425 | 9.53175 | .115473 |
| 8.67 | 75.1689 | 651.714 | 2.94449 | 9.31128 | 2.05434 | 4.42595 | 9.53542 | .115340 |
| 8.68 | 75.3424 | 653.972 | 2.94618 | 9.31665 | 2.05513 | 4.42765 | 9.53908 | .115207 |
| 8.69 | 75.5161 | 656.235 | 2.94788 | 9.32202 | 2.05592 | 4.42935 | 9.54274 | .115075 |
| 8.70 | 75.6900 | 658.503 | 2.94958 | 9.32738 | 2.05671 | 4.43105 | 9.54640 | .114943 |
| 8.71 | 75.8641 | 660.776 | 2.95127 | 9.33274 | 2.05750 | 4.43274 | 9.55006 | .114811 |
| 8.72 | 76.0384 | 663.055 | 2.95296 | 9.33809 | 2.05828 | 4.43444 | 9.55371 | .114679 |
| 8.73 | 76.2129 | 665.339 | 2.95466 | 9.34345 | 2.05907 | 4.43614 | 9.55736 | .114548 |
| 8.74 | 76.3876 | 667.628 | 2.95635 | 9.34880 | 2.05986 | 4.43783 | 9.56101 | .114417 |
| 8.75 | 76.5625 | 669.922 | 2.95804 | 9.35414 | 2.06064 | 4.43952 | 9.56466 | .114286 |
| 8.76 | 76.7376 | 672.221 | 2.95973 | 9.35949 | 2.06143 | 4.44121 | 9.56830 | .114155 |
| 8.77 | 76.9129 | 674.526 | 2.96142 | 9.36483 | 2.06221 | 4.44290 | 9.57194 | .114025 |
| 8.78 | 77.0884 | 676.836 | 2.96311 | 9.37017 | 2.06299 | 4.44459 | 9.57557 | .113895 |
| 8.79 | 77.2641 | 679.151 | 2.96479 | 9.37550 | 2.06378 | 4.44627 | 9.57921 | .113766 |
| 8.80 | 77.4400 | 681.472 | 2.96648 | 9.38083 | 2.06456 | 4.44796 | 9.58284 | .113636 |
| 8.81 | 77.6161 | 683.798 | 2.96816 | 9.38616 | 2.06534 | 4.44964 | 9.58647 | .113507 |
| 8.82 | 77.7924 | 686.129 | 2.96985 | 9.39149 | 2.06612 | 4.45133 | 9.59009 | .113379 |
| 8.83 | 77.9689 | 688.465 | 2.97153 | 9.39681 | 2.06690 | 4.45301 | 9.59372 | .113250 |
| 8.84 | 78.1456 | 690.807 | 2.97321 | 9.40213 | 2.06768 | 4.45469 | 9.59734 | .113122 |
| 8.85 | 78.3225 | 693.154 | 2.97489 | 9.40744 | 2.06846 | 4.45637 | 9.60095 | .112994 |
| 8.86 | 78.4996 | 695.506 | 2.97658 | 9.41276 | 2.06924 | 4.45805 | 9.60457 | .112867 |
| 8.87 | 78.6769 | 697.864 | 2.97825 | 9.41807 | 2.07002 | 4.45972 | 9.60818 | .112740 |
| 8.88 | 78.8544 | 700.227 | 2.97993 | 9.42338 | 2.07080 | 4.46140 | 9.61179 | .112613 |
| 8.89 | 79.0321 | 702.595 | 2.98161 | 9.42868 | 2.07157 | 4.46307 | 9.61540 | .112486 |
| 8.90 | 79.2100 | 704.969 | 2.98329 | 9.43398 | 2.07235 | 4.46474 | 9.61900 | .112360 |
| 8.91 | 79.3881 | 707.348 | 2.98496 | 9.43928 | 2.07313 | 4.46642 | 9.62260 | .112233 |
| 8.92 | 79.5664 | 709.732 | 2.98664 | 9.44458 | 2.07390 | 4.46809 | 9.62620 | .112108 |
| 8.93 | 79.7449 | 712.122 | 2.98831 | 9.44987 | 2.07468 | 4.46976 | 9.62980 | .111982 |
| 8.94 | 79.9236 | 714.517 | 2.98998 | 9.45516 | 2.07545 | 4.47142 | 9.63339 | .111857 |
| 8.95 | 80.1025 | 716.917 | 2.99166 | 9.46044 | 2.07622 | 4.47309 | 9.63698 | .111732 |
| 8.96 | 80.2816 | 719.323 | 2.99333 | 9.46573 | 2.07700 | 4.47476 | 9.64057 | .111607 |
| 8.97 | 80.4609 | 721.734 | 2.99500 | 9.47101 | 2.07777 | 4.47642 | 9.64415 | .111483 |
| 8.98 | 80.6404 | 724.151 | 2.99666 | 9.47629 | 2.07854 | 4.47808 | 9.64774 | .111359 |
| 8.99 | 80.8201 | 726.573 | 2.99833 | 9.48156 | 2.07931 | 4.47974 | 9.65132 | .111235 |
| 9.00 | 81.0000 | 729.000 | 3.00000 | 9.48683 | 2.08008 | 4.48140 | 9.65489 | .111111 |

| $n$ | $n^2$ | $n^3$ | $\sqrt{n}$ | $\sqrt{10n}$ | $\sqrt[3]{n}$ | $\sqrt[3]{10n}$ | $\sqrt[3]{100n}$ | $\frac{1}{n}$ |
|---|---|---|---|---|---|---|---|---|
| 9.01 | 81.1801 | 731.433 | 3.00167 | 9.49210 | 2.08085 | 4.48306 | 9.65847 | .110988 |
| 9.02 | 81.3604 | 733.871 | 3.00333 | 9.49737 | 2.08162 | 4.48472 | 9.66204 | .110865 |
| 9.03 | 81.5409 | 736.314 | 3.00500 | 9.50263 | 2.08239 | 4.48638 | 9.66561 | .110742 |
| 9.04 | 81.7216 | 738.763 | 3.00666 | 9.50789 | 2.08316 | 4.48803 | 9.66918 | .110619 |
| 9.05 | 81.9025 | 741.218 | 3.00832 | 9.51315 | 2.08393 | 4.48968 | 9.67274 | .110497 |
| 9.06 | 82.0836 | 743.677 | 3.00998 | 9.51840 | 2.08470 | 4.49134 | 9.67630 | .110375 |
| 9.07 | 82.2649 | 746.143 | 3.01164 | 9.52365 | 2.08546 | 4.49299 | 9.67986 | .110254 |
| 9.08 | 82.4464 | 748.613 | 3.01330 | 9.52890 | 2.08623 | 4.49464 | 9.68342 | .110132 |
| 9.09 | 82.6281 | 751.089 | 3.01496 | 9.53415 | 2.08699 | 4.49629 | 9.68697 | .110011 |
| 9.10 | 82.8100 | 753.571 | 3.01662 | 9.53939 | 2.08776 | 4.49794 | 9.69052 | .109890 |
| 9.11 | 82.9921 | 756.058 | 3.01828 | 9.54463 | 2.08852 | 4.49959 | 9.69407 | .109770 |
| 9.12 | 83.1744 | 758.551 | 3.01993 | 9.54987 | 2.08929 | 4.50123 | 9.69762 | .109649 |
| 9.13 | 83.3569 | 761.048 | 3.02159 | 9.55510 | 2.09005 | 4.50288 | 9.70116 | .109529 |
| 9.14 | 83.5396 | 763.552 | 3.02324 | 9.56033 | 2.09081 | 4.50452 | 9.70470 | .109409 |
| 9.15 | 83.7225 | 766.061 | 3.02490 | 9.56556 | 2.09158 | 4.50616 | 9.70824 | .109290 |
| 9.16 | 83.9056 | 768.575 | 3.02655 | 9.57079 | 2.09234 | 4.50780 | 9.71177 | .109170 |
| 9.17 | 84.0889 | 771.095 | 3.02820 | 9.57601 | 2.09310 | 4.50945 | 9.71531 | .109051 |
| 9.18 | 84.2724 | 773.621 | 3.02985 | 9.58123 | 2.09386 | 4.51108 | 9.71884 | .108933 |
| 9.19 | 84.4561 | 776.152 | 3.03150 | 9.58645 | 2.09462 | 4.51272 | 9.72236 | .108814 |
| 9.20 | 84.6400 | 778.688 | 3.03315 | 9.59166 | 2.09538 | 4.51436 | 9.72589 | .108696 |
| 9.21 | 84.8241 | 781.230 | 3.03480 | 9.59687 | 2.09614 | 4.51599 | 9.72941 | .108578 |
| 9.22 | 85.0084 | 783.777 | 3.03645 | 9.60208 | 2.09690 | 4.51763 | 9.73293 | .108460 |
| 9.23 | 85.1929 | 786.330 | 3.03809 | 9.60729 | 2.09765 | 4.51926 | 9.73645 | .108342 |
| 9.24 | 85.3776 | 788.889 | 3.03974 | 9.61249 | 2.09841 | 4.52089 | 9.73996 | .108225 |
| 9.25 | 85.5625 | 791.453 | 3.04138 | 9.61769 | 2.09917 | 4.52252 | 9.74348 | .108108 |
| 9.26 | 85.7476 | 794.023 | 3.04302 | 9.62289 | 2.09992 | 4.52415 | 9.74699 | .107991 |
| 9.27 | 85.9329 | 796.598 | 3.04467 | 9.62808 | 2.10068 | 4.52578 | 9.75049 | .107875 |
| 9.28 | 86.1184 | 799.179 | 3.04631 | 9.63328 | 2.10144 | 4.52740 | 9.75400 | .107759 |
| 9.29 | 86.3041 | 801.765 | 3.04795 | 9.63846 | 2.10219 | 4.52903 | 9.75750 | .107643 |
| 9.30 | 86.4900 | 804.357 | 3.04959 | 9.64365 | 2.10294 | 4.53065 | 9.76100 | .107527 |
| 9.31 | 86.6761 | 806.954 | 3.05123 | 9.64883 | 2.10370 | 4.53228 | 9.76450 | .107411 |
| 9.32 | 86.8624 | 809.558 | 3.05287 | 9.65401 | 2.10445 | 4.53390 | 9.76799 | .107296 |
| 9.33 | 87.0489 | 812.166 | 3.05450 | 9.65919 | 2.10520 | 4.53552 | 9.77148 | .107181 |
| 9.34 | 87.2356 | 814.781 | 3.05614 | 9.66437 | 2.10595 | 4.53714 | 9.77497 | .107066 |
| 9.35 | 87.4225 | 817.400 | 3.05778 | 9.66954 | 2.10671 | 4.53876 | 9.77846 | .106952 |
| 9.36 | 87.6096 | 820.026 | 3.05941 | 9.67471 | 2.10746 | 4.54038 | 9.78195 | .106838 |
| 9.37 | 87.7969 | 822.657 | 3.06105 | 9.67988 | 2.10821 | 4.54199 | 9.78543 | .106724 |
| 9.38 | 87.9844 | 825.294 | 3.06268 | 9.68504 | 2.10896 | 4.54361 | 9.78891 | .106610 |
| 9.39 | 88.1721 | 827.936 | 3.06431 | 9.69020 | 2.10971 | 4.54522 | 9.79239 | .106496 |
| 9.40 | 88.3600 | 830.584 | 3.06594 | 9.69536 | 2.11045 | 4.54684 | 9.79586 | .106383 |
| 9.41 | 88.5481 | 833.238 | 3.06757 | 9.70052 | 2.11120 | 4.54845 | 9.79933 | .106270 |
| 9.42 | 88.7364 | 835.897 | 3.06920 | 9.70567 | 2.11195 | 4.55006 | 9.80280 | .106157 |
| 9.43 | 88.9249 | 838.562 | 3.07083 | 9.71082 | 2.11270 | 4.55167 | 9.80627 | .106045 |
| 9.44 | 89.1136 | 841.232 | 3.07246 | 9.71597 | 2.11344 | 4.55328 | 9.80974 | .105932 |
| 9.45 | 89.3025 | 843.909 | 3.07409 | 9.72111 | 2.11419 | 4.55488 | 9.81320 | .105820 |
| 9.46 | 89.4916 | 846.591 | 3.07571 | 9.72625 | 2.11494 | 4.55649 | 9.81666 | .105708 |
| 9.47 | 89.6809 | 849.278 | 3.07734 | 9.73139 | 2.11568 | 4.55809 | 9.82012 | .105597 |
| 9.48 | 89.8704 | 851.971 | 3.07896 | 9.73653 | 2.11642 | 4.55970 | 9.82357 | .105485 |
| 9.49 | 90.0601 | 854.670 | 3.08058 | 9.74166 | 2.11717 | 4.56130 | 9.82703 | .105374 |
| 9.50 | 90.2500 | 857.375 | 3.08221 | 9.74679 | 2.11791 | 4.56290 | 9.83048 | .105263 |

| $n$ | $n^2$ | $n^3$ | $\sqrt{n}$ | $\sqrt{10\,n}$ | $\sqrt[3]{n}$ | $\sqrt[3]{10\,n}$ | $\sqrt[3]{100\,n}$ | $\frac{1}{n}$ |
|---|---|---|---|---|---|---|---|---|
| 9.51 | 90.4401 | 860.085 | 3.08383 | 9.75192 | 2.11865 | 4.56450 | 9.83392 | .105153 |
| 9.52 | 90.6304 | 862.801 | 3.08545 | 9.75705 | 2.11940 | 4.56610 | 9.83737 | .105042 |
| 9.53 | 90.8209 | 865.523 | 3.08707 | 9.76217 | 2.12014 | 4.56770 | 9.84081 | .104932 |
| 9.54 | 91.0116 | 868.251 | 3.08869 | 9.76729 | 2.12088 | 4.56930 | 9.84425 | .104822 |
| 9.55 | 91.2025 | 870.984 | 3.09031 | 9.77241 | 2.12162 | 4.57089 | 9.84769 | .104712 |
| 9.56 | 91.3936 | 873.723 | 3.09192 | 9.77753 | 2.12236 | 4.57249 | 9.85113 | .104603 |
| 9.57 | 91.5849 | 876.467 | 3.09354 | 9.78264 | 2.12310 | 4.57408 | 9.85456 | .104493 |
| 9.58 | 91.7764 | 879.218 | 3.09516 | 9.78775 | 2.12384 | 4.57568 | 9.85799 | .104384 |
| 9.59 | 91.9681 | 881.974 | 3.09677 | 9.79285 | 2.12458 | 4.57727 | 9.86142 | .104275 |
| 9.60 | 92.1600 | 884.736 | 3.09839 | 9.79796 | 2.12532 | 4.57886 | 9.86485 | .104167 |
| 9.61 | 92.3521 | 887.504 | 3.10000 | 9.80306 | 2.12605 | 4.58045 | 9.86827 | .104058 |
| 9.62 | 92.5444 | 890.277 | 3.10161 | 9.80816 | 2.12679 | 4.58203 | 9.87169 | .103950 |
| 9.63 | 92.7369 | 893.056 | 3.10322 | 9.81326 | 2.12753 | 4.58362 | 9.87511 | .103842 |
| 9.64 | 92.9296 | 895.841 | 3.10483 | 9.81835 | 2.12826 | 4.58521 | 9.87853 | .103734 |
| 9.65 | 93.1225 | 898.632 | 3.10644 | 9.82344 | 2.12900 | 4.58679 | 9.88195 | .103627 |
| 9.66 | 93.3156 | 901.429 | 3.10805 | 9.82853 | 2.12974 | 4.58838 | 9.88536 | .103520 |
| 9.67 | 93.5089 | 904.231 | 3.10966 | 9.83362 | 2.13047 | 4.58996 | 9.88877 | .103413 |
| 9.68 | 93.7024 | 907.039 | 3.11127 | 9.83870 | 2.13120 | 4.59154 | 9.89217 | .103306 |
| 9.69 | 93.8961 | 909.853 | 3.11288 | 9.84378 | 2.13194 | 4.59312 | 9.89558 | .103199 |
| 9.70 | 94.0900 | 912.673 | 3.11448 | 9.84886 | 2.13267 | 4.59470 | 9.89898 | .103093 |
| 9.71 | 94.2841 | 915.499 | 3.11609 | 9.85393 | 2.13340 | 4.59628 | 9.90238 | .102987 |
| 9.72 | 94.4784 | 918.330 | 3.11769 | 9.85901 | 2.13414 | 4.59786 | 9.90578 | .102881 |
| 9.73 | 94.6729 | 921.167 | 3.11929 | 9.86408 | 2.13487 | 4.59943 | 9.90918 | .102775 |
| 9.74 | 94.8676 | 924.010 | 3.12090 | 9.86914 | 2.13560 | 4.60101 | 9.91257 | .102669 |
| 9.75 | 95.0625 | 926.859 | 3.12250 | 9.87421 | 2.13633 | 4.60258 | 9.91596 | .102564 |
| 9.76 | 95.2576 | 929.714 | 3.12410 | 9.87927 | 2.13706 | 4.60416 | 9.91935 | .102459 |
| 9.77 | 95.4529 | 932.575 | 3.12570 | 9.88433 | 2.13779 | 4.60573 | 9.92274 | .102354 |
| 9.78 | 95.6484 | 935.441 | 3.12730 | 9.88939 | 2.13852 | 4.60730 | 9.92612 | .102250 |
| 9.79 | 95.8441 | 938.314 | 3.12890 | 9.89444 | 2.13925 | 4.60887 | 9.92950 | .102145 |
| 9.80 | 96.0400 | 941.192 | 3.13050 | 9.89949 | 2.13997 | 4.61044 | 9.93288 | .102041 |
| 9.81 | 96.2361 | 944.076 | 3.13209 | 9.90454 | 2.14070 | 4.61200 | 9.93626 | .101937 |
| 9.82 | 96.4324 | 946.966 | 3.13369 | 9.90959 | 2.14143 | 4.61357 | 9.93964 | .101833 |
| 9.83 | 96.6289 | 949.862 | 3.13528 | 9.91464 | 2.14216 | 4.61513 | 9.94301 | .101729 |
| 9.84 | 96.8256 | 952.764 | 3.13688 | 9.91968 | 2.14288 | 4.61670 | 9.94638 | .101626 |
| 9.85 | 97.0225 | 955.672 | 3.13847 | 9.92472 | 2.14361 | 4.61826 | 9.94975 | .101523 |
| 9.86 | 97.2196 | 958.585 | 3.14006 | 9.92975 | 2.14433 | 4.61983 | 9.95311 | .101420 |
| 9.87 | 97.4169 | 961.505 | 3.14166 | 9.93479 | 2.14506 | 4.62139 | 9.95648 | .101317 |
| 9.88 | 97.6144 | 964.430 | 3.14325 | 9.93982 | 2.14578 | 4.62295 | 9.95984 | .101215 |
| 9.89 | 97.8121 | 967.362 | 3.14484 | 9.94485 | 2.14651 | 4.62451 | 9.96320 | .101112 |
| 9.90 | 98.0100 | 970.299 | 3.14643 | 9.94987 | 2.14723 | 4.62607 | 9.96655 | .101010 |
| 9.91 | 98.2081 | 973.242 | 3.14802 | 9.95490 | 2.14795 | 4.62762 | 9.96991 | .100908 |
| 9.92 | 98.4064 | 976.191 | 3.14960 | 9.95992 | 2.14867 | 4.62918 | 9.97326 | .100807 |
| 9.93 | 98.6049 | 979.147 | 3.15119 | 9.96494 | 2.14940 | 4.63073 | 9.97661 | .100705 |
| 9.94 | 98.8036 | 982.108 | 3.15278 | 9.96995 | 2.15012 | 4.63229 | 9.97996 | .100604 |
| 9.95 | 99.0025 | 985.075 | 3.15436 | 9.97497 | 2.15084 | 4.63384 | 9.98331 | .100503 |
| 9.96 | 99.2016 | 988.048 | 3.15595 | 9.97998 | 2.15156 | 4.63539 | 9.98665 | .100402 |
| 9.97 | 99.4009 | 991.027 | 3.15753 | 9.98499 | 2.15228 | 4.63694 | 9.98999 | .100301 |
| 9.98 | 99.6004 | 994.012 | 3.15911 | 9.98999 | 2.15300 | 4.63849 | 9.99333 | .100200 |
| 9.99 | 99.8001 | 997.003 | 3.16070 | 9.99500 | 2.15372 | 4.64004 | 9.99667 | .100100 |
| 10.00 | 100.000 | 1000.00 | 3.16228 | 10.0000 | 2.15443 | 4.64159 | 10.0000 | .100000 |

*Square Root.*—EXAMPLE.— (*a*) $\sqrt{3.1416} = ?$ (*b*) $\sqrt{2342.9} = ?$

SOLUTION.—(*a*) In this case, the decimal point need not be moved. In the table under $n^2$ find $3.1329 = 1.77^2$ and $3.1684 = 1.78^2$, one of these numbers being a little less and the other a little greater than the given number 3.1416. The first three figures of the required root are 177. $31{,}684 - 31{,}329 = 355$ is the first difference; 31,416 (the number itself) $- 31{,}329 = 87$ is the second difference. $87 \div 355 = .245$, or .25, which gives the fourth and fifth figures of the root. Hence, $\sqrt{3.1416} = 1.7725$.

(*b*) Pointing off and placing the decimal point between the first and second periods, the number appears 23.4290. Under $n^2$ find $23.4256 = 4.84^2$ and $23.5225 = 4.85^2$. The first three figures of the root are 484. The first difference is $235{,}225 - 234{,}256 = 969$; the second difference is $234{,}290 - 234{,}256 = 34$; $34 \div 969 = .035$, or .04, which gives the fourth and fifth figures of the root. Since the integral part of the number 23′42.9 contains two periods, the integral part of the root contains two figures, or $\sqrt{2342.9} = 48.404$.

*Cube Root.*—EXAMPLE.—(*a*) $\sqrt[3]{.0000062417} = ?$
(*b*) $\sqrt[3]{50932676} = ?$

SOLUTION.—(*a*) Pointed off, the number appears .000′006′241′700, and with the decimal point placed between the first and second periods of the significant part, gives 6.2417. Under $n^3$ find $6.22950 = 1.84^3$ and $6.33163 = 1.85^3$. The first three figures of the root are 1.84. The first difference is 10,213, and the second difference is 1,220; $1{,}220 \div 10{,}213 = .119$, or .12, which gives the fourth and fifth figures. There is one cipher period after the decimal point in the number; hence, $\sqrt[3]{.0000062417} = .018412$.

(*b*) Replace all after the sixth figure with ciphers, making the sixth figure 1 greater when the seventh figure is 5 or greater; that is, $\sqrt[3]{50932700}$ and $\sqrt[3]{50932676}$ will be the same. Placing the decimal point between the first and second periods gives 50.9327. Under $n^3$ find $50.6530 = 3.70^3$ and $51.0648 = 3.71^3$. The first three figures of the root are 370.

The second difference 2,797 ÷ the first difference, 4,118 = .679 or .68. Hence, $\sqrt[3]{50932676} = 370.68$.

**Squares.**—If the given number contains less than four significant figures, the significant figures of the square or cube can be found under $n^2$ or $n^3$ opposite the given number under $n$. The decimal point can be located by the fact that if the column headed $\sqrt{10n}$ is used, the square will contain twice as many figures as the number to be squared, while if the column headed $\sqrt{n}$ is used, the square will contain twice as many figures as the number to be squared, less 1. If the number contains an integral part, the principle is applied to the integral part only; if the number is wholly decimal, the square will have twice as many ciphers, or twice as many plus 1, following the decimal point as in the number itself, depending on whether $\sqrt{10n}$ or $\sqrt{n}$ column is used.

To square a number containing more than three significant figures, place the decimal point between the first and second significant figures and find in the column headed $\sqrt{n}$ or $\sqrt{10n}$ two consecutive numbers, one a little greater and the other a little less than the given number. The remainder of the work is exactly as described for extracting roots. The square will contain twice as many figures as the number itself, or twice as many less 1, according to whether the column headed $\sqrt{10n}$ or $\sqrt{n}$ is used. The number of ciphers following the decimal point in the square of a number wholly decimal is indicated in the same way.

EXAMPLE 1.—(*a*) $273.42^2 = ?$ (*b*) $.052436^2 = ?$

SOLUTION.—(*a*) Placing the decimal point between the first and second significant figures, the number is 2.7342, which occurs between $2.73313 = \sqrt{7.47}$ and $2.73496 = \sqrt{7.48}$, found under $\sqrt{n}$. The first three figures of the square are 747. The second difference 107 ÷ the first difference 183 = .584, or .58. Hence, $273.42^2 = 74{,}758$.

(*b*) With the position of the decimal point changed, the number is 5.2436, which is between $5.23450 = \sqrt{2.74}$ and $5.24404 = \sqrt{2.75}$, both under $\sqrt{10n}$. The first three significant figures of the root are 2.74 and the second difference

910 ÷ the first difference 954 = .953, or .95, the next two figures. The number has one cipher following the decimal point, and the column headed $\sqrt{10n}$ is used; hence, $.052436^2 = .0027495$.

**Cubes.**—To cube a number, proceed in the same way, but use a column headed $\sqrt[3]{n}$, $\sqrt[3]{10n}$, or $\sqrt[3]{100n}$. If the number contains an integral part, the number of figures in the integral part of the cube will be three times as many as in the given number if the column headed $\sqrt[3]{100n}$ is used; it will be three times as many less 1 if the column headed $\sqrt[3]{10n}$ is used; and it will be three times as many less 2 if the column headed $\sqrt[3]{n}$ is used. If the number is wholly decimal, the number of ciphers following the decimal point in the cube will be three times, three times plus 1, or three times plus 2, as many as in the given number, depending on whether $\sqrt[3]{100n}$, $\sqrt[3]{10n}$, or $\sqrt[3]{n}$ column is used.

EXAMPLE 2.—(*a*) $129.684^3 = ?$ (*b*) $7.6442^3 = ?$ (*c*) $.032425^3 = ?$

SOLUTION.—(*a*) With the position of the decimal point changed, the number 1.29684 is between $1.29664 = \sqrt[3]{2.18}$ and $1.29862 = \sqrt[3]{2.19}$, found under $\sqrt[3]{n}$. The second difference 20 ÷ the first difference 198 = .101+, or .10. Hence, the first five significant figures are 21810; the number of figures in the integral part of the cube is $3 \times 3 - 2 = 7$; and $129.684^3 = 2{,}181{,}000$, correct to five significant figures.

(*b*) 7.64420 occurs between $7.64032 = \sqrt[3]{446}$ and $7.64603 = \sqrt[3]{447}$. The first difference is 571; the second difference is 388; and 388 ÷ 571 = .679+, or .68. Hence, the first five significant figures are 44668; the number of ciphers following the decimal point is $3 \times 0 = 0$; and $7.6442^3 = 446.68$, correct to five significant figures.

(*c*) 3.2425 falls between $3.24278 = \sqrt[3]{10 \times 3.41}$ and $3.23961 = \sqrt[3]{10 \times 3.40}$. The first difference is 317; the second difference is 289; 289 ÷ 317 = .911+, or .91. Hence, the first five significant figures are 34091; the number of ciphers following the decimal point is $3 \times 1 + 1 = 4$; and $.032425^3 = .000034091$, correct to five significant figures.

**Reciprocals.**—The table gives the reciprocals of all numbers expressed by three significant figures correct to six significant figures. The number of ciphers following the decimal point in the reciprocal of a number is 1 less than the number of figures in the integral parts of the number; and if the number is entirely decimal, the number of figures in the integral part of the reciprocal is 1 greater than the number of ciphers following the decimal point in the number. For example, the reciprocal of 3370 = .000296736 and of .00348 = 287.356.

The following examples show the process when the number contains more than three significant figures:

EXAMPLE.—The reciprocal (*a*) of 379.426 = ? (*b*) of .0004692 = ?

SOLUTION.—(*a*) .379426 falls between $.378788=\frac{1}{2.64}$ and $.380228=\frac{1}{2.63}$. The first difference is 380,228 − 378,788 = 1,440; the second difference is 380,228 − 379,426 = 802; 802 ÷ 1,440 = .557, or .56. Hence, the first five significant figures are 26356, and the reciprocal of 379.426 is .0026356, to five significant figures.

(*b*) .469200 falls between $.469484=\frac{1}{2.13}$ and $.467290=\frac{1}{2.14}$. The first difference is 2,194; the second difference is 284; 284 ÷ 2,194 = .129+, or .13. Hence, $\frac{1}{.0004692}=2{,}131.3$, correct to five significant figures.

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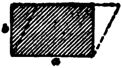
## MENSURATION

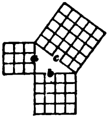
In the following formulas, unless otherwise stated, the letters have the meanings here given:

$A$ = area of a plane figure;
$d$ = diameter;
$r$ = radius;
$p$ = perimeter, or circumference;
$\pi$ = ratio of any circumference to its diameter.

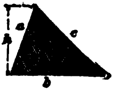
## POLYGONS



Rectangle and parallelogram
$A = ab$

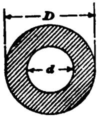


Right-angled triangle
$c^2 = a^2 + b^2$
$A = \frac{1}{2}ab$



Any triangle
$A = \frac{1}{2}bh$

## CIRCLES

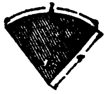


The circle: $p = \pi d = 2\pi r$

$$A = \pi r^2 = \frac{\pi d^2}{4}$$

The ring: $A = \frac{\pi}{4}(D^2 - d^2)$

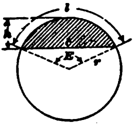
The sector:



$$A = \tfrac{1}{2}lr = \frac{E}{360}\pi r^2 = .008727r^2E$$

$E$ = angle; $l$ = length of arc

The segment:



$$A = \tfrac{1}{2}[lr - c(r-h)] = \frac{E}{360}\pi r^2 - \frac{c}{2}(r-h)$$

$$l = \frac{\pi r E}{180} = .0175rE$$

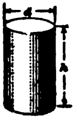
$$c = \text{chord} = 2\sqrt{2hr - h^2}$$

## ELLIPSE

$$p = \pi\sqrt{\frac{D^2+d^2}{2} - \frac{(D-d)^2}{8.8}}$$

$$A = \frac{\pi}{4}Dd$$

## SOLIDS

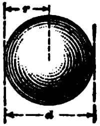


The cylinder: Convex surface $= \pi dh$

Both end surfaces $= \frac{\pi d^2}{2}$

Volume $= \frac{\pi d^2}{4}h$

**The sphere:**




$$\text{Surface} = \pi d^2 = 4\pi r^2$$

$$\text{Volume} = \frac{1}{6}\pi d^3 = \frac{4}{3}\pi r^3$$

## USEFUL NUMBERS

$\frac{1}{\pi} = .3183$ $\pi^2 = 9.8696$ $\frac{1}{\pi^2} = .1013$

$\sqrt{\pi} = 1.7725$ $\frac{1}{\sqrt{\pi}} = .5642$

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# MECHANICS

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## FALLING BODIES

Let $g$ = 32.16 = constant acceleration due to the attraction of the earth;

$t$ = number of seconds that the body falls;

$v$ = velocity in feet per second at the end of the time $t$;

$h$ = distance, in feet, that the body falls during the time $t$.

Then,

$$v = gt = \frac{2h}{t} = \sqrt{2gh} = 8.02\sqrt{h};$$

$$h = \frac{vt}{2} = \frac{gt^2}{2} = \frac{v^2}{2g} = .015547v^2;$$

$$t = \frac{v}{g} = \frac{2h}{v} = \sqrt{\frac{2h}{g}} = .24938\sqrt{h}.$$

If $h$ is in centimeters and $v$ in centimeters per second, then $g = 981$ at Paris.

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## CENTRIFUGAL FORCE

$F$ = centrifugal force, in pounds;

$W$ = weight of revolving body, in pounds;

$m$ = mass of body $= \frac{W}{g}$;

$r$ = distance from axis of motion to center of gravity of body, in feet;

$N$ = number of revolutions per minute;
$v$ = velocity, in feet per second.

$$F = \frac{Wv^2}{gr} = \frac{Mv^2}{r} = .00034WrN^2.$$

In calculating the centrifugal force of flywheels, it is customary to neglect the arms and take $r$ equal to the mean radius of the rim; in such cases, $W$ is taken as one-half the weight of the rim. The result thus obtained, divided by $\pi$, is approximately the force tending to burst the flywheel rim.

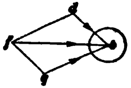
EXAMPLE.—What is the force tending to burst a flywheel rim weighing 7 tons, making 150 rev. per min., and having a mean radius of 5 ft.?

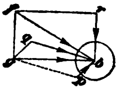
SOLUTION.—

$$F = \frac{.00034\times(\frac{1}{2}\times 7\times 2{,}000)5\times 150^2}{3.1416} = 85{,}227 \text{ lb.}$$

## PARALLELOGRAM OF FORCES

Let $db$ and $qb$ represent the magnitudes and directions of two forces that act to move the body $b$. By completing the parallelogram, there will be obtained a diagonal force $fb$, whose magnitude and direction are equal to the effect produced by $db$ and $qb$. $fb$ is called the resultant of $db$ and $qb$. If three or more forces act in different directions to move a body $b$, find the resultant of any two of them, and consider it as a single force. Between this and the next force find a second resultant. Thus, $pb$, $qb$, and $rb$ are magnitudes and directions of the forces. $pb+qb+rb=gb+rb=fb$, which is the resultant in the magnitude and direction of the three forces $pb$, $qb$, and $rb$.





## WORK AND POWER

Work is the overcoming of resistance through a distance. The unit of work is the *foot-pound;* that is, it equals 1 lb. raised vertically 1 ft. The amount of work done is equal to

the resistance in pounds multiplied by the distance in feet through which it is overcome. If a body is lifted, the resistance is the weight or the overcoming of the attraction of gravity, the work done being the weight in pounds multiplied by the height of the lift in feet. If a body moves in a horizontal direction, the work done is the friction overcome, or the force needed to move a resistant body or combination of bodies, multiplied by the distance moved through.

Power is the rate of doing work, or the quantity of work done in unit time. The ordinary unit of mechanical power is the horsepower, which is equivalent to 33,000 ft.-lb. per min., or 550 ft.-lb. per sec.

The work necessary to be done in raising a body weighing $W$ lb. through a height of $h$ ft. equals $W\,h$ ft.-lb. The total work that any moving body is capable of doing in being brought to rest equals its kinetic energy, or $\frac{Wv^2}{2g} = \frac{1}{2}\,mv^2$.

The kinetic energy of a 200,000-lb. train running at 40 mi. per hr. (58.7 ft. per sec.) is $200{,}000 \times 58.7^2 \div (2 \times 32.16) = 10{,}714{,}220$ ft.-lb.; the retarding force necessary to stop the train within 2,000 ft. is $10{,}714{,}220 \div 2{,}000 = 5357.1$ lb., and the average power required to stop the train in $\frac{1}{2}$ min. is $10{,}714{,}220 \div \frac{1}{2} = 21{,}428{,}440$ ft.-lb. per min. or $21{,}428{,}440 \div 33{,}000 = 649.3$ H. P.

## BELTS, SHAFTING, ETC.

To find the angle of contact of a belt on each pulley:

Let $D$ = diameter of the larger pulley, in inches;
$d$ = diameter of the smaller pulley, in inches;
$l$ = distance between the pulley centers, in inches;
$a$ = $\frac{1}{2}$ the arc of contact on the smaller pulley.

Then,
$$\operatorname{Cos} a = \frac{D - d}{2\,l}.$$

From a table of natural cosines the angle $a$ can be found and $2\,a$ = the arc of contact on the smaller pulley; $360° - 2a$ = the arc of contact on the larger pulley. In calculating belts, only the arc of contact on the smaller pulley need be considered.

To find the length $L$ of a belt,

$$L = \frac{D+d}{2} \times 3\tfrac{1}{4} + 2l, \text{ approximately.}$$

NOTE.—These formulas apply only to ordinary open belts and not to crossed belts.

EXAMPLE.—A 12-in. pulley and a 60-in. pulley with centers 15 ft. apart are connected by an open belt. (*a*) Find the arc of contact of the belt on each pulley. (*b*) Find the length of the belt.

SOLUTION.—(*a*) 15 ft. = 180 in. $(60 - 12) \div (2 \times 180) = .1333 = \cos 82° 20'$.

Arc of contact on smaller pulley $= 2 \times 82° 20' = 164° 40'$.

Arc of contact on larger pulley $= 360 - 164° 40' = 195° 20'$.

(*b*) $\frac{60+12}{2} \times 3\tfrac{1}{4} + 2 \times 180 = 477$ in. $= 39$ ft. 9 in.

The following formulas give conservative results:

Let $C$ = allowable effective pull, in pounds per inch, width of belt (see table);

$H$ = horsepower to be transmitted;

$W$ = width of belt, in inches;

$V$ = belt speed, in feet per minute.

Then, $W = \frac{33{,}000H}{VC}$; $H = \frac{VCW}{33{,}000}$

**ALLOWABLE BELT PULL**

| Arc Covered by Belt | | Allowable Pull per Inch Width in Pounds | |
|---|---|---|---|
| Degrees | Fraction of Circumference | Single belt | Double belt |
| 90 | .250 | 23 | 32.9 |
| 112½ | .312 | 27.4 | 39.2 |
| 120 | .333 | 28.8 | 41.2 |
| 135 | .375 | 31.3 | 44.7 |
| 150 | .417 | 33.8 | 48.3 |
| 157½ | .437 | 34.9 | 49.9 |
| 180 | .500 | 38.1 | 54.5 |

Single belting is $\frac{3}{16}$ in. thick; four-ply cotton belting is generally considered equivalent to single belting. To install one pulley directly over the other, so that the belt runs vertical should be avoided if possible; it is better that the angle between the belt and the floor does not exceed 45°, and the bottom side of the belt should be the driving side. The distance between pulley centers depends on the size of the pulleys and of the belt; it should be great enough so that the belt will run with a slight sag and a gently undulating motion, but not great enough to cause excessive sag and an unsteady flapping motion of the belt. In general, the centers of small pulleys carrying light narrow belts should be about 15 ft. apart and the belt sag 1½ to 2 in.; for large pulleys and heavy belts the distance should be 20 to 30 ft. and the sag 2½ to 5 in.

Loose-running belts will last much longer than tight ones, and will be less likely to cause heating and wear of pulley bearings. High-speed belts are less likely to slip than low-speed belts; hence, pulleys should be selected so as to make the belt speeds high, provided they do not exceed 3,500 ft. per min. for laced belts and 5,000 ft. per min. for endless belts. Leather belts should be run with the grain, or hair, side next to the pulley; they should be kept clean, dry, and free from grease and lubricating oil. A dry, husky, leather belt can be made soft and pliable by the application of a coat of melted tallow and beeswax; this should be done only when the belt becomes dry and hard.

## SHAFTING

The diameter of a shaft may be found by the following formulas. The first is used when great stiffness is required and the shafts are very long; the second when strength only is required to be considered; and the third for calculating the diameters of steel shafts for dynamos.

$d$ = diameter of shaft, in inches;
$H$ = horsepower transmitted;
$W$ = kilowatts output;
$N$ = number of revolutions per minute;
$c$ = constant in formula (1):

$c'$ = constant in formula (2);
$k$ = constant in formula (3).

$$d = c\sqrt[4]{\frac{H}{N}} \quad (1) \qquad d = c'\sqrt[4]{\frac{H}{N}} \quad (2) \qquad d = k\sqrt[4]{\frac{W}{N}} \quad (3)$$

$c$ = 5.26 for cast iron; 4.75 for wrought iron; 3.96 for steel;
$c'$ = 4.02 for cast iron; 3.63 for wrought iron; 3.03 for steel.
$k$ = .9 to 1 for 1- to 10-kilowatt dynamos;
$k$ = 1.1 to 1.4 for 50- to 500-kilowatt dynamos.

Note.—To extract the fourth root, extract the square root twice.

## ROPES AND CHAINS

$D$ = diameter of the rope in inches = diameter of iron from which the link in chain is made;
$W$ = safe load in tons of 2,000 lb.
For common hemp rope, $W = \frac{1}{3} D^2$.
For iron-wire rope, $W = \frac{8}{3} D^2$.
For steel-wire rope, $W = \frac{14}{3} D^2$.
For close-link wrought-iron chain, $W = 6 D^2$.
For stud-link wrought-iron chain, $W = 9 D^2$.

# ELECTRICITY AND MAGNETISM

## ELECTRICAL UNITS, SYMBOLS, AND QUANTITIES

The fundamental units, from which are derived the units used in electricity and magnetism, are the *centimeter* as the unit of length, the *gram* as the unit of mass, and the *second* as the unit of time. A system of units derived from these fundamental units is called the *centimeter-gram-second*, or *C. G. S., system.* The C. G. S. unit of velocity, sometimes called the *kine* is 1 centimeter per second; the C. G. S. unit of force, called the *dyne*, is the force required to produce an acceleration of 1 kine per second in a body having a mass of 1 gram; the C. G. S. unit of work or energy, called the *erg*, is the work done by a force of 1 dyne working through a distance of 1 centimeter.

Two systems of units are derived from the fundamental units: the *electrostatic units*, based on the force exerted between two quantities of electricity; and the *electromagnetic units*, based on the force exerted between two magnetic poles or between a current and a magnetic pole. The ratio of the electrostatic to the electromagnetic units is some multiple or submultiple of the velocity $v$ of light in air, which is $3 \times 10^{10}$ cm. per sec.

The electrostatic units are those of quantity, current, electromotive force, resistance, capacity, and inductivity. These units have not been named.

The electromagnetic units may be considered in two classes, electric units and magnetic units. The C. G. S. electromagnetic units have not been named and, as they are inconvenient in magnitude for practical purposes, a so-called practical system of units has been adopted. Practical electromagnetic units are equal to the C. G. S. electromagnetic units multiplied or divided by some power of 10. The practical units are used to express quantity of electricity, strength of electric current, electromotive force, resistance, work, power, inductance, and capacity.

The magnetic units that would correspond to the practical electric units are not used and have not been definitely named on account of their inconvenient magnitudes. The names of all the practical electromagnetic units, except the mho, have been adopted by some international conventions and their use legalized by most of the important nations.

**Numerical Expression of Electrical Units.**—The expression of electrical units often requires large numbers, and it is customary to use the multiple 10 with an index to indicate the power to which it is raised. The sign of the index indicates whether the designated power of 10 is to be used as a multiplier or as a divisor. For example, $7\times10^2=700$, but $7\times10^{-2}=.07$; $v\times10^{-1}=\frac{v}{10}$; $v^2\times10^{-9}=\frac{v^2}{1{,}000{,}000{,}000}$; $723\times10^{-4}=\frac{723}{10{,}000}=.0723$; etc.

## LIST OF IMPORTANT SYMBOLS

In the two tables following, $l$ represents a length or distance, $F$ a force, $v$ a velocity, $T$ the number of turns in a coil or circuit, $t$ time, $W$ work, and $A$ an area.

In this work, the following meanings will be understood unless otherwise specified:

$A$ = area in square centimeters;
**A** = area in square inches;
$\mathfrak{B}$ = magnetic density per square centimeter;
**B** = magnetic density per square inch;
$C$ = capacity in farads or microfarads;
$D$, $d$ = diameters;
$E$, $e$ = electromotive force, in volts;
$F$ = force, usually in dynes;
$\mathfrak{F}$ = magnetomotive force;
$G$ = conductance;
$\mathfrak{H}$ = intensity of magnetic field per square centimeter;
**H** = intensity of magnetic field per square inch;
H. P. = horsepower;
$I$ = current, in amperes;
$\mathfrak{J}$ = intensity of magnetization;
$J$ = work, in joules;

## MAGNETIC UNITS

| Magnetic Quantities | Symbol | Defining Equation | Names of C. G. S. Units |
|---|---|---|---|
| Strength of pole | $m$ | $m=\sqrt{Fl^2}$ | Has no name |
| Magnetic moment | $\mathfrak{M}$ | $\mathfrak{M}=ml$ | Has no name |
| Intensity of magnetization | $\mathfrak{J}$ | $\mathfrak{J}=\frac{m}{A}$ | Has no name |
| Magnetizing force or field density | $\mathfrak{H}$ | $\mathfrak{H}=\frac{F}{m}=\frac{m}{l^2}$ | Gauss, or 1 line of force per sq. cm. |
| Susceptibility | $\kappa$ | $\kappa=\frac{\mathfrak{J}}{\mathfrak{H}}$ | Has no name |
| Magnetomotive force | $\mathfrak{F}$ | $\mathfrak{F}=\mathfrak{H}l$ or $\mathfrak{F}=\frac{W}{m}$ | Gilbert (Not internationally accepted) |
| Magnetic density or magnetic induction | $\mathfrak{B}$ | $\mathfrak{B}=4\pi\mathfrak{J}+\mathfrak{H}$ | Gauss, or 1 line of force per sq. cm. |
| Magnetic flux | $\Phi$ | $\Phi=\mathfrak{B}A$ | Maxwell, or 1 line of force |
| Permeability | $\mu$ | $\mu=\frac{\mathfrak{B}}{\mathfrak{H}}$ | Has no name |
| Reluctance | $\mathfrak{R}$ | $\mathfrak{R}=\frac{\mathfrak{F}}{\Phi}$ or $\mathfrak{R}=\frac{l}{A\mu}$ | Oersted (Not internationally accepted) |

## ELECTRICAL UNITS

| Electrical Quantities | Symbol | Defining Equation | Names of Practical Electromagnetic Units | Quantities by Which to Multiply Practical Electromagnetic Units to Reduce to C. G. S. Electromagnetic Units | Quantities by Which to Multiply Practical Electromagnetic Units to Reduce to C. G. S. Electrostatic Units $v=3\times10^{10}$ |
|---|---|---|---|---|---|
| Current........ | $I$ or $i$ | $I=\frac{F}{l\mathcal{H}}$ | Ampere | $10^{-1}$ | $v\times10^{-1}=3\times10^{9}$ |
| Quantity of electricity... | $Q$ or $q$ | $Q=It$ or $Q=\sqrt{Fl^2}$ | Coulomb | $10^{-1}$ | $v\times10^{-1}=3\times10^{9}$ |
| Electromotive force........ | $E$ or $e$ | $E=\mathcal{H}lv=\frac{\Phi}{t}$ or $E=\frac{W}{Q}$ | Volt | $10^{8}$ | $10^{8}\div v=\frac{1}{3}\times10^{-2}$ |
| Resistance.... | $R$ | $R=\frac{E}{I}$ or $R=\frac{W}{I^2t}$ | Ohm | $10^{9}$ | $10^{9}\div v^2=\frac{1}{9}\times10^{-11}$ |
| Resistivity.... | $\rho$ | $\rho=\frac{RA}{l}$ | Ohm | | |

## ELECTRICAL UNITS—(*Continued*)

| Electrical Quantities | Symbol | Defining Equation | Names of Practical Electromagnetic Units | Quantities by Which to Multiply Practical Electromagnetic Units to Reduce to C. G. S. Electromagnetic Units | Quantities by Which to Multiply Practical Electromagnetic Units to Reduce to C. G. S. Electrostatic Units $v=3\times10^{10}$ |
|---|---|---|---|---|---|
| Conductance... | $G$ | $G=\frac{1}{R}$ | Mho (Not internationally accepted) | | |
| Conductivity.. | $\gamma$ | $\gamma=\frac{1}{\rho}$ | Mho (Not internationally accepted) | | |
| Work or energy | $W$ or $J$ | $J=EIt$ | Joule | $10^7$ ergs | |
| Power........ | $P$ | $P=EI$ | Watt | $10^7$ ergs per sec. | |
| Capacity...... | $C$ | $C=\frac{Q}{E}$ | Farad | $10^{-9}$ | $v^2\times10^{-9}=9\times10^{11}$ |
| Inductivity.... | $K$ | $K=\frac{Q}{Q'}$ (air as dielectric) (other dielectric) or $K=\frac{4\pi Cl}{A}$ | A number | | |
| Inductance (self)....... | $L$ | $L=\frac{\Phi T}{I}$ | Henry | $10^9$ | $10^9\div v^2=\frac{1}{9}\times10^{-11}$ |
| Inductance (mutual).... | $M$ | $M=\frac{\Phi T}{I}$ | Henry | $10^9$ | $10^9\div v^2=\frac{1}{9}\times10^{-11}$ |

K. W. = power, in kilowatts;
$l$ = length, in centimeters;
**l** = length, in inches;
$L$ = inductance or coefficient of self-induction, in henrys;
$\mathfrak{M}$ = magnetic moment;
$M$ = mutual inductance;
$m$ = strength of pole;
$P$ = power, in watts;
$Q$ = quantity of electricity, in coulombs,
$\mathfrak{R}$ = reluctance;
$R$ = resistance, in ohms;
$\rho$ = resistivity;
$t$ = time, in seconds;
$v$ = volume, in cubic centimeters, or velocity, in centimeters per second;
**V** = volume, in cubic inches;
$W$ = work;
$\Phi$ = total magnetic flux;
$\mu$ = permeability.

## PRACTICAL ELECTROMAGNETIC UNITS

**Current** ($I$).—The strength of current $I$ is the rate at which electricity is flowing through a conductor, and is analogous to the rate of flow of water through a pipe in gallons per second.

The unit strength of current, called the *ampere*, is represented sufficiently well for practical use by the unvarying current that, when passed through a specified solution of nitrate of silver in water, deposits silver at the rate of .001118 gram per sec.

A *milliampere* is equal to $\frac{1}{1,000}$ or .001 ampere.

**Quantity of Electricity** ($Q$).—The quantity of electricity that passes through a circuit is comparable to the quantity of water that flows through a pipe, and equals the product of the rate of flow and the time; that is,

$$Q = It$$

If $I$ is 1 ampere and $t$ is 1 second, $Q$ is 1 *coulomb*, which is the practical unit quantity of electricity. If 5 amperes is flowing through a wire, then, in 30 seconds, $5 \times 30 = 150$ coulombs of electricity will pass. One coulomb will deposit

.001118 gram of silver out of a neutral solution of silver nitrate consisting of 15 parts by weight of silver nitrate and 85 parts of water.

**Electromotive Force** (*E. M. F.*, or *E*).—Electromotive force, or electric pressure, is that which causes electricity to flow in a closed circuit. The practical unit of E. M. F. is the *volt*, which is the E. M. F. that will cause a current of 1 ampere to flow through a resistance of 1 ohm. The volt is represented sufficiently well for practical use by $\frac{1000}{1434}$ of the E. M. F. between the electrodes of a Carhart-Clarke standard cell at a temperature of 15° C. A *kilovolt* = 1,000 volts, a *millivolt* = .001 volt, and a *microvolt* = .000001 volt.

**Resistance** (*R.*)—All substances offer resistance to the passage of electricity through them, the amount of the resistance depending on the substance and on its shape; that is, on the length and cross-section. The resistance of all metals increases with an increase in the temperature; while the resistance of carbon, insulating materials, and electrolytic solutions decreases with an increase in their temperatures.

The practical *unit of resistance* is the *ohm*. A conductor has a resistance of 1 ohm when the pressure required to send 1 ampere through it is 1 volt. In other words, the drop, or fall, in pressure through a resistance of 1 ohm, when a current of 1 ampere is flowing, is 1 volt. The *microhm* = .000001 ohm. The *megohm* = 1,000,000 ohms. The ohm is one of the few electrical units for which a material standard can be used. Different standards have been used, all based on the resistance of a column of mercury at 0° C., having a cross-sectional area of 1 sq. mm. and a different length for each standard, as follows:

1. The *international ohm*, now universally recognized as the standard, has a column of mercury 106.3 cm. in length.

2. The *legal ohm*, in use previous to 1893, but now superseded by the international ohm, has a column of mercury 106 cm. in length.

3. The *British Association unit* (B. A. U.). which preceded the legal ohm but which is no longer in use, has a column of mercury 104.8 cm. in length.

## RESISTANCES AND TEMPERATURE COEFFICIENTS OF METALS*

| Metal | Specific Resistance ρ† (Microhms per Centimeter Cube) | Resistance of 1 Mil-Foot in Ohms | | Temperature Coefficient per Degree C. Between 0° and 100° C. | Temperature Coefficient per Degree F. Between 32° and 212° F. | Percentage Conductivity | Relative Resistance |
|---|---|---|---|---|---|---|---|
| | At 0° C. or 32° F. | 0° C. or 32° F. | 23.8° C. or 75° F. | | | | |
| Silver,[2] pure annealed.. | 1.468 | 8.831 | 9.674 | .004000 | .002220 | 108.60 | .925 |
| Copper,[2] pure annealed. | 1.561 | 9.390 | | .004280 | .002380 | 102.10 | .980 |
| Copper,[1] annealed ..... | 1.594 | 9.590 | 10.505 | .004020 | .002230 | 100.00 | 1.000 |
| Silver, hard-drawn..... | 1.629 | 9.799 | | | | 97.80 | 1.022 |
| Copper,[1] hard-drawn... | 1.631 | 9.810 | 10.745 | .004020 | .002230 | 97.80 | 1.022 |
| Gold[2] (99.9% pure).... | 2.197 | 13.216 | | .003770 | .002090 | 72.55 | 1.378 |
| Aluminum (99.5% pure) | 2.530 | 15.219 | | | | 63.00 | |
| Aluminum[2] (commer- | | | | | | | 1.587 |
| cial—97.5% pure)... | 2.665 | 16.031 | | .004350 | .002420 | 59.80 | 1.672 |
| Magnesium[2].......... | 4.355 | 26.197 | | .003810 | .002120 | 36.60 | 2.732 |
| Zinc[2] (very pure)...... | 5.751 | 34.595 | | .004060 | .002260 | 27.72 | 3.608 |
| Iron,[2] approximately pure.............. | 9.065 | 54.529 | | .006250 | .003470 | 17.50 | 5.714 |
| Iron "E. B. B." iron wire | 9.759 | 58.702 | 65.190 | .004630 | .002570 | 16.20 | 6.173 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Cadmium[2] (pure) | 10.023 | 60.292 | | .004190 | .002320 | 15.90 | 6.289 |
| Palladium[2] (pure) | 10.219 | 61.471 | | .003540 | .001970 | 15.60 | 6.410 |
| Platinum[2] (pure) | 10.917 | 65.670 | | .003669 | .002038 | 14.60 | 6.845 |
| Iron, "B. B." iron wire | 11.085 | 68.680 | 76.270 | .004630 | .002570 | 13.50 | 7.407 |
| Nickel[2] | 12.323 | 74.128 | | .006220 | .003460 | 12.94 | 7.726 |
| Tin[2] (pure) | 13.048 | 78.489 | | .004400 | .002450 | 12.22 | 8.184 |
| Steel (wire) | 13.495 | 81.179 | 90.150 | .004630 | .002570 | 11.60 | 8.621 |
| Thallium[2] (pure) | 17.633 | 106.070 | | .003980 | .002210 | 9.04 | 11.060 |
| Lead[2] (pure) | 20.380 | 122.590 | 134.610 | .004110 | .002280 | 7.82 | 12.790 |
| Antimony (pressed) | 35.400 | 212.950 | | .004100 | | 4.50 | 22.220 |
| Mercury[2] (pure) | 94.070 | 565.870 | 610.370 | .000720 | .000400 | 1.69 | 59.170 |
| Bismuth (pressed) | 130.800 | 786.810 | | .003540 | | 1.22 | 81.970 |

* The resistances are given in international ohms † This is the resistance of a piece 1 cm. long and 1 sq. cm. in sectional area, at 0° C., in microhms. [1]Determined by Matthiessen and taken as the standard. [2]Determined by Fleming and Dewar. According to the American Institute of Electrical Engineers, the temperature coefficient of pure commercial copper should be .0042 per degree C

## RESISTANCES AND TEMPERATURE COEFFICIENTS OF ALLOYS†

| Substance | Specific Resistance $\rho$‡ (Microhms per Centimeter Cube) At 0° C. or 32° F. | Resistance of 1 Mil-Foot in Ohms at 0° C. or 32° F. | Temperature Coefficient per Degree Centigrade | Temperature Coefficient per Degree Fahrenheit | Percentage Conductivity | Relative Resistance |
|---|---|---|---|---|---|---|
| Brass | 7.200 | 43.310 | | | 22.15 | 4.515 |
| Phosphor-bronze, commercial—*Cu*, *Sn*, *P* | 8.479 | 51.005 | .000640 | .000356 | 18.80 | 5.319 |
| Aluminum bronze | 12.300 | 73.989 | .001000 | .000556 | 12.96 | 7.714 |
| Platinum rhodium,[2] *Pt* 90, *Rh* 10 | 21.142 | 127.180 | .001430* | .000795* | 7.54 | 13.260 |
| German silver,[3] *Cu* 50, *Zn* 35, *Ni* 15 | 21.250 | 127.800 | .000400 | .000220 | 7.50 | 17.300 |
| Platinum silver,[3] *Pt* 66⅔, *Ag* 33⅓ | 24.900 | 149.800 | .000310 | .000170 | 6.40 | 15.600 |
| German silver,[2] *Cu* 60, *Zn* 25, *Ni* 15 | 29.982 | 180.350 | .000273* | .000152* | 5.32 | 18.800 |
| Platinum iridium,[2] *Pt* 80, *Ir* 20 | 30.896 | 185.850 | .000822* | .000457* | 5.16 | 19.380 |

| | | | | | | |
|---|---|---|---|---|---|---|
| Platinum silver,[2] *Pt* 33⅓, *Ag* 66⅔ | 31.582 | 189.980 | .000243* | .000135* | 5.05 | 19.800 |
| Platinoid,[2] *Cu* 59, *Zn* 25.5, *Ni* 14, *W* (tungsten) 1.5 | 41.731 | 251.030 | .000310* | .000172* | 3.82 | 26.180 |
| German silver,[3] *Cu* 55, *Zn* 20, *Ni* 35 | 45.540 | 271.100 | .000330 | .000180 | 3.50 | 28.600 |
| Manganin,[2] *Cu* 84, *Ni* 4, *Mn* 12 | 46.678 | 280.790 | .000000* | | 3.41 | 29.330 |
| Constantan, *Cu* 58, *Ni* 41, *Mn* 1 | {50 52} | {300.77 312.80} | ±.000010 | .000005 | {3.19 3.07} | {31.35 32.57} |
| Reostene | 76.468 | 459.990 | .001100* | .000610* | 2.08 | 48.080 |
| Gray cast iron, *C* 3.46; graphite, 2.06; *Mn* .173; *S* .042; *Si* 2.04; *P* .151 | 114.000 | 684.000 | | | | |
| Carbon, arc light | {4400 8600} | {26500 51700} | .000520* | .000289* | {.0360 .0186} | {2778 5376} |

*These are the temperature coefficients at 15° C. or 59° F.; the others are mean temperature coefficients between the freezing and boiling temperatures of water. †Where the proportions are not given, the experimenters merely stated that they were made of the usual proportions. As this is not very definite, we cannot give the proportions. ‡This is the resistance of a piece 1 cm. long and 1 sq. cm. in sectional area at 0° C. in microhms. [2]Determined by Fleming and Dewar. [3]Given by Jackson. *Pt* = platinum; *Ag* = silver; etc.

The relative values of these units, as accepted by United States Bureau of Standards, are as follows:

1 international ohm = 1.01348 B. A. U.
1 international ohm = 1.00283 legal ohms
1 legal ohm = .997178 international ohm
1 legal ohm = 1.0106 B. A. U.
1 B. A. U. = .986699 international ohm
1 B. A. U. = .98949 legal ohm

The legal ohm has been extensively used, and many resistance coils still in use were calibrated in legal ohms; but nearly all instruments containing resistance coils that were made since about 1893 have been calibrated in international ohms.

The *resistivity*, or *specific resistance*, of a substance is usually defined as the resistance, at 32° F. or 0° C., of a piece of the substance 1 cm. long and 1 sq. cm. in sectional area. If $l$ is the length and $a$ is the sectional area of a piece of a substance whose resistivity is $\rho$, at a given temperature, then the resistance $R$ of the piece at the same temperature may be determined by the formula

$$R = \frac{\rho \times l}{a}$$

## SPECIFIC RESISTANCE OF INSULATORS

| Substance | Specific Resistance $\rho$ |
|---|---|
| Mica | 84 tregohms |
| Gutta percha | 449 tregohms |
| Hard rubber | 28 quegohms |
| Paraffin (solid) | 34 quegohms |
| Paraffin oil | 8 tregohms |
| Porcelain | 540 quegohms |
| Flint glass | 16,700 quegohms |
| Olive oil | 1 tregohm |
| Lard oil | 350 begohms |
| Benzine | 14 tregohms |
| Wood tar | 1,670 tregohms |
| Ozokerite (crude) | 450 tregohms |

The *resistivity per meter-gram* means the resistance of a piece of a substance 1 meter long (uniform in sectional area) and having a mass of 1 gram; this is the resistivity expressed in terms of the length and mass. If $k$ represents the length-mass resistivity, then, a conductor $l$ meters in length and having a mass of $m$ grams will have a resistance of

$$R=\frac{k\times l^2}{m}$$

The *mile-ohm* is a circular wire 1 mi. long having a resistance of 1 ohm. The *weight per mile-ohm* is a convenient standard for expressing the conducting quality of wires; the higher the conductivity of a metal, the less its weight per mile-ohm. The mile-ohm = weight per mile × resistance per mile. The expression that the weight per mile-ohm of a certain grade of copper is 888 lb. at 60° F. means that a wire 1 mi. long made of this copper and having a resistance of 1 ohm at 60° F. weighs 888 lb.

The weight per mile-ohm of pure copper is 859 lb. Calling the conductivity of pure copper 100, the percentage conductivity $x$ of copper weighing 888 lb. per mile-ohm may be determined as follows:

$$x : 100 = 859 : 888$$

or

$$x=\frac{859}{888}\times 100 = 96.73$$

in which $x$ = percentage conductivity.

The following formulas are useful:

$$\text{Weight of a given wire per mile} = \frac{\text{weight per mile-ohm}}{\text{resistance per mile}}$$

$$\text{Resistance per mile} = \frac{\text{weight per mile-ohm}}{\text{weight per mile}}$$

## PHYSICAL AND ELECTRICAL PROPERTIES OF METALS AND ALLOYS

*(By H. F. Parshall, M. Inst. C. E., and H. M. Hobart, S. B., in "Engineering.")*

The following table gives some physical and electrical properties of various metals and alloys. In nearly every case the name of the observer is stated. No attempt has been made to reconcile divergent measurements. The merit

## PHYSICAL AND ELECTRICAL PROPERTIES OF METALS AND ALLOYS

| Material | Specific Resistance at 0° C. (Microhms per Cent. Cube) | Microhms per Cubic Inch at 0° C. | Resistance of Wire 1 Ft. Long and .001 In. Dia. Ohms at 0° C. | Per Cent. Increase of Resistance per Deg. Cent. | Melting Point Deg. Cent. | Specific Heat Mean | Ultimate Tensile Strength. Pounds per Square Inch | Specific Gravity | Weight of 1 Cu. In. Pound |
|---|---|---|---|---|---|---|---|---|---|
| Aluminum (Neuhausen), 99% *Al*. Dewar and Fleming | 2.56 | 1.01 | 15.4 | .423 | 600 | .21 | | 2.6 | .094 |
| Aluminum (commercial), 97.5% *Al*. Dewar and Fleming | 2.67 | 1.05 | 16.0 | .435 | 600 | .21 | | 2.6 | .094 |
| Aluminum (annealed). Matthiessen | 2.89 | 1.14 | 17.4 | .139 | 600 | .21 | | 2.6 | .094 |
| Aluminum, 94%; copper, 6%. Dewar and Fleming | 2.90 | 1.14 | 17.4 | .381 | | | | | |
| Aluminum, 94%; copper, 6% (annealed). Charpentier | 3.11 | 1.23 | 18.7 | | | | | 2.95 | .107 |
| Aluminum, 94%; copper, 6% (hard). Charpentier | 3.33 | 1.31 | 20.0 | | | | | 2.95 | .107 |
| Aluminum, 94%; silver, 6%. Dewar and Fleming | 4.64 | 1.83 | 27.8 | .238 | | | | | |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Aluminum Bronze, *Cu* (90%) *Al* (10%). C. Limb...... | 12.6 | 4.96 | 75.5 | .105 | | | | 7.7 | .278 |
| Antimony (compressed). Matthiessen............ | 35.2 | 13.9 | 211 | .389 | 440 | .049 | | 6.7 | .242 |
| Bessemer soft steel, *C* (.045); *Mn* (.200); *S* (.030); *Sl* (0); *P* (.040). Hopkinson | 10.5 | 4.14 | 63.0 | | | .117 | | 7.8 | .282 |
| Bismuth (compressed). Matthiessen............ | 130 | 51.2 | 780 | .354 | 260 | .030 | | 9.8 | .354 |
| Cadmium (pure). Dewar and Fleming............ | 10.0 | 3.93 | 60.0 | .419 | | | | 8.60 | .310 |
| Chrome bronze, copper, tin, and chromium. Hospitalier | 1.64 | .645 | 9.84 | | | | 64,000 | 8.9 | .321 |
| Chrome bronze, copper, tin, and chromium.......... | 4.71 | 1.85 | 28.3 | | | | 107,000 | | |
| Chrome bronze, copper, tin, and chromium. Hospitalier | 7.80 | 3.07 | 46.8 | | | | 150,000 | 8.9 | .321 |
| Chrome steel (annealed), *C*, 687; *Mn*, .28; *S*, .02; *Sl*, .134; *P*, .043; *Cr*, 1.195. Hopkinson............. | 17.9 | 7.05 | 108 | | | | | | |
| Chrome steel (annealed), *C*, .532; *Mn*, .393; *S*, .02; *Sl*, .22; *P*, .04; *Cr*, .621. Hopkinson................ | 19.4 | 7.65 | 117 | | | | | | |
| Electrolytic copper (annealed). Lagarde....... | 1.54 | .605 | 9.25 | .445 | 1050 | .093 | | 9.05 | .327 |
| Electrolytic copper (annealed). Dewar and Fleming................... | 1.56 | .614 | 9.35 | .428 | 1050 | .093 | | 8.91 | .322 |
| Copper (annealed). Matthiessen.................... | 1.59 | .625 | 9.54 | .388 | 1050 | .093 | | 8.9 | .321 |

## PHYSICAL AND ELECTRICAL PROPERTIES OF METALS AND ALLOYS—(*Continued*)

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Copper, 50%; silver, 50% Abbott | 1.84 | .725 | 11.1 | | | | | | |
| Copper, 96%; silicon, 4% Abbott | 2.11 | .830 | 12.7 | | | | | | |
| Copper, 88%; silicon, 12% Abbott | 2.94 | 1.16 | 17.7 | | | | | | |
| Copper, 99.29%; zinc, 71% R. Haas | 1.83 | .720 | 11.0 | .373 | | | | | |
| Copper, 90.9%; zinc, 9.1% R. Haas | 3.64 | 1.43 | 21.8 | .204 | | | | | |
| Zinc, 99.5%; copper, 5% R. Haas | 5.88 | 2.31 | 35.3 | .385 | | .095 | | 7.1 | .256 |
| Copper, 65.8%; zinc, 34.2% R. Haas | 6.30 | 2.48 | 37.8 | .158 | | | | | |
| Cast copper | 4.65 | 1.83 | 27.9 | | | | | | |
| Copper, 90%; lead, 10% Abbott | 5.28 | 2.08 | 31.7 | | | | | | |
| Copper, 97%; aluminum, 3%. Dewar and Fleming | 8.84 | 3.48 | 53.0 | .090 | | | | | |
| Copper, 87%; *Ni*, 6.5%; *Al*, 6.5%. Dewar and Fleming | 14.9 | 5.87 | 89.5 | .0645 | | | | | |
| Copper, 90%; arsenic, 10% Abbott | 17.6 | 6.94 | 106 | | | | | | |
| Copper, 75%; nickel, 25% Feussner and Lindeck | 34.2 | 13.5 | 205 | .019 | | | | | |
| German silver. *Cu* (60); *Zn* (25); *Ni* (15). Feussner and Lindeck | 30.0 | 11.8 | 180 | .036 | | | | | |
| Gold (annealed), Matthiessen | 2.04 | .803 | 12.3 | .365 | 1100 | .032 | | 19.3 | .695 |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Gold, 99.9% (pure). Dewar and Fleming | 2.20 | .865 | 13.2 | .377 | 1200 | .032 | | 19.3 | .695 |
| Gold, 90%; silver, 10%. Dewar and Fleming | 6.28 | 2.47 | 37.7 | .124 | | | | | |
| Gold, 67%; silver, 33% (alloy). Matthiessen | 10.8 | 4.25 | 64.8 | .065 | | | | | |
| Iron (very pure). Dewar and Fleming | 9.07 | 3.57 | 54.5 | .625 | | .113 | | 7.8 | .282 |
| Iron with .25% $Mn$ and .01% $S$. Dewar and Fleming | 10.5 | 4.14 | 63.0 | .544 | | .113 | | 7.8 | .282 |
| White cast iron, $C$, 2.04; graphite, O, $Mn$, .386; $S$, .467; $Si$, .764; $P$, .458. Hopkinson | 56.6 | 22.3 | 340 | | 1130 | | | 7.20 | .260 |
| Spiegeleisen—$C$, 4.5%; $Mn$, 7.97%; $S$ traces, $Si$, .502% $P$, .128%. Hopkinson | 105 | 41.4 | 630 | | | | | | |
| Grey cast iron—$C$, 3.46; graphite, 2.06; $Mn$, .173; $S$, .042; $Si$, 2.04; $P$, .151. Hopkinson | 114 | 44.9 | 684 | | 1220 | | | 7.20 | .260 |
| Wrought iron (annealed). Hopkinson | 13.8 | 5.44 | 82.8 | | | | | 7.8 | .282 |
| Lead (compressed) Matthiessen | 19.5 | 7.68 | 117 | .387 | 330 | .032 | | 11.4 | .410 |
| Lead (pure). Dewar and Fleming | 20.4 | 8.04 | 123 | .411 | 330 | .032 | | 11.4 | .410 |
| Magnesium. Dewar and Fleming | 4.36 | 1.72 | 26.2 | .381 | | .25 | | 1.74 | .063 |
| Manganese steel (annealed), $C$, .674; $Mn$, 4.73; $S$, .023; $Si$, .608; $P$, .078. Hopkinson | 39.3 | 15.5 | 236 | | 1260 | | | 7.8 | .282 |

## PHYSICAL AND ELECTRICAL PROPERTIES OF METALS AND ALLOYS—(*Continued*)

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Copper, 84%; manganese, 12%; *Nl*, 4% (manganine). Dewar and Fleming | 46.7 | 18.4 | 281 | .00 | | | | 8.9 | .321 |
| Copper, 73%; manganese, 24%; nickel, 3%. Feussner and Lindeck........ | 47.7 | 18.8 | 287 | .003 | | | | 8.9 | .321 |
| Copper, 80.5%; manganese; 16.5%; nickel, 3% (manganine). Tests by G. E.Co. | 49.0 | 19.3 | 294 | .0 | | | | 8.9 | .321 |
| Copper, 83.4%; *Mn*, 15.2%; *Fe*, 1.4%. Tests by G. E. Co.................... | 50.0 | 19.7 | 300 | .0 | | | | 8.9 | .321 |
| Copper, 79.5%; *Mn*, 19.7%; *Fe*, .8%. Tests by G. E. Co.................... | 65.5 | 25.8 | 393 | .0 | | | | | |
| Manganese steel (annealed), *C*, 1.298; *Mn*, 8.74; *S*, .024; *Si*, .094; *P*, .072. Hopkinson............. | 63.2 | 24.9 | 380 | | 1260 | | | 7.8 | .282 |
| Manganese steel (Hadfield), *C*, 1.005; *Mn*, 12.36; *S*, .038; *Si*, .204; *P*, .070. Hopkinson............. | 65.5 | 25.8 | 393 | | 1260 | | | 7.8 | .282 |
| Manganese steel (Hadfield), 12% *Mn*. Dewar and Fleming................ | 67.1 | 26.4 | 401 | .127 | 1260 | | | 7.8 | .282 |
| Manganese steel (Hadfield's Hecla Foundry), *C*, 1.001; *Mn*, 11.40; *P*, .059. Tests by G. E. Co............ | 69.0 | 27.1 | 414 | .135 | 1260 | | | 7.8 | .282 |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **Manganese** steel. **Hospitalier** | 75.0 | 29.5 | 450 | .136 | 1260 | | 230,000 | 7.8 | .282 |
| **Copper**, 70%; **manganese**, 30%. **Feussner and Lindeck** | 101.0 | 39.8 | 605 | .004 | | | | | |
| **Mercury**. **Matthiessen** | 94.3 | 37.1 | 566 | .072 | | .032 | | 13.6 | .490 |
| **Nickel**. **Dewar and Fleming** | 12.3 | 4.85 | 73.7 | .62 | 1500 | .109 | | 8.9 | .321 |
| Nickel (annealed). Matthiessen | 12.4 | 4.89 | 74.4 | .50 | 1500 | .109 | | 8.9 | .321 |
| Nickel steel (Hadfield), 4.35%; nickel. Dewar and Fleming | 29.5 | 11.6 | 177 | .201 | | | | | |
| Nickeline. Lange & Co., Berlin | 40.0 | 15.8 | 240 | | | | | | |
| Palladium (pure). Dewar and Fleming | 10.2 | 4.02 | 61.1 | .354 | | | | | |
| Platinum, 67%; silver, 33% (alloy). Matthiessen | 24.2 | 9.54 | 145 | .133 | | | | | |
| Platinum, 80%; iridium, 20%. Dewar and Fleming | 30.9 | 12.2 | 186 | .082 | | | | | |
| Platinoid. Dewar and Fleming | 41.7 | 16.4 | 251 | .031 | | | | 8.8 | .318 |
| Platinoid-martino. Dewar and Fleming | 43.6 | 17.2 | 262 | | | | | 8.8 | .318 |
| Platinoid-martino | 33.0 | 13.0 | 198 | .024 | | | | | |
| Platinum (soft annealed, pure) | 8.25 | 3.24 | 49.5 | | 1775 | .032 | | 21.2 | .765 |
| Platinum (annealed). Matthiessen | 8.98 | 3.53 | 53.9 | .247 | 1775 | .032 | | 21.2 | .765 |

## PHYSICAL AND ELECTRICAL PROPERTIES OF METALS AND ALLOYS—(*Continued*)

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Platinum (pure) wire .0259 cm. in diam. Dewar and Fleming | 11.0 | 4.34 | 66.0 | .35 | 1775 | .032 | | 21.2 | .765 |
| Platinum, 90%; rhodium, 10%. Dewar and Fleming | 21.1 | 8.30 | 127 | .143 | | | | | |
| Platinum, 90%; iridium, 10% (alloy). Matthiessen | 21.6 | 8.50 | 130 | .133 | | | | | |
| Phosphor-bronze, with 9% phosphorus. Abbott | 32.5 | 12.8 | 195 | | | | | | |
| Phosphor-bronze (copper, tin, and phosphorus). Hospitalier | 1.6 | .630 | 9.6 | .394 | | | 64,000 | 8.9 | .321 |
| Phosphor-bronze (copper, tin, and phosphorus). Hospitalier | 5.6 | 2.20 | 33.6 | .394 | | | 117,000 | 8.9 | .321 |
| Phosphor-bronze, with 10% of tin. Abbott | 24.6 | 9.69 | 148 | | | | | | |
| Pure electrolytic (annealed) silver. Dewar and Fleming | 1.47 | .579 | 8.82 | .400 | 950 | .056 | | 10.5 | .379 |
| Silver (annealed). Matthiessen | 1.49 | .586 | 8.94 | .377 | 950 | .056 | | 10.5 | .379 |
| Silverine, *Cu* (77), *Ni* (17), *Fe* (2), *Zn* (2), *CO* (2). Dewar and Fleming | 2.06 | .810 | 12.4 | .285 | | | | | |
| Silver, 80%; palladium 20% Dewar and Fleming | 15.0 | 5.90 | 90.0 | | | | | | |
| Silver, 66%; platinum, 33% Dewar and Fleming | 31.6 | 12.4 | 190 | .0213 | | | | | |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Silicon-bronze (copper, tin, and silicon). Hospitalier | 1.67 | .657 | 10.0 | .152 | | | 64,000 | 8.9 | .321 |
| Silicon-bronze (copper, tin, and silicon). Hospitalier | 2.69 | 1.06 | 16.2 | | | | 93,000 | 8.9 | .321 |
| Silicon-bronze (copper, tin, and silicon). Hospitalier | 5.76 | 2.27 | 31.6 | | | | 107,000 | 8.9 | .321 |
| Silicon-bronze (copper, tin, and silicon). Hospitalier | 7.80 | 3.07 | 46.8 | | | | 143,000 | 8.9 | .321 |
| Silicon steel (annealed), *C*, .685; *Mn*, .694; *S*, .024; *Si*, 3.44; *P*, .133. Hopkinson.................... | 61.9 | 24.3 | 372 | | | | | | |
| Thallium (pure). Dewar and Fleming................ | 17.6 | 6.94 | 106 | .398 | | | | | |
| Tin (pure). Dewar and Fleming................ | 13.1 | 5.16 | 78.5 | .440 | 230 | .056 | | 7.3 | .264 |
| Tin (compressed). Matthiessen.................... | 13.1 | 5.16 | 78.5 | .365 | 230 | .055 | | 7.3 | .264 |
| Tungsten steel (annealed), *C*, 1.36; *Mn*, .36; *S*, 0; *Si*, .043; *P*, .047; tungsten, 4.65. Hopkinson........ | 22.5 | 8.86 | 135.0 | | | | | | |
| Whitworth soft steel (annealed), *C*, .090; *Mn*, .153; *S*, .016; *Si*, .0; *P*, .042. Hopkinson............. | 10.8 | 4.25 | 64.8 | | | .117 | | 7.8 | .282 |
| Zinc (very pure). Dewar and Fleming................ | 5.75 | 2.26 | 34.5 | .406 | 415 | .095 | | 7.1 | .256 |
| Zinc (compressed) Matthiessen.................... | 5.80 | 2.28 | 34.8 | .365 | 415 | .095 | | 7.1 | .256 |

## RESISTANCES AND TEMPERATURE COEFFICIENTS OF ELECTROLYTES

(*Kohlrausch, Wiedemann's Annalen*)

| Composition | Per Cent. | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 50 | 60 | 70 | 80 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Nitric Acid $HNO_3$ | Ohms | 3.90 | 2.18 | 1.64 | 1.41 | 1.31 | 1.28 | 1.31 | 1.37 | 1.59 | 1.96 | 2.54 | 3.76 |
| | Temp.Coef. | 1.50 | 1.40 | 1.40 | 1.40 | 1.40 | 1.40 | 1.40 | 1.50 | 1.6 | 1.6 | 1.5 | 1.3 |
| Hydrochloric Acid $HCl$ | Ohms | 2.55 | 1.59 | 1.35 | 1.32 | 1.39 | 1.52 | 1.70 | 1.95 | | | | |
| | Temp.Coef. | 1.60 | 1.60 | 1.60 | 1.50 | 1.50 | 1.50 | 1.50 | 1.50 | | | | |
| Sulphuric Acid $H_2SO_4$ | Ohms | 4.82 | 2.57 | 1.85 | 1.54 | 1.40 | 1.36 | 1.39 | 1.48 | 1.87 | 2.70 | 4.66 | 9.13 |
| | Temp.Coef. | 1.20 | 1.30 | 1.40 | 1.50 | 1.50 | 1.60 | 1.70 | 1.80 | 1.90 | 2.10 | 2.60 | 3.53 |
| Silver Nitrate $AgNO_3$ | Ohms | 39.30 | 21.40 | 14.70 | 11.60 | 9.50 | 8.11 | 7.18 | 6.44 | 5.44 | 4.80 | | |
| | Temp.Coef. | 2.20 | 2.20 | 2.20 | 2.10 | 2.10 | 2.10 | 2.10 | 2.10 | 2.10 | 2.10 | | |
| Caustic Potash $KOH$ | Ohms | 5.84 | 3.19 | 2.36 | 2.01 | 1.86 | 1.85 | 1.97 | 2.23 | | | | |
| | Temp.Coef. | 1.90 | 1.90 | 1.90 | 2.00 | 2.10 | 2.30 | 2.40 | 2.70 | | | | |
| Zinc Sulphate $ZnSO_4$ | Ohms | 52.30 | 31.40 | 24.10 | 21.90 | 21.40 | 22.90 | 28.50 | | | | | |
| | Temp.Coef. | 2.20 | 2.30 | 2.30 | 2.40 | 2.60 | 3.00 | 4.00 | | | | | |

| | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Copper Sulphate $CuSO_4$ | Ohms | 52.30 | 31.40 | 24.10 | | | | | | | | | |
| | Temp.Coef. | 2.20 | 2.30 | 2.30 | | | | | | | | | |
| Magnesium Sulphate $MgSO_4$ | Ohms | 39.30 | 24.10 | *20.90* | 20.90 | 24.10 | | | | | | | |
| | Temp.Coef. | 2.30 | 2.40 | 2.50 | 2.70 | 2.90 | | | | | | | |
| Sodium Sulphate $Na_2SO_4$ | Ohms | 24.80 | 14.70 | 11.30 | | | | | | | | | |
| | Temp.Coef. | 2.40 | 2.50 | 2.60 | | | | | | | | | |
| Alum $KAl(SO_4)_2$ | Ohms | 39.30 | | | | | | | | | | | |
| | Temp.Coef. | 2.00 | | | | | | | | | | | |
| Sodium Chloride $NaCl$ | Ohms | 14.90 | 8.33 | 6.15 | 5.14 | 4.70 | | | | | | | |
| | Temp.Coef. | 2.20 | 2.10 | 2.10 | 2.20 | 2.30 | | | | | | | |
| Sal Ammoniac $NH_4Cl$ | Ohms | 10.90 | 5.67 | 3.89 | 2.98 | 2.50 | | | | | | | |
| | Temp.Coef. | 2.00 | 1.00 | 1.70 | 1.60 | 1.50 | | | | | | | |

of the table is that it presents in compact form recent information previously scattered through a large number of publications and technical journals.

In the last table the first horizontal line gives the per cent. by weight of the substance dissolved in water. The *specific resistance* of each substance (opposite the word "Ohms") is given in ohms at 18° C. between opposite parallel faces of a cube of the electrolyte 1 centimeter on a side. Opposite *temperature coefficient* is given the per cent. decrease of resistance per ohm for each degree increase of temperature. The resistance also varies with the density of the solution. The resistance of the best conducting sulphuric-acid solution is about 1,000,000 times that of copper.

**Conductance** ($G$).—Conductance is that property of a substance in virtue of which it conducts an electric current. The conductance of a piece of any material 1 cm. long and 1 sq. cm. in cross-section is called its *specific conductance*, or *conductivity*, and is represented by the Greek letter $\gamma$ (gamma). The word *mho*, which is ohm spelled backwards, has been proposed as the name of the unit of conductance and conductivity, but it has not been generally accepted. Conductance is the reciprocal of resistance, and conductivity is the reciprocal of resistivity. Thus, if the resistance of 2 cm. of a piece of any material having a uniform sectional area of 1 sq. cm. is 4 ohms, its resistivity is 2 ohms, its conductance $\frac{1}{4}$ mho, and its conductivity $\frac{1}{2}$ mho. Percentage conductivity of a substance is the ratio of its conductivity to that of the standard at the same temperature. The conductivity of Matthiessen's pure copper at 0° C. is usually taken as the standard, i. e., 100%.

## RESISTANCE OF CIRCUITS

**Resistances in Series.**—When a number of resistances are connected in series, the total resistance is equal to their sum.

**Resistances in Parallel.**—The joint resistance $R$ of any number of resistances $r_1$, $r_2$, etc., in parallel, may be determined by the following formula, in the denominator of which

there should be as many terms as there are resistances in parallel:

$$R = \frac{1}{\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} + \frac{1}{r_4} + \text{etc.}}$$

## TEMPERATURE COEFFICIENT

The change in the resistance of a substance per ohm per degree change of temperature is known as the *temperature coefficient*. If $R_0$ is the resistance of a piece of wire at 0° C. and $a$ is the temperature coefficient of the substance, its resistance $R_t$ at $t^0$ may be calculated by the formula

$$R_t = R_0\,(1 + at)$$

If the resistance $R_0$ is not known, but the resistance $R_1$ at a temperature $t_1$ is known, and it is desired to determine the resistance $R_2$ at a temperature $t_2$, the following formula may be used:

$$R_2 = \frac{R_1(1 + at_2)}{(1 + at_1)}$$

The value of the temperature coefficient $a$ taken from any table must be for Fahrenheit or centigrade scales, according to which is used in expressing $t_1$ and $t_2$. The American Institute of Electrical Engineers considers .0042 as the best value of $a$ for commercial copper; for ordinary work it is quite customary to use the approximate value .004.

## TEMPERATURE COEFFICIENTS FOR COPPER WIRE

The following table gives factors by which the known resistance of good commercial copper at any temperature from 31.5° F to 85° F., inclusive, may be multiplied to give its resistance at 75° F.; or, if the resistance at 75° F. is known, the resistance at any other temperature can be found by dividing the resistance at 75° by the factor corresponding to the other temperature. Also, if the resistance at any temperature is known, the resistance at any other temperature can be found by means of the factors; for example, multiplying a known resistance at 32° F. by the corresponding factor 1.1026 and dividing the product by the factor for 55° F. 1.0454, gives the resistance at 55° F.

## TEMPERATURE FACTORS FOR COPPER WIRE

| Temperature Degrees F. | Factor | Temperature Degrees F. | Factor | Temperature Degrees F. | Factor | Temperature Degrees F. | Factor |
|---|---|---|---|---|---|---|---|
| 85.0 | .9787 | 71.5 | 1.0077 | 58.0 | 1.0384 | 44.5 | 1.0708 |
| 84.5 | .9797 | 71.0 | 1.0088 | 57.5 | 1.0395 | 44.0 | 1.0720 |
| 84.0 | .9808 | 70.5 | 1.0099 | 57.0 | 1.0407 | 43.5 | 1.0733 |
| 83.5 | .9818 | 70.0 | 1.0110 | 56.5 | 1.0419 | 43.0 | 1.0745 |
| 83.0 | .9820 | 69.5 | 1.0121 | 56.0 | 1.0430 | 42.5 | 1.0757 |
| 82.5 | .9839 | 69.0 | 1.0132 | 55.5 | 1.0442 | 42.0 | 1.0770 |
| 82.0 | .9850 | 68.5 | 1.0144 | 55.0 | 1.0454 | 41.5 | 1.0783 |
| 81.5 | .9861 | 68.0 | 1.0155 | 54.5 | 1.0466 | 41.0 | 1.0795 |
| 81.0 | .9871 | 67.5 | 1.0166 | 54.0 | 1.0478 | 40.5 | 1.0808 |
| 80.5 | .9882 | 67.0 | 1.0177 | 53.5 | 1.0490 | 40.0 | 1.0821 |
| 80.0 | .9892 | 66.5 | 1.0188 | 53.0 | 1.0501 | 39.5 | 1.0833 |
| 79.5 | .9903 | 66.0 | 1.0200 | 52.5 | 1.0513 | 39.0 | 1.0846 |
| 79.0 | .9914 | 65.5 | 1.0211 | 52.0 | 1.0525 | 38.5 | 1.0858 |
| 78.5 | .9924 | 65.0 | 1.0222 | 51.5 | 1.0537 | 38.0 | 1.0871 |
| 78.0 | .9935 | 64.5 | 1.0233 | 51.0 | 1.0549 | 37.5 | 1.0884 |
| 77.5 | .9046 | 64.0 | 1.0245 | 50.5 | 1.0561 | 37.0 | 1.0897 |
| 77.0 | .9950 | 63.5 | 1.0257 | 50.0 | 1.0573 | 36.5 | 1.0910 |
| 76.5 | .9967 | 63.0 | 1.0268 | 49.5 | 1.0585 | 36.0 | 1.0922 |
| 76.0 | .9978 | 62.5 | 1.0279 | 49.0 | 1.0598 | 35.5 | 1.0935 |
| 75.5 | .9989 | 62.0 | 1.0291 | 48.5 | 1.0610 | 35.0 | 1.0948 |
| 75.0 | 1.0000 | 61.5 | 1.0302 | 48.0 | 1.0622 | 34.5 | 1.0961 |
| 74.5 | 1.0011 | 61.0 | 1.0314 | 47.5 | 1.0634 | 34.0 | 1.0974 |
| 74.0 | 1.0022 | 60.5 | 1.0325 | 47.0 | 1.0646 | 33.5 | 1.0987 |
| 73.5 | 1.0033 | 60.0 | 1.0337 | 46.5 | 1.0659 | 33.0 | 1.1000 |
| 73.0 | 1.0044 | 59.5 | 1.0349 | 46.0 | 1.0671 | 32.5 | 1.1013 |
| 72.5 | 1.0055 | 59.0 | 1.0360 | 45.5 | 1.0683 | 32.0 | 1.1026 |
| 72.0 | 1.0066 | 58.5 | 1.0372 | 45.0 | 1.0695 | 31.5 | 1.1039 |

NOTE.—This table, which is given by Kempe in his "Handbook of Electrical Testing," is calculated from the exact formula $R_t = R_{32}\ [1 + .0023708\ (t - 32°) + .00000034548\ (t - 32°)^2]$, for pure, or good commercial, copper, as determined by Clark, Ford, and Taylor, in which $t$ is expressed in degrees Fahrenheit.

**Crocker's Method.**—Dr. F. B. Crocker gives, for finding resistance of copper at any temperature $t°\ C$ the formula

$$R_t = R_0(1 + .004t + .0000024t^2)$$

This formula is easy to apply and gives very accurate results up to 100° C., being only .1% above those given by Matthiessen's formula for conductance $G$, which is as follows:

$$G_t = G_0(1 - .0038901t + .000009009t^2)$$

**Kennelly's Method.**—In applying the formula for resistance, it is usually necessary to work to or from $R_0$; to avoid this when changing from one temperature $t°$ to another $t° + u°$, Kennelly's method, which is as follows, may be used:

$$R_{t+u} = R_t(1 + au)$$

$R_t$ is the resistance at $t°$ C., and $a$ is the temperature coefficient at $t°$; the temperature coefficient varies with the initial temperature $t°$, and is given in the accompanying table. For example, if the resistance $R_t$, when $t = 25°$ C., is known, and the resistance at 30° is desired, then $u = 5°$ C., and

| $t°C$ | $a$ |
|---|---|
| 0 | .0042 |
| 12 | .0040 |
| 25 | .0038 |
| 40 | .0036 |

$$R_{30} = R_{25}\ (1 + .0038 \times 5) = 1.019\ R_{25}$$

In North America, the table and formulas given on page 88 are considered the best.

## SIZES AND RESISTANCES OF WIRES

In expressing diameters of wires, .001 in. is called 1 *mil* and the square of the diameter of a wire in mils is called its area in *circular mils*. A wire 1 ft. long and 1 mil in diameter is 1 *mil-foot*. Resistance per mil-foot is a unit much used.

The resistance $R$ of any conductor varies directly as the length of the conductor, and inversely as the sectional area. For a cylindrical wire

$$R = \frac{m \times l}{d^2}$$

in which $m$ is the resistance per mil-foot, $l$ is the length in feet, and $d$ is the diameter in mils, $d^2$ being the sectional area in circular mils.

## WIRE GAUGES

The Brown and Sharpe (B. & S.) gauge or American wire gauge (A. W. G.), as it is sometimes called, is generally used in the United States. Other gauges and their comparative diameters are also given in the following table, dimensions of wires being given in decimal parts of an inch.

## VARIOUS WIRE GAUGES

| Number of Wire Gauge | American, or Brown & Sharpe (B. & S.) | Birmingham, or Stubs (B. W. G.) | Washburn & Moen Mfg. Co., Worcester, Mass. | Trenton Iron Co., Trenton, N. J. | G. W. Prentiss, Holyoke, Mass. | Old English, From Brass Mfrs' List | British Standard (S. W. G.) | Number of Wire Gauge |
|---|---|---|---|---|---|---|---|---|
| 0000000 | | | | | | | .500 | 0000000 |
| 000000 | | | .460 | | | | .464 | 000000 |
| 00000 | | | .430 | .450 | | | .432 | 00000 |
| 0000 | .46000 | .454 | .393 | .400 | | | .400 | 0000 |
| 000 | .40964 | .425 | .362 | .360 | .3586 | | .372 | 000 |
| 00 | .36480 | .380 | .331 | .330 | .3282 | | .348 | 00 |
| 0 | .32486 | .340 | .307 | .305 | .2994 | | .324 | 0 |
| 1 | .28930 | .300 | .283 | .285 | .2777 | | .300 | 1 |
| 2 | .25763 | .284 | .263 | .265 | .2591 | | .276 | 2 |
| 3 | .22942 | .259 | .244 | .245 | .2401 | | .252 | 3 |
| 4 | .20431 | .238 | .225 | .225 | .2230 | | .232 | 4 |
| 5 | .18194 | .220 | .207 | .205 | .2047 | | .212 | 5 |
| 6 | .16202 | .203 | .192 | .190 | .1885 | | .192 | 6 |
| 7 | .14428 | .180 | .177 | .175 | .1758 | | .176 | 7 |
| 8 | .12849 | .165 | .162 | .160 | .1605 | | .160 | 8 |
| 9 | .11443 | .148 | .148 | .145 | .1471 | | .144 | 9 |
| 10 | .10189 | .134 | .135 | .130 | .1351 | | .128 | 10 |
| 11 | .090742 | .120 | .120 | .1175 | .1205 | | .116 | 11 |
| 12 | .080808 | .109 | .105 | .1050 | .1065 | | .104 | 12 |
| 13 | .071961 | .095 | .0920 | .0925 | .0928 | | .0920 | 13 |
| 14 | .064084 | .083 | .0800 | .0800 | .0816 | .08300 | .0800 | 14 |
| 15 | .057068 | .072 | .0720 | .0700 | .0726 | .07200 | .0720 | 15 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 16 | .050820 | .065 | .0630 | .0610 | .0627 | .06500 | .0640 | 16 |
| 17 | .045257 | .058 | .0540 | .0525 | .0546 | .05800 | .0560 | 17 |
| 18 | .040303 | .049 | .0470 | .0450 | .0478 | .04900 | .0480 | 18 |
| 19 | .035890 | .042 | .0410 | .0400 | .0411 | .04000 | .0400 | 19 |
| 20 | .031961 | .035 | .0350 | .0350 | .0351 | .03500 | .0360 | 20 |
| 21 | .028462 | .032 | .0320 | .0310 | .0321 | .03150 | .0320 | 21 |
| 22 | .025347 | .028 | .0280 | .0280 | .0290 | .02950 | .0280 | 22 |
| 23 | .022571 | .025 | .0250 | .0250 | .0261 | .02700 | .0240 | 23 |
| 24 | .020100 | .022 | .0230 | .0225 | .0231 | .02500 | .0220 | 24 |
| 25 | .017900 | .020 | .0200 | .0200 | .0212 | .02300 | .0200 | 25 |
| 26 | .015940 | .018 | .0180 | .0180 | .0194 | .02050 | .0180 | 26 |
| 27 | .014195 | .016 | .0170 | .0170 | .0182 | .01875 | .0164 | 27 |
| 28 | .012641 | .014 | .0160 | .0160 | .0170 | .01650 | .0148 | 28 |
| 29 | .011257 | .013 | .0150 | .0150 | .0163 | .01550 | .0136 | 29 |
| 30 | .010025 | .012 | .0140 | .0140 | .0156 | .01375 | .0124 | 30 |
| 31 | .008928 | .010 | .0130 | .0130 | .0146 | .01225 | .0116 | 31 |
| 32 | .007950 | .009 | .0120 | .0120 | .0136 | .01125 | .0108 | 32 |
| 33 | .007080 | .008 | .0110 | .0110 | .0130 | .01025 | .0100 | 33 |
| 34 | .006305 | .007 | .0100 | .0100 | .0118 | .00950 | .0092 | 34 |
| 35 | .005615 | .005 | .0095 | .0095 | .0109 | .00900 | .0084 | 35 |
| 36 | .005000 | .004 | .0090 | .0090 | .0100 | .00750 | .0076 | 36 |
| 37 | .004453 | | .0085 | .0085 | .0095 | .00650 | .0068 | 37 |
| 38 | .003965 | | .0080 | .0080 | .0090 | .00575 | .0066 | 38 |
| 39 | .003531 | | .0075 | .0075 | .0083 | .00500 | .0052 | 39 |
| 40 | .003145 | | .0070 | .0070 | .0078 | .00450 | .0048 | 40 |
| 41 | | | | | | | .0044 | 41 |
| 42 | | | | | | | .0040 | 42 |

## TEMPERATURE COEFFICIENTS FOR COPPER

(*i is the initial temperature in degrees C; a, the temperature coefficient in per cent. per ohm per degree C.*)

| *i* | *a* | *i* | *a* | *i* | *a* |
|---|---|---|---|---|---|
| 0 | .4200 | 17 | .3920 | 34 | .3675 |
| 1 | .4182 | 18 | .3905 | 35 | .3662 |
| 2 | .4165 | 19 | .3890 | 36 | .3648 |
| 3 | .4148 | 20 | .3875 | 37 | .3635 |
| 4 | .4131 | 21 | .3860 | 38 | .3622 |
| 5 | .4114 | 22 | .3845 | 39 | .3609 |
| 6 | .4097 | 23 | .3830 | 40 | .3596 |
| 7 | .4080 | 24 | .3815 | 41 | .3583 |
| 8 | .4063 | 25 | .3805 | 42 | .3570 |
| 9 | .4047 | 26 | .3786 | 43 | .3557 |
| 10 | .4031 | 27 | .3772 | 44 | .3545 |
| 11 | .4015 | 28 | .3758 | 45 | .3532 |
| 12 | .3999 | 29 | .3744 | 46 | .3520 |
| 13 | .3983 | 30 | .3730 | 47 | .3508 |
| 14 | .3967 | 31 | .3716 | 48 | .3495 |
| 15 | .3951 | 32 | .3702 | 49 | .3483 |
| 16 | .3936 | 33 | .3689 | 50 | .3471 |

The American Institute of Electrical Engineers gives this table and following formulas for calculating resistance of copper wire at a temperature $u^\circ$ C. above an initial temperature $i^\circ$ C. and the rise in degrees C. above an initial temperature $i^\circ$ C.:

$$R_{i+u}=R_i\left(1+\frac{au}{100}\right)$$

$$u=(238.1+i)\left(\frac{R_{i+u}}{R_i}-1\right)$$

in which $a$ is the temperature coefficient given in the table corresponding to the initial temperature $i^\circ$ C., and $u$ is the rise in temperature above the initial temperature $i^\circ$ C.

## COPPER WIRE

The specific gravity of pure annealed copper at 60° F. is 8.89 to 8.91. One cubic inch of it weighs .32 lb., and its melting point is about 2,100° F. By the process of hard drawing, the tensile strength of copper is greatly increased

without greatly decreasing its conductivity. Since the conductivity vaiies, even with a variation of less than .02 of 1% of impurity, scarcely two samples can be obtained with exactly the same conductivity. Authorities very seldom agree on the specific resistance or temperature coefficient of copper.

**Matthiessen's Standards.**—Copper-wire tables are usually based on the grade of copper used by Matthiessen in determining the resistance of copper. The following are based on his measurements on copper with a specific gravity of 8.89.

| Dimensions | Resistance at 0° C. International Ohms |
|---|---|
| Mil-foot soft copper | 9.590 |
| Meter-gram soft copper | .141729 |
| Meter-millimeter soft copper | .02030 |
| Centimeter cube soft copper | .000001594 |
| Meter-gram hard-drawn copper | .1449 |
| Ratio hard- to soft-drawn copper | 1.0226 |

## COPPER-WIRE TABLES

In the copper-wire tables to follow, the values in the columns marked † at the top are taken from the table prepared by the American Institute of Electrical Engineers and are correct to one part in two thousand. These values were computed for Matthiessen's standard copper from the data in the preceding table, and from temperature coefficients of resistance for 20° C. = 1.07968, for 50° C. = 1.20625, and for 80° C = 1.33681; 1 ft. = .3048028 m.; 1 lb. = 453.59256 g.

Matthiessen's standard of resistivity may be permanently recognized, but the temperature coefficient that he introduced, and which is here used, may in future undergo slight revision. The values in the columns marked with a * were computed especially for this pocketbook from data given in other parts of the table and the ratio of resistivity of hard to soft copper.

The average of a number of the most reliable determinations gives the resistance of a meter-gram of pure annealed

## SIZE AND WEIGHT OF ANNEALED COPPER WIRE

(*B. & S. Gauge, Specific Gravity, 8.89*)

| B. & S. Gauge | Diameter in Mils $d$ † | Area in Circular Mils $d^2$ † | Area in Sq. In. $\frac{d^2 \times .7854}{1,000,000}$ * | Pounds per 1,000 Feet † | Pounds per Mile * | Feet per Pound † |
|---|---|---|---|---|---|---|
| 0000 | 460.00 | 211,600 | .16619 | 640.5 | 3,381.4 | 1.561 |
| 000 | 409.64 | 167,805 | .13179 | 508.0 | 2,682.2 | 1.969 |
| 00 | 364.80 | 133,079 | .10452 | 402.8 | 2,126.8 | 2.482 |
| 0 | 324.86 | 105,534 | .082887 | 319.5 | 1,686.9 | 3.130 |
| 1 | 289.30 | 83,694 | .065732 | 253.3 | 1,337.2 | 3.947 |
| 2 | 257.63 | 66,373 | .052128 | 200.9 | 1,060.6 | 4.977 |
| 3 | 229.42 | 52,634 | .041339 | 159.3 | 841.09 | 6.276 |
| 4 | 204.31 | 41,742 | .032784 | 126.4 | 667.39 | 7.914 |
| 5 | 181.94 | 33,102 | .025999 | 100.2 | 529.06 | 9.980 |
| 6 | 162.02 | 26,250 | .020618 | 79.46 | 419.55 | 12.58 |
| 7 | 144.28 | 20,816 | .016351 | 63.02 | 332.75 | 15.87 |
| 8 | 128.49 | 16,509 | .012967 | 49.98 | 263.89 | 20.01 |
| 9 | 114.43 | 13,094 | .010283 | 39.63 | 209.24 | 25.23 |
| 10 | 101.89 | 10,381 | .0081548 | 31.43 | 165.95 | 31.82 |
| 11 | 90.742 | 8,234.0 | .0064656 | 24.93 | 131.63 | 40.12 |
| 12 | 80.808 | 6,529.9 | .0051287 | 19.77 | 104.39 | 50.59 |
| 13 | 71.961 | 5,178.4 | .0040672 | 15.68 | 82.791 | 63.79 |
| 14 | 64.084 | 4,106.8 | .0032254 | 12.43 | 76.191 | 80.44 |
| 15 | 57.068 | 3,256.7 | .0025579 | 9.858 | 52.050 | 101.4 |

| | | | | | | |
|---|---|---|---|---|---|---|
| 16 | 50.820 | 2,582.9 | .0020285 | 7.818 | 41.277 | 127.9 |
| 17 | 45.257 | 2,048.2 | .0016087 | 6.200 | 32.736 | 161.3 |
| 18 | 40.303 | 1,624.3 | .0012757 | 4.917 | 25.960 | 203.4 |
| 19 | 35.890 | 1,288.1 | .0010117 | 3.899 | 20.595 | 256.5 |
| 20 | 31.961 | 1,021.5 | .00080231 | 3.092 | 16.324 | 323.4 |
| 21 | 28.462 | 810.10 | .00063626 | 2.452 | 12.946 | 407.8 |
| 22 | 25.347 | 642.40 | .00050457 | 1.945 | 10.268 | 514.2 |
| 23 | 22.571 | 509.45 | .00040015 | 1.542 | 8.142 | 684.4 |
| 24 | 20.100 | 404.01 | .00031733 | 1.223 | 6.457 | 817.6 |
| 25 | 17.900 | 320.40 | .00025166 | .9699 | 5.121 | 1020 |
| 26 | 15.940 | 254.10 | .00019958 | .7692 | 4.061 | 1300 |
| 27 | 14.195 | 201.50 | .00015827 | .6100 | 3.221 | 1639 |
| 28 | 12.641 | 159.79 | .00012551 | .4837 | 2.554 | 2067 |
| 29 | 11.257 | 126.72 | .000099536 | .3836 | 2.025 | 2607 |
| 30 | 10.025 | 100.50 | .000078936 | .3042 | 1.606 | 3287 |
| 31 | 8.928 | 79.70 | .000062599 | .2413 | 1.274 | 4145 |
| 32 | 7.950 | 63.21 | .000049643 | .1913 | 1.010 | 5227 |
| 33 | 7.080 | 50.13 | .000039368 | .1517 | .801 | 6591 |
| 34 | 6.305 | 39.75 | .000031221 | .1203 | .635 | 8311 |
| 35 | 5.615 | 31.52 | .000024759 | .09543 | .504 | 10480 |
| 36 | 5.000 | 25.00 | .000019635 | .07568 | .400 | 13210 |
| 37 | 4.453 | 19.82 | .000015574 | .06001 | .317 | 16660 |
| 38 | 3.965 | 15.72 | .000012345 | .04759 | .251 | 21010 |
| 39 | 3.531 | 12.47 | .0000097923 | .03774 | .199 | 26500 |
| 40 | 3.145 | 9.89 | .0000077634 | .02993 | .158 | 33410 |

## RESISTANCE OF ANNEALED COPPER WIRE

*(B. & S. Gauge)*

| B. & S. Gauge | Pounds per Ohm | | | Feet per Ohm | | |
|---|---|---|---|---|---|---|
| | At 20° C. or 68° F. † | At 50° C. or 122° F. † | At 80° C. or 176° F. † | At 20° C. or 68° F. † | At 50° C. or 122° F. † | At 80° C. or 176° F † |
| 0000 | 13,090 | 11,720 | 10,570 | 20,440 | 18,290 | 16,510 |
| 000 | 8,232 | 7,369 | 6,647 | 16,210 | 14,510 | 13,090 |
| 00 | 5,177 | 4,634 | 4,182 | 12,850 | 11,500 | 10,380 |
| 0 | 3,256 | 2,914 | 2,630 | 10,190 | 9,123 | 8,232 |
| 1 | 2,048 | 1,833 | 1,654 | 8,083 | 7,235 | 6,528 |
| 2 | 1,288 | 1,153 | 1,040 | 6,410 | 5,738 | 5,177 |
| 3 | 810.0 | 725.0 | 654.2 | 5,084 | 4,550 | 4,106 |
| 4 | 509.4 | 455.9 | 411.4 | 4,031 | 3,608 | 3,256 |
| 5 | 320.4 | 286.7 | 258.7 | 3,197 | 2,862 | 2,582 |
| 6 | 201.5 | 180.3 | 162.7 | 2,535 | 2,269 | 2,048 |
| 7 | 126.7 | 113.4 | 102.3 | 2,011 | 1,800 | 1,624 |
| 8 | 79.69 | 71.33 | 64.36 | 1,595 | 1,427 | 1,288 |
| 9 | 50.12 | 44.86 | 40.48 | 1,265 | 1,132 | 1,021 |
| 10 | 31.52 | 28.21 | 25.46 | 1,003 | 897.6 | 809.9 |
| 11 | 19.82 | 17.74 | 16.01 | 795.3 | 711.8 | 642.3 |
| 12 | 12.47 | 11.16 | 10.07 | 630.7 | 564.5 | 509.4 |
| 13 | 7.840 | 7.017 | 6.332 | 500.1 | 447.7 | 404.0 |
| 14 | 4.931 | 4.413 | 3.982 | 396.6 | 355.0 | 320.3 |

| | | | | | | |
|---|---|---|---|---|---|---|
| 15 | 3.101 | 2.776 | 2.504 | 314.5 | 281.5 | 254.0 |
| 16 | 1.950 | 1.746 | 1.575 | 249.4 | 223.3 | 201.5 |
| 17 | 1.226 | 1.098 | .9906 | 197.8 | 177.1 | 159.8 |
| 18 | .7713 | .6904 | .6230 | 156.9 | 140.4 | 126.7 |
| 19 | .4851 | .4342 | .3918 | 124.4 | 111.4 | 100.5 |
| 20 | .3051 | .2731 | .2464 | 98.66 | 88.31 | 79.68 |
| 21 | .1919 | .1717 | .1550 | 78.24 | 70.03 | 63.19 |
| 22 | .1207 | .1080 | .09746 | 62.05 | 55.54 | 50.11 |
| 23 | .07589 | .06793 | .06129 | 49.21 | 44.04 | 39.74 |
| 24 | .04773 | .04272 | .03855 | 39.02 | 34.93 | 31.52 |
| 25 | .03002 | .02687 | .02424 | 31.29 | 28.01 | 24.99 |
| 26 | .01888 | .01690 | .01525 | 24.54 | 21.97 | 19.82 |
| 27 | .01187 | .01063 | .009588 | 19.46 | 17.42 | 15.72 |
| 28 | .007466 | .006683 | .006030 | 15.43 | 13.82 | 12.47 |
| 29 | .004696 | .004203 | .003792 | 12.24 | 10.96 | 9.886 |
| 30 | .002953 | .002643 | .002385 | 9.707 | 8.688 | 7.840 |
| 31 | .001857 | .001662 | .001500 | 7.698 | 6.890 | 6.217 |
| 32 | .001168 | .001045 | .0009436 | 6.105 | 5.464 | 4.930 |
| 33 | .0007346 | .0006575 | .0005933 | 4.841 | 4.333 | 3.910 |
| 34 | .0004620 | .0004135 | .0003731 | 3.839 | 3.436 | 3.101 |
| 35 | .0002905 | .0002601 | .0002347 | 3.045 | 2.725 | 2.459 |
| 36 | .0001827 | .0001636 | .0001476 | 2.414 | 2.161 | 1.950 |
| 37 | .0001149 | .0001029 | .00009281 | 1.915 | 1.714 | 1.547 |
| 38 | .00007210 | .00006454 | .00005824 | 1.519 | 1.359 | 1.226 |
| 39 | .00004545 | .00004068 | .00003671 | 1.204 | 1.078 | .9726 |
| 40 | .00002858 | .00002559 | .00002309 | .9550 | .8548 | .7713 |

## RESISTANCE OF ANNEALED COPPER WIRE

*(B. & S. Gauge)*

| B. & S. Gauge | Ohms per Pound | | | Ohms per 1,000 Feet | | |
|---|---|---|---|---|---|---|
| | At 20° C. or 68° F. † | At 50° C. or 122° F. † | At 80° C. or 176° F. † | At 20° C. or 68° F. † | At 50° C. or 122° F. † | At 80° C. or 176° F. † |
| 0000 | .00007639 | .00008535 | .00009459 | .04893 | .05467 | .06058 |
| 000 | .0001215 | .0001357 | .0001504 | .06170 | .06404 | .07640 |
| 00 | .0001931 | .0002158 | .0002391 | .07780 | .08692 | .09633 |
| 0 | .0003071 | .0003431 | .0003803 | .09811 | .1096 | .1215 |
| 1 | .0004883 | .0005456 | .0006046 | .1237 | .1382 | .1532 |
| 2 | .0007765 | .0008675 | .0009614 | .1560 | .1743 | .1932 |
| 3 | .001235 | .001379 | .001529 | .1967 | .2198 | .2435 |
| 4 | .001963 | .002193 | .002431 | .2480 | .2771 | .3071 |
| 5 | .003122 | .003487 | .003865 | .3128 | .3495 | .3873 |
| 6 | .004963 | .005545 | .006145 | .3944 | .4406 | .4883 |
| 7 | .007892 | .008817 | .009772 | .4973 | .5556 | .6158 |
| 8 | .01255 | .01402 | .01554 | .6271 | .7007 | .7765 |
| 9 | .01995 | .02229 | .02471 | .7908 | .8835 | .9791 |
| 10 | .03173 | .03545 | .03928 | .9972 | 1.114 | 1.235 |
| 11 | .05045 | .05636 | .06246 | 1.257 | 1.405 | 1.557 |
| 12 | .08022 | .08962 | .09932 | 1.586 | 1.771 | 1.963 |
| 13 | .1276 | .1425 | 1579 | 1.999 | 2.234 | 2.476 |
| 14 | .2028 | .2266 | .2511 | 2.521 | 2.817 | 3.122 |

| | | | | | | |
|---|---|---|---|---|---|---|
| 15 | .3225 | .3603 | .3993 | 3.179 | 3.552 | 3.936 |
| 16 | .5128 | .5729 | .6349 | 4.009 | 4.479 | 4.964 |
| 17 | .8153 | .9109 | 1.010 | 5.055 | 5.648 | 6.259 |
| 18 | 1.296 | 1.448 | 1.605 | 6.374 | 7.122 | 7.892 |
| 19 | 2.061 | 2.303 | 2.552 | 8.038 | 8.980 | 9.952 |
| 20 | 3.278 | 3.662 | 4.058 | 10.14 | 11.32 | 12.55 |
| 21 | 5.212 | 5.823 | 6.453 | 12.78 | 14.28 | 15.83 |
| 22 | 8.287 | 9.259 | 10.26 | 16.12 | 18.01 | 19.96 |
| 23 | 13.18 | 14.72 | 16.32 | 20.32 | 22.71 | 25.16 |
| 24 | 20.95 | 23.41 | 25.94 | 25.63 | 28.63 | 31.73 |
| 25 | 33.32 | 37.22 | 41.25 | 32.31 | 36.10 | 40.01 |
| 26 | 52.97 | 59.18 | 65.59 | 40.75 | 45.52 | 50.45 |
| 27 | 84.23 | 94.11 | 104.3 | 51.38 | 57.40 | 63.62 |
| 28 | 133.9 | 149.6 | 165.8 | 64.79 | 72.39 | 80.22 |
| 29 | 213.0 | 237.9 | 263.7 | 81.70 | 91.28 | 101.2 |
| 30 | 338.6 | 378.3 | 419.3 | 103.0 | 115.1 | 127.6 |
| 31 | 538.4 | 601.6 | 666.7 | 129.9 | 145.1 | 160.8 |
| 32 | 856.2 | 956.5 | 1060 | 163.8 | 183.0 | 202.8 |
| 33 | 1361 | 1521 | 1685 | 206.6 | 230.8 | 255.8 |
| 34 | 2165 | 2418 | 2680 | 260.5 | 291.0 | 322.5 |
| 35 | 3441 | 3845 | 4262 | 328.4 | 366.9 | 406.7 |
| 36 | 5473 | 6114 | 6776 | 414.2 | 462.7 | 512.9 |
| 37 | 8702 | 9722 | 10770 | 522.2 | 583.5 | 646.6 |
| 38 | 13870 | 15490 | 17170 | 658.5 | 735.7 | 815.4 |
| 39 | 22000 | 24580 | 27240 | 830.4 | 927.7 | 1,028 |
| 40 | 34980 | 39080 | 43320 | 1,047 | 1,170 | 1,296 |

## RESISTANCE OF ANNEALED AND HARD-DRAWN COPPER WIRE

*(B. & S. Gauge)*

| B. & S. Gauge | Annealed Ohms per Mile | Hard-Drawn. At 20° C. or 68° F. | | |
|---|---|---|---|---|
| | At 20° C. or 68° F. | Ohms per Pound * | Ohms per 1,000 Feet * | Ohms per Mile * |
| 0000 | .25835 | .00007812 | .050036 | .26419 |
| 000 | .32577 | .0001242 | .063094 | .33314 |
| 00 | .41079 | .0001975 | .079558 | .42007 |
| 0 | .51802 | .0003140 | .10033 | .52973 |
| 1 | .65314 | .0004993 | .12649 | .66790 |
| 2 | .82368 | .0007940 | .15953 | .84230 |
| 3 | 1.0386 | .001263 | .20114 | 1.0621 |
| 4 | 1.3094 | .002007 | .25361 | 1.3392 |
| 5 | 1.6516 | .003193 | .31987 | 1.6889 |
| 6 | 2.0825 | .005075 | .40332 | 2.1295 |
| 7 | 2.6258 | .008070 | .50854 | 2.6850 |
| 8 | 3.3111 | .01283 | .64127 | 3.3859 |
| 9 | 4.1753 | .02040 | .80876 | 4.2769 |
| 10 | 5.2657 | .03245 | 1.0199 | 5.3848 |
| 11 | 6.6369 | .05159 | 1.2854 | 6.7869 |
| 12 | 8.3741 | .08203 | 1.6218 | 8.5633 |

| | | | | |
|---|---|---|---|---|
| 13 | 10.555 | .1305 | 2.0443 | 10.794 |
| 14 | 13.311 | .2074 | 2.5779 | 13.612 |
| 15 | 16.785 | .3298 | 3.2508 | 17.165 |
| 16 | 21.168 | .5244 | 4.0996 | 21.646 |
| 17 | 26.691 | .8337 | 5.1692 | 27.294 |
| 18 | 33.655 | 1.325 | 6.5183 | 34.416 |
| 19 | 42.441 | 2.108 | 8.2196 | 43.400 |
| 20 | 53.539 | 3.352 | 10.372 | 54.749 |
| 21 | 67.479 | 5.330 | 13.069 | 78.004 |
| 22 | 85.114 | 8.444 | 16.484 | 87.038 |
| 23 | 107.29 | | | |
| 24 | 135.53 | | | |
| 25 | 170.59 | | | |
| 26 | 215.16 | | | |
| 27 | 271.29 | | | |
| 28 | 342.09 | | | |
| 29 | 431.37 | | | |
| 30 | 543.84 | | | |
| 31 | 685.87 | | | |
| 32 | 864.87 | | | |
| 33 | 1,090.8 | | | |
| 34 | 1,375.5 | | | |
| 35 | 1,734.0 | | | |
| 36 | 2,187.0 | | | |
| 37 | 2,757.3 | | | |
| 38 | 3,476.8 | | | |
| 39 | 4,384.5 | | | |
| 40 | 5,528.2 | | | |

NOTE.—In these tables, the resistances are all based on Matthiessen's Standard.

## SIZE AND WEIGHT OF ANNEALED COPPER WIRE

*(B. W. G. or Stubbs Gauge; Specific Gravity, 8.89)*

| Gauge No. (B. W. G.) | Diameter in Mils $d$ | Area in Circular Mils $d^2$ † | Pounds per 1,000 Feet † | Pounds per Mile * | Pounds per Ohm At 20° C. or 68° F. † | Pounds per Ohm At 50° C. or 122° F. † | Pounds per Ohm At 80° C. or 176° F. † |
|---|---|---|---|---|---|---|---|
| 0000 | 454 | 206,116 | 623.9 | 3,294 | 12,420 | 11,120 | 10,570 |
| 000 | 425 | 180,625 | 546.8 | 2,887 | 9,538 | 8,537 | 7,702 |
| 00 | 380 | 144,400 | 437.1 | 2,308 | 6,096 | 5,456 | 4,924 |
| 0 | 340 | 115,600 | 349.9 | 1,847 | 3,907 | 3,497 | 3,155 |
| 1 | 300 | 90,000 | 272.4 | 1,438 | 2,368 | 2,120 | 1,913 |
| 2 | 284 | 80,656 | 244.1 | 1,289 | 1,902 | 1,702 | 1,536 |
| 3 | 259 | 67,081 | 203.1 | 1,072 | 1,316 | 1,178 | 1,063 |
| 4 | 238 | 56,644 | 171.5 | 905 | 938.0 | 839.6 | 757.6 |
| 5 | 220 | 48,400 | 146.5 | 773 | 684.9 | 613.0 | 553.1 |
| 6 | 203 | 41,209 | 124.7 | 659 | 496.5 | 444.4 | 401.0 |
| 7 | 180 | 32,400 | 98.08 | 518 | 306.9 | 274.7 | 247.9 |
| 8 | 165 | 27,225 | 82.41 | 435 | 216.7 | 194.0 | 175.0 |
| 9 | 148 | 21,904 | 66.30 | 350 | 140.3 | 125.6 | 113.3 |
| 10 | 134 | 17,956 | 54.35 | 287 | 94.26 | 84.37 | 76.13 |
| 11 | 120 | 14,400 | 43.59 | 230 | 60.62 | 54.26 | 48.96 |
| 12 | 109 | 11,881 | 35.96 | 190 | 41.27 | 36.94 | 33.33 |
| 13 | 95 | 9,025 | 27.32 | 144 | 23.81 | 21.31 | 19.23 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 14 | 83 | 6,889 | 20.85 | 110 | 13.87 | 12.42 | 11.21 |
| 15 | 72 | 5,184 | 15.69 | 83.0 | 7.857 | 7.032 | 6.346 |
| 16 | 65 | 4,225 | 12.79 | 68.0 | 5.219 | 4.671 | 4.215 |
| 17 | 58 | 3,364 | 10.18 | 54.0 | 3.308 | 2.961 | 2.672 |
| 18 | 49 | 2,401 | 7.268 | 38.4 | 1.658 | 1.509 | 1.361 |
| 19 | 42 | 1,764 | 5.340 | 28.2 | .9097 | .8143 | .7347 |
| 20 | 35 | 1,225 | 3.708 | 19.6 | .4387 | .3927 | .3543 |
| 21 | 32 | 1,024 | 3.100 | 16.4 | .3066 | .2744 | .2476 |
| 22 | 28 | 784 | 2.373 | 12.5 | .1797 | .1608 | .1451 |
| 23 | 25 | 625 | 1.892 | 10.0 | .1142 | .1022 | .09224 |
| 24 | 22 | 484 | 1.465 | 7.70 | .06849 | .06130 | .05531 |
| 25 | 20 | 400 | 1.211 | 6.40 | .04678 | .04187 | .03778 |
| 26 | 18 | 324 | 9.808 | 5.20 | .03069 | .02747 | .02479 |
| 27 | 16 | 256 | .7749 | 4.10 | .01916 | .01715 | .01548 |
| 28 | 14 | 196 | .5933 | 3.10 | .01123 | .01005 | .009071 |
| 29 | 13 | 169 | .5116 | 2.70 | .008350 | .007474 | .006744 |
| 30 | 12 | 144 | .4359 | 2.30 | .006062 | .005426 | .004896 |
| 31 | 10 | 100 | .3027 | 1.60 | .002924 | .002617 | .002361 |
| 32 | 9 | 81 | .2452 | 1.30 | .001918 | .001717 | .001549 |
| 33 | 8 | 64 | .1937 | 1.02 | .001197 | .001072 | .0009672 |
| 34 | 7 | 49 | .1483 | .780 | .0007019 | .0006283 | .0005669 |
| 35 | 5 | 25 | .07568 | .400 | .0001827 | .0001636 | .0001476 |
| 36 | 4 | 16 | .04843 | .256 | .00007484 | .00006699 | .00006045 |

## LENGTH AND RESISTANCE OF ANNEALED COPPER WIRE

*(B. W. G. or Stubbs Gauge)*

| Gauge No. (B. W. G.) | Feet per Pound † | Feet per Ohm | | | Ohms per Pound | | |
|---|---|---|---|---|---|---|---|
| | | At 20° C. or 68° F. † | At 50° C. or 122° F. † | At 80° C. or 176° F. † | At 20° C. or 68° F. † | At 50° C. or 122° F. † | At 80° C. or 176° F. † |
| 0000 | 1.603 | 19,910 | 17,820 | 16,080 | .00008051 | .00008996 | .00009969 |
| 000 | 1.829 | 17,450 | 15,620 | 14,090 | .0001048 | .0001171 | .0001298 |
| 00 | 2.288 | 13,950 | 12,480 | 11,260 | .0001640 | .0001833 | .0002031 |
| 0 | 2.858 | 11,160 | 9,993 | 9,017 | .0002560 | .0002860 | .0003169 |
| 1 | 3.671 | 8,692 | 7,780 | 7,020 | .0004223 | .0004718 | .0005228 |
| 2 | 4.096 | 7,790 | 6,973 | 6,292 | .0005258 | .0005874 | .0006510 |
| 3 | 4.925 | 6,479 | 5,799 | 5,233 | .0007601 | .0008492 | .0009412 |
| 4 | 5.832 | 5,471 | 4,897 | 4,419 | .001066 | .001191 | .001320 |
| 5 | 6.826 | 4,675 | 4,184 | 3,775 | .001460 | .001631 | .001808 |
| 6 | 8.017 | 3,980 | 3,562 | 3,215 | .002014 | .002250 | .002494 |
| 7 | 10.20 | 3,129 | 2,801 | 2,527 | .003258 | .003640 | .004034 |
| 8 | 12.13 | 2,629 | 2,354 | 2,124 | .004615 | .005156 | .005714 |
| 9 | 15.08 | 2,116 | 1,894 | 1,709 | .007129 | .007965 | .008827 |
| 10 | 18.40 | 1,734 | 1,552 | 1,401 | .01061 | .01185 | .01314 |
| 11 | 22.94 | 1,391 | 1,245 | 1,123 | .01650 | .01843 | .02042 |
| 12 | 27.81 | 1,147 | 1,027 | 926.9 | .02423 | .02707 | .03000 |
| 13 | 36.60 | 871.7 | 780.2 | 704.0 | .04199 | .04692 | .05200 |
| 14 | 47.95 | 665.4 | 595.5 | 537.4 | .07207 | .08052 | .08924 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 15 | 63.73 | 500.7 | 448.1 | 404.4 | .1273 | .1422 | .1576 |
| 16 | 78.19 | 408.1 | 365.2 | 329.6 | .1916 | .2141 | .2373 |
| 17 | 98.23 | 324.9 | 290.8 | 262.4 | .3023 | .3377 | .3742 |
| 18 | 137.6 | 231.9 | 207.6 | 187.3 | .5933 | .6629 | .7346 |
| 19 | 187.3 | 170.4 | 152.5 | 137.6 | 1.099 | 1.228 | 1.361 |
| 20 | 269.7 | 118.3 | 105.9 | 95.56 | 2.279 | 2.547 | 2.822 |
| 21 | 322.6 | 98.10 | 88.52 | 79.88 | 3.262 | 3.644 | 4.039 |
| 22 | 421.4 | 75.72 | 67.78 | 61.16 | 5.565 | 6.217 | 6.890 |
| 23 | 528.6 | 60.36 | 54.03 | 48.75 | 8.756 | 9.783 | 10.84 |
| 24 | 682.6 | 46.75 | 41.84 | 37.75 | 14.60 | 16.31 | 18.08 |
| 25 | 825.9 | 38.63 | 34.58 | 31.20 | 21.38 | 23.88 | 26.47 |
| 26 | 1,020 | 31.29 | 28.01 | 25.27 | 32.58 | 36.40 | 40.34 |
| 27 | 1,290 | 24.73 | 22.13 | 19.97 | 52.19 | 58.31 | 64.62 |
| 28 | 1,685 | 18.93 | 16.94 | 15.29 | 89.04 | 99.48 | 110.2 |
| 29 | 1,955 | 16.32 | 14.61 | 13.18 | 119.8 | 133.8 | 148.3 |
| 30 | 2,294 | 13.91 | 12.45 | 11.23 | 165.0 | 184.3 | 204.2 |
| 31 | 3,304 | 9.658 | 8.645 | 7.800 | 342.0 | 382.1 | 423.5 |
| 32 | 4,078 | 7.823 | 7.002 | 6.318 | 521.3 | 582.5 | 645.5 |
| 33 | 5,162 | 6.181 | 5.533 | 4.992 | 835.1 | 933.0 | 1,034 |
| 34 | 6,742 | 4.733 | 4.236 | 3.822 | 1,425 | 1,792 | 1,764 |
| 35 | 13,210 | 2.414 | 2.161 | 1.950 | 5,473 | 6,114 | 6,776 |
| 36 | 20,650 | 1.545 | 1.383 | 1.248 | 13,360 | 14,930 | 16,540 |

## RESISTANCE OF ANNEALED AND HARD-DRAWN COPPER WIRE

*(B. W. G. or Stubbs Gauge)*

| Gauge No. (B. W. G.) | Annealed | | | | Hard-Drawn. At 20° C. or 68° F. | | |
|---|---|---|---|---|---|---|---|
| | Ohms per 1,000 Ft. | | | Ohms per Mile At 20° C. or 68° F. * | Ohms per Pound * | Ohms per 1,000 Feet * | Ohms per Mile * |
| | At 20° C. or 68° F. † | At 50° C. or 122° F. † | At 80° C. or 176° F. † | | | | |
| 0000 | .05023 | .05612 | .06220 | .2652 | .00008233 | .05137 | .2712 |
| 000 | .05732 | .06404 | .07097 | .3026 | .0001072 | .05862 | .3095 |
| 00 | 07170 | .08011 | .08878 | .3786 | .0001677 | .07332 | .3871 |
| 0 | .08957 | .1001 | .1109 | .4729 | .0002618 | .09159 | .4836 |
| 1 | .1150 | .1285 | .1424 | .6072 | .0004318 | .1176 | .6209 |
| 2 | .1284 | .1434 | .1589 | .6780 | .0005377 | .1313 | .6833 |
| 3 | .1543 | .1724 | .1911 | .8147 | .0007773 | .1578 | .8332 |
| 4 | .1828 | .2042 | .2263 | .9652 | .001090 | .1869 | .9868 |
| 5 | .2139 | .2390 | .2649 | 1.129 | .001493 | .2187 | 1.155 |
| 6 | .2513 | .2807 | .3111 | 1.327 | .002060 | .2570 | 1.357 |
| 7 | .3196 | .3570 | .3957 | 1.687 | .003331 | .3268 | 1.726 |
| 8 | .3803 | .4249 | .4709 | 2.008 | .004719 | .3889 | 2.053 |
| 9 | .4727 | .5281 | .5853 | 2.496 | .007290 | .4834 | 2.552 |
| 10 | .5766 | .6442 | .7140 | 3.044 | .01085 | .5896 | 3.113 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 11 | .7190 | .8033 | .8903 | 3.796 | .01687 | .7352 | 3.882 |
| 12 | .8715 | .9736 | 1.079 | 4.602 | .02479 | .8912 | 4.706 |
| 13 | 1.147 | 1.282 | 1.420 | 6.056 | .04294 | 1.173 | 6.193 |
| 14 | 1.503 | 1.679 | 1.861 | 7.936 | .07370 | 1.537 | 8.115 |
| 15 | 1.997 | 2.231 | 2.473 | 10.54 | .1302 | 2.042 | 10.78 |
| 16 | 2.451 | 2.738 | 3.034 | 12.94 | .1959 | 2.506 | 13.23 |
| 17 | 3.078 | 3.439 | 3.811 | 16.52 | .3101 | 3.148 | 16.62 |
| 18 | 4.312 | 4.818 | 5.339 | 22.77 | .6067 | 4.409 | 23.28 |
| 19 | 5.870 | 6.558 | 7.267 | 30.99 | 1.124 | 6.003 | 31.70 |
| 20 | 8.452 | 9.443 | 10.47 | 44.63 | 2.331 | 8.643 | 45.64 |
| 21 | 10.11 | 11.30 | 12.52 | 53.38 | 3.336 | 10.34 | 54.60 |
| 22 | 13.21 | 14.75 | 16.35 | 69.75 | 5.691 | 13.51 | 71.33 |
| 23 | 16.57 | 18.51 | 20.51 | 87.49 | 8.954 | 16.94 | 89.44 |
| 24 | 21.39 | 23.90 | 26.49 | 112.9 | 14.93 | 21.87 | 115.5 |
| 25 | 25.88 | 28.92 | 32.05 | 136.6 | 21.86 | | |
| 26 | 31.96 | 35.70 | 39.57 | 168.7 | 33.32 | | |
| 27 | 40.45 | 45.19 | 50.08 | 213.6 | 53.37 | | |
| 28 | 52.83 | 59.02 | 65.41 | 278.9 | 91.05 | | |
| 29 | 61.27 | 68.45 | 75.86 | 323.5 | 122.5 | | |
| 30 | 71.90 | 80.33 | 89.03 | 379.6 | 168.7 | | |
| 31 | 103.5 | 115.7 | 128.2 | 546.5 | 349.7 | | |
| 32 | 127.8 | 144.8 | 158.3 | 674.8 | 533.1 | | |
| 33 | 161.8 | 180.7 | 200.3 | 854.3 | 854.0 | | |
| 34 | 211.3 | 236.1 | 261.6 | 1,116 | 1,457 | | |
| 35 | 414.2 | 462.7 | 512.9 | 2,187 | 5,597 | | |
| 36 | 647.1 | 723.0 | 801.1 | 3,417 | 13,660 | | |

## APPROXIMATE WEIGHTS OF WEATHER-PROOF WIRE

*(American Electrical Works)*

TRIPLE-BRAIDED INSULATION

| Size | Feet per Pound | Pounds per 1,000 Ft. | Pounds per Mile | Ampere Capacity Allowed by Fire Under-writers |
|---|---|---|---|---|
| 0000 | 1.34 | 742 | 3,920 | 312 |
| 000 | 1.64 | 609 | 3,215 | 262 |
| 00 | 2.05 | 487 | 2,570 | 220 |
| 0 | 2.59 | 386 | 2,040 | 185 |
| 1 | 3.25 | 308 | 1,625 | 156 |
| 2 | 4.10 | 244 | 1,289 | 131 |
| 3 | 5.15 | 194 | 1,025 | 110 |
| 4 | 6.26 | 160 | 845 | 92 |
| 5 | 7.46 | 134 | 710 | 77 |
| 6 | 9.00 | 111 | 585 | 65 |
| 8 | 13.00 | 73 | 385 | 46 |
| 10 | 20.00 | 50 | 265 | 32 |
| 12 | 29.00 | 35 | 182 | 23 |
| 14 | 38.00 | 26 | 137 | 16 |
| 16 | 48.00 | 21 | 113 | 8 |
| 18 | 67.00 | 15 | 81 | 5 |

DOUBLE-BRAIDED INSULATION

| Size | Feet per Pound | Pounds per 1,000 Ft. | Pounds per Mile | Ampere Capacity Allowed by Fire Under-writers |
|---|---|---|---|---|
| 0000 | 1.40 | 711 | 3,754 | 312 |
| 000 | 1.75 | 570 | 3,010 | 262 |
| 00 | 2.29 | 436 | 2,300 | 220 |
| 0 | 2.81 | 355 | 1,875 | 185 |
| 1 | 3.56 | 281 | 1,482 | 156 |
| 2 | 4.49 | 223 | 1,175 | 131 |
| 3 | 5.45 | 184 | 969 | 110 |
| 4 | 6.82 | 147 | 774 | 92 |
| 5 | 9.10 | 110 | 580 | 77 |
| 6 | 10.35 | 97 | 510 | 65 |
| 8 | 15.52 | 64 | 340 | 46 |
| 10 | 22.00 | 45 | 237 | 32 |
| 12 | 40.00 | 25 | 132 | 23 |
| 14 | 56.00 | 18 | 95 | 16 |
| 16 | 76.00 | 13 | 69 | 8 |
| 18 | 100.00 | 10 | 53 | 5 |

commercial copper as .1486 ohm at 60° F. In England, Matthiessen's values, .150822 ohm for a meter-gram of annealed high-conductivity commercial copper and .153858 ohm for a meter-gram of hard-drawn high-conductivity commercial copper, both at 60° F. and having a temperature coefficient of .00238 per degree F., are considered as standards.

## STANDARD WEATHER-PROOF FEED-WIRE

(*Roebling's*)

| Circular Mils | Outside Diameters Inches | Weights Pounds 1,000 Ft. | Weights Pounds Mile | Approximate Length on Reels Feet | Carrying Capacity, National Board Fire Underwriters |
|---|---|---|---|---|---|
| 1,000,000 | $1\frac{1}{2}$ | 3,550 | 18,744 | 800 | 1,000 |
| 900,000 | $1\frac{13}{32}$ | 3,215 | 16,975 | 800 | 920 |
| 800,000 | $1\frac{11}{32}$ | 2,880 | 15,206 | 850 | 840 |
| 750,000 | $1\frac{5}{16}$ | 2,713 | 14,325 | 850 | |
| 700,000 | $1\frac{9}{32}$ | 2,545 | 13,438 | 900 | 760 |
| 650,000 | $1\frac{1}{4}$ | 2,378 | 12,556 | 900 | |
| 600,000 | $1\frac{7}{32}$ | 2,210 | 11,668 | 1,000 | 680 |
| 550,000 | $1\frac{3}{16}$ | 2,043 | 10,787 | 1,200 | |
| 500,000 | $1\frac{1}{8}$ | 1,875 | 9,900 | 1,320 | 590 |
| 450,000 | $1\frac{3}{32}$ | 1,703 | 8,992 | 1,400 | |
| 400,000 | $1\frac{1}{16}$ | 1,530 | 8,078 | 1,450 | 500 |
| 350,000 | 1 | 1,358 | 7,170 | 1,500 | |
| 300,000 | $\frac{15}{16}$ | 1,185 | 6,257 | 1,600 | 400 |
| 250,000 | $\frac{29}{32}$ | 1,012 | 5,343 | 1,600 | |

Carl Herring advocates the following values: resistance of 1 mil-foot at 15° C., 10.0275 international ohms, as given by Prof. Lindeck for pure copper, and 10.1478 international ohms for Matthiessen's standard copper; resistivity (per centimeter cube) at 15° C., 1.667 microhms, as given by Prof. Lindeck for pure copper, and 1.687 microhms for Matthiessen's standard copper.

Joints in aluminum and hard-drawn copper telephone and telegraph line wires should always be made with McIntire or similar sleeves made of the same metal as the wire. When making a McIntire sleeve joint, pass each end of the wire through the sleeve until it extends ½ inch beyond the end of the sleeve, then place a steel tie-wrench or connector on each end of the sleeve, the outside of the tool to be ½ inch from

**HARD-DRAWN COPPER WIRE**

| Diameter in Mils | Gauge and Number | Weight per Mile in Pounds | Resistance per Mile in Ohms at 60° F. |
|---|---|---|---|
| 165 | 8 B. W. G. | 435 | 1.9742 |
| 162 | 6 B. & S. G. | 419 | 2.0481 |
| 160 | 8 N. B. S. G. | 409 | 2.0998 |
| 148 | 9 B. W. G. | 350 | 2.4541 |
| 144.3 | 7 B. & S. G. | 331 | 2.5925 |
| 144 | 9 N. B. S. G. | 331 | 2.5925 |
| 134 | 10 B. W. G. | 287 | 2.9835 |
| 128.5 | 8 B. & S. G. | 262 | 3.2810 |
| 128 | 10 N. B. S. G. | 262 | 3.2810 |
| 120 | 11 B. W. G. | 230 | 3.7330 |
| 116 | 11 N. B. S. G. | 215 | 3.9948 |
| 114.4 | 9 B. & S. G. | 208 | 4.1363 |
| 109 | 12 B. W. G. | 190 | 4.5244 |
| 104 | 12 N. B. S. G. | 173 | 4.9701 |
| 101.9 | 10 B. & S. G. | 166 | 5.1665 |
| 95 | 13 B. W. G. | 144 | 5.9558 |
| 92 | 13 N. B. S. G. | 135 | 6.3518 |
| 90.74 | 11 B. & S. G. | 132 | 6.4891 |
| 83 | 14 B. W. G. | 110 | 7.8038 |
| 80.81 | 12 B. & S. G. | 105 | 8.1946 |
| 80 | 14 N. B. S. G. | 102 | 8.4005 |

N. B. S. stands for the New British Standard wire gauge.

the end of the sleeve, after which 3 to 4½ complete turns, depending on the size of the wire, should be made, using great care to keep the sleeve absolutely straight. For No. 8 B. W. G. wire give 4½ turns, for sizes more extensively used give 3 turns and use sleeves of proper size to fit the wire. Full-length and half-length sleeves are made, the former for through line joints and the latter for branch joints.

## HARD-DRAWN COPPER WIRE

| Number and Gauge | Diameters in Mils | | | Weights per Mile | | | Breaking Weights | | Weights of Coils | | Conductivity | | Twist in 6 In. | Per Cent. Elongation in 5 Ft. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Required | Maximum | Minimum | Required | Maximum | Minimum | Actual Required | Per Square Inch | Maximum | Minimum | Required | Minimum | | |
| 8 B.W.G. | 165.0 | 166.0 | 164.0 | 436.4 | 441.7 | 431.1 | 1,328 | 62,108 | 218 | 152 | 97 | 96 | 30 | 1.14 |
| 12 N.B.S.* | 104.0 | 104.7 | 103.3 | 173.4 | 175.7 | 171.1 | 549 | 64,600 | 219 | 151 | 97 | 96 | 40 | 1.00 |
| 10 B. & S. | 101.9 | 102.8 | 101.0 | 165.0 | 168.0 | 162.0 | 540 | 64,800 | 218 | 152 | 97 | 96 | 40 | .99 |
| 12 B. & S. | 80.8 | 81.3 | 80.3 | 104.7 | 106.0 | 103.4 | 336 | 65,500 | 72 | 52 | 97 | 96 | 44 | .95 |
| 14 B. & S. | 64.0 | 65.0 | 63.0 | 65.0 | 67.5 | 63.0 | 220 | 68,200 | | | 97 | 96 | 47 | .91 |
| 16 B.W.G. | 65.0 | 65.5 | 64.5 | | 68.8 | 66.7 | 220 | 66,200 | | | | 96 | | .91 |
| 14 N.B.S. | 80.0 | 80.5 | 79.5 | | 103.9 | 101.3 | 330 | 65,600 | | | | 96 | | .94 |
| 13 N.B.S. | 92.0 | 92.6 | 91.4 | | 137.5 | 133.9 | 433 | 65,100 | | | | 96 | | .97 |
| 10 N.B.S. | 128.0 | 128.8 | 127.2 | | 265.9 | 259.4 | 820 | 63,700 | | | | 96 | | 1.06 |
| 10 B.W.G. | 134.0 | 134.9 | 133.1 | | 291.7 | 284.0 | 894 | 63,400 | | | | 96 | | 1.07 |

*N. B. S. stands for the New British Standard wire gauge for which S. W. G. is sometimes used.

## TENSILE STRENGTH OF COPPER WIRE

| Nos. B. & S. Gauge | Breaking Weight in Pounds | | Nos. B. & S. Gauge | Breaking Weight in Pounds | |
|---|---|---|---|---|---|
| | Hard-Drawn | Annealed | | Hard-Drawn | Annealed |
| 0000 | 8,310 | 5,650 | 9 | 617 | 349 |
| 000 | 6,580 | 4,480 | 10 | 489 | 277 |
| 00 | 5,226 | 3,553 | 11 | 388 | 219 |
| 0 | 4,558 | 2,818 | 12 | 307 | 174 |
| 1 | 3,746 | 2,234 | 13 | 244 | 138 |
| 2 | 3,127 | 1,772 | 14 | 193 | 109 |
| 3 | 2,480 | 1,405 | 15 | 153 | 87 |
| 4 | 1,967 | 1,114 | 16 | 133 | 69 |
| 5 | 1,559 | 883 | 17 | 97 | 55 |
| 6 | 1,237 | 700 | 18 | 77 | 43 |
| 7 | 980 | 555 | 19 | 61 | 34 |
| 8 | 778 | 440 | 20 | 48 | 27 |

## DATA ON DOUBLE SILK-COVERED COPPER WIRE

| B. & S. Gauge No. | $o$ = Ohms per Cubic Inch | $u$ | Pounds per Cubic Inch |
|---|---|---|---|
| 20 | .76 | .79 | .24 |
| 22 | 2.0 | .69 | .23 |
| 24 | 5.0 | .62 | .21 |
| 26 | 12.0 | .55 | .19 |
| 28 | 25.0 | .49 | .17 |
| 30 | 54.0 | .43 | .14 |
| 32 | 105.0 | .37 | .12 |
| 34 | 195.0 | .31 | .08 |
| 36 | 355.0 | .25 | .075 |
| 38 | 630.0 | .19 | .06 |
| 40 | 1,050.0 | .13 | .05 |

NOTE.—$u$ is the portion of the total volume that is occupied by the copper alone, the difference $1-u$ being the portion occupied by the insulation.

## IRON WIRE

There are three grades of iron wire; namely, Extra Best Best (E. B. B.), which has the highest conductivity and is the most uniform in quantity, being both tough and pliable; Best Best (B. B.), which is less uniform and tough, lower in conductivity, frequently sold as E. B. B.; and Best, which is the poorest grade made, being still less uniform, more brittle, and lowest in conductivity.

## MECHANICAL AND ELECTRICAL TESTS OF IRON WIRE OF AMERICAN MANUFACTURE

The column headed "Percentage Conductivity" in the following table gives the percentages that the conductivities of the various samples bear to the conductivity of pure copper. "Percentage of Elongation" means the percentage of the length the wire elongated before breaking. The column headed "Relative Breaking Stress" gives the number of feet of its own length that each sample was able to sustain.

**Specifications.**—Iron wire for use on telegraph and telephone lines should conform to the following specifications of the Western Union Telegraph Company:

1. The wire must be soft and pliable, and be capable of elongating 15%, without breaking, after being galvanized.
2. Great tensile strength is not required, but the wire must not break under a less strain than 2½ times its weight, in pounds per mile.
3. Tests for ductility should be made as follows: The piece of wire will be gripped by two vises, 6 in. apart, and twisted; the full number of twists must be distinctly visible on the 6-in. piece between the vises, and the number of twists must not be less than 15.
4. The weight per mile for the different gauge wires must be: for No. 4 B. W. G., 730 lb.; No. 6, 540 lb.; No. 8, 380 lb.; No. 9, 320 lb.; No. 10, 250 lb.; or as near these figures as practicable.
5. The electrical resistance of the wire, in ohms per mile, at a temperature of 68° F., must not exceed the quotient arising from dividing the constant number 4,800 by the

MECHANICAL AND ELECTRICAL TESTS OF IRON WIRE OF AMERICAN MANUFACTURE

| Sample Mark and B. W. G. No. | Mechanical | | | | | Electrical | |
|---|---|---|---|---|---|---|---|
| | Weight per Mile Pounds | Percentage of Elongation | Number of Twists That 6 In. Will Stand | Actual Breaking Stress Pounds | Relative Breaking Stress | Percentage Conductivity | Resistance per Mile in Ohms, at 60° F. |
| E. B. B. 12 | 190.83 | 11.50 | 15.00 | 417.50 | 11,552.20 | 14.40 | 30.50 |
| E. B. B. 8 | 381.66 | 17.70 | 26.50 | 937.50 | 12,930.50 | 17.30 | 12.67 |
| E. B. B. 11 | 222.64 | 17.20 | 21.50 | 577.50 | 13,639.40 | 15.60 | 24.20 |
| 151 9½ | 282.80 | 10.00 | 26.50 | 770.00 | 14,375.90 | 21.90 | 16.10 |
| E. B. B. 10 | 254.44 | 17.70 | 28.50 | 697.50 | 14,478.10 | 17.80 | 18.42 |
| 146 9½ | 287.50 | 16.00 | 29.00 | 832.50 | 15,288.86 | 21.90 | 16.10 |
| E. B. B. 6 | 508.88 | 11.40 | 21.50 | 1,587.50 | 16,462.40 | 17.70 | 9.21 |
| E. B. B. 9 | 318.05 | 19.30 | 17.50 | 1,007.50 | 16,725.10 | 16.90 | 15.54 |
| Nashua 8 | 381.66 | 15.10 | 26.50 | 1,535.00 | 21,183.00 | 14.70 | 15.00 |
| M. S. plain 6 | 528.00 | 10.40 | 19.50 | 2,137.50 | 21,375.00 | 13.50 | 11.78 |
| 443 8 | 378.10 | 10.00 | 31.00 | 1,635.00 | 22,301.40 | 16.50 | 16.10 |
| A. H. 9½ | 293.50 | 16.00 | 27.50 | 1,257.50 | 22,635.00 | 15.10 | 22.70 |

DIMENSIONS AND RESISTANCE OF IRON WIRE

| No. B. W. G. | Diameter in Mils = $d$ | Area in Circular Mils = $d^2$ | Weight in Pounds | | Breaking Strength in Pounds | | Resistance per Mile at 68 F. | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | 1,000 Ft. | 1 Mi. | Iron | Steel | E. B. B. | B. B. | Steel |
| 0 | 340 | 115,600 | 304.0 | 1,607 | 4,821 | 9,079 | 2.93 | 3.42 | 4.05 |
| 1 | 300 | 90,000 | 237.0 | 1,251 | 3,753 | 7,068 | 3.76 | 4.40 | 5.20 |
| 2 | 284 | 80,656 | 212.0 | 1,121 | 3,363 | 6,335 | 4.19 | 4.91 | 5.80 |
| 3 | 259 | 67,081 | 177.0 | 932 | 2,796 | 5,268 | 5.04 | 5.90 | 6.97 |
| 4 | 238 | 56,644 | 149.0 | 787 | 2,361 | 4,449 | 5.97 | 6.99 | 8.26 |
| 5 | 220 | 48,400 | 127.0 | 673 | 2,019 | 3,801 | 6.99 | 8.18 | 9.66 |
| 6 | 203 | 41,209 | 109.0 | 573 | 1,719 | 3,237 | 8.21 | 9.60 | 11.35 |
| 7 | 180 | 32,400 | 85.0 | 450 | 1,350 | 2,545 | 10.44 | 12.21 | 14.43 |
| 8 | 165 | 27,225 | 72.0 | 378 | 1,134 | 2,138 | 12.42 | 14.53 | 17.18 |
| 9 | 148 | 21,904 | 58.0 | 305 | 915 | 1,720 | 15.44 | 18.06 | 21.35 |
| 10 | 134 | 17,956 | 47.0 | 250 | 750 | 1,410 | 18.83 | 22.04 | 26.04 |
| 11 | 120 | 14,400 | 38.0 | 200 | 600 | 1,131 | 23.48 | 27.48 | 32.47 |
| 12 | 109 | 11,881 | 31.0 | 165 | 495 | 933 | 28.46 | 33.30 | 39.36 |
| 13 | 95 | 9,025 | 24.0 | 125 | 375 | 709 | 37.47 | 43.85 | 51.82 |
| 14 | 83 | 6,889 | 18.0 | 96 | 288 | 541 | 49.08 | 57.44 | 67.88 |
| 15 | 72 | 5,184 | 13.7 | 72 | 216 | 407 | 65.23 | 76.33 | 90.21 |
| 16 | 65 | 4,225 | 11.1 | 59 | 177 | 332 | 80.03 | 93.66 | 110.70 |
| 17 | 58 | 3,364 | 8.9 | 47 | 141 | 264 | 100.50 | 120.40 | 139.00 |
| 18 | 49 | 2,401 | 6.3 | 33 | 99 | 189 | 140.80 | 164.80 | 194.80 |

## IRON AND STEEL WIRE

(*Weight per Mile-Ohm*)

| Name of Wire | Weight per Mile-Ohm | |
|---|---|---|
| | Roebling's Sons Co. | Washburn & Moen |
| Extra Best Best | 4,700 | 5,000 |
| Best Best | 5,500 | 6,200 |
| Best | 6,000 | |
| Steel | 6,500 | 6,500 |

## COMPARISON OF PROPERTIES OF COPPER AND ALUMINUM

| Properties | Aluminum | Copper |
|---|---|---|
| Conductivity (for equal sizes) | .54 to .63 | 1 |
| Weight (for equal sizes) | .33 | 1 |
| Weight (for equal length and resistance) | .48 | 1 |
| Price—aluminum, 29c.; copper, 16c.; (bare line wire) | 1.81 | 1 |
| Price—(Equal resistance and length, bare line wire) | .868 | 1 |
| Temperature coefficient per degree F. | .002138 | .002155 |
| Resistance of mil-foot (20° C.) | 18.73 | 10.5 |
| Specific gravity | 2.5 to 2.68 | 8 89 to 8.93 |
| Tensile strength (hard-drawn) per square inch | 40,000 | 60,000 |
| Tensile strength (for equal weight and resistance) | 58,000 | 60,000 |
| Coefficient of expansion per degree F. | .0000231 | .0000093 |

## COMPARATIVE DATA—ALUMINUM AND COPPER

| Property | Pure Copper | Aluminum | | |
|---|---|---|---|---|
| | | A 0 | A 75 | A 2 |
| Conductivity | 100 | 62 | 58 | 54 |
| Comparative section of equal conductivity | 100 | 156.4 | 167.0 | 180.0 |
| Comparative weights of same lengths of equal conductivity | 100 | 47 | 50.2 | 54.0 |

weight of the wire, in pounds per mile. The coefficient .003 will be allowed for each degree F. in reducing to standard temperature

6. The wire must be well galvanized, and be capable of withstanding the following tests: Several samples to be selected at random and immersed in a saturated solution of copper sulphate for 70 sec., then removed and wiped dry and clean; this operation to be repeated three more times and if, then, the wire remains black as after the first immersion, there being no appearance of a copper deposit, the samples are well galvanized. Any appearance of a copper deposit shows that the film of zinc forming the galvanizing covering was too thin and has been removed by combining with the sulphuric acid of the solution and forming zinc sulphate.

### ALLOYED WIRE

*Phono-electric wire* is made by the Bridgeport Brass Company of an alloy containing 98.55% copper, 1.4% tin, and .5% silicon. In its manufacture, the silicon is nearly all slagged off, only .05% remaining. It is claimed to have a tensile strength from 40 to 45% greater than that of hard-drawn copper. Its conductivity is only 40% of pure copper. It is exceedingly tough, as a 6-in. piece of No. 8 will stand 50 complete turns, instead of 30 for hard-drawn copper. It is used for trolley wire and for long telephone-line spans.

RESISTANCE OF PURE ALUMINUM WIRE AT 75° F.*

| B. & S. Gauge No. | Ohms per 1,000 Ft. | Ohms per Mile | Feet per Ohm | Ohms per Pound |
|---|---|---|---|---|
| 0000 | .08177 | .43172 | 12,229.8 | .00042714 |
| 000 | .10310 | .54440 | 9,699.0 | .00067022 |
| 00 | .13001 | .68645 | 7,692.0 | .0010812 |
| 0 | .16385 | .86515 | 6,245.4 | .0016739 |
| 1 | .20672 | 1.09150 | 4,637.35 | .0027272 |
| 2 | .26077 | 1.37637 | 3,836.22 | .0043441 |
| 3 | .32872 | 1.7357 | 3,036.12 | .0069057 |
| 4 | .41448 | 2.1885 | 2,412.60 | .010977 |
| 5 | .52268 | 2.7597 | 1,913.22 | .017456 |
| 6 | .65910 | 3.4802 | 1,517.22 | .027758 |
| 7 | .83118 | 4.3885 | 1,203.12 | .044138 |
| 8 | 1.06802 | 5.5355 | 964.180 | .070179 |
| 9 | 1.32135 | 6.9767 | 756.780 | .11156 |
| 10 | 1.66667 | 8.8000 | 600.000 | .17467 |
| 11 | 2.1012 | 11.0947 | 475.908 | .28211 |
| 12 | 2.6497 | 13.990 | 377.412 | .44856 |
| 13 | 3.3412 | 17.642 | 299.298 | .71478 |
| 14 | 4.3180 | 22.800 | 231.582 | 1.1623 |
| 15 | 5.1917 | 27.462 | 192.612 | 1.7600 |
| 16 | 6.6985 | 35.368 | 149.286 | 2.8667 |
| 17 | 8.4472 | 44.602 | 118.380 | 4.5588 |
| 18 | 10.6518 | 56.242 | 93.8820 | 7.2490 |

| | | | | |
|---|---|---|---|---|
| 19 | 13.8148 | 72.942 | 72.3840 | 12.192 |
| 20 | 16.938 | 89.430 | 59.0406 | 18.328 |
| 21 | 21.358 | 112.767 | 46.8222 | 29.142 |
| 22 | 26.920 | 142.138 | 37.1466 | 46.316 |
| 23 | 33.962 | 179.32 | 29.4522 | 73.686 |
| 24 | 42.825 | 226.12 | 23.3508 | 117.17 |
| 25 | 54.000 | 285.12 | 18.5184 | 186.28 |
| 26 | 68.113 | 359.65 | 14.6814 | 296.32 |
| 27 | 85.865 | 453.37 | 11.6460 | 485.56 |
| 28 | 108.277 | 571.70 | 9.2358 | 749.02 |
| 29 | 136.535 | 720.90 | 7.3242 | 1,191.0 |
| 30 | 172.17 | 908.98 | 5.8087 | 1,893.9 |
| 31 | 212.12 | 1,119.98 | 4.7144 | 2,941.5 |
| 32 | 273.97 | 1,445.45 | 3.6528 | 4,788.9 |
| 33 | 345.13 | 1,822.3 | 2.8974 | 7,610.7 |
| 34 | 435.38 | 2,298.8 | 2.2969 | 12,109. |
| 35 | 548.92 | 2,898.2 | 1.8218 | 19,251. |
| 36 | 692.07 | 3,654.2 | 1.4449 | 30,600 |
| 37 | 872.93 | 4,609.2 | 1.1456 | 48,661. |
| 38 | 1,100.62 | 5,811.2 | .9086 | 76,658. |
| 39 | 1,387.47 | 7,325.8 | .7207 | 121,881. |
| 40 | 1,749.50 | 9,236.8 | .5716 | 193,835. |

*Calculated on the basis of Matthiessen's standard, viz.: 1 mi. of pure copper wire of $\frac{1}{16}$ in. diameter equals 13.59 ohms at 15.5° C. or 59.9° F.

## RESISTANCE, TENSILE STRENGTH, AND WEIGHT OF ALUMINUM LINE WIRE

| No. in B. & S. Gauge | Diameter in Mils $d$ | Circular Mils $d^2$ | Area in Square Inches $\frac{d^2 \times .7854}{100,000,000}$ | Grade A 0 | | Grade A 75 | | Grade A 2 | | Pounds per Mile Sp. Gr. 2.68 Water, 62,355 lb. per Cu. Ft. | Pounds per Mile of Aluminum Having Same Resistance as Copper Wire of Size Given. Grade A 75 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | Resistance per 1,000 Ft. at 75° F. | Tensile Strength, Pounds per Square Inch | Resistance per 1,000 Ft. at 75° F. | Tensile Strength, Pounds per Square Inch | Resistance per 1,000 Ft. at 75° F. | Tensile Strength, Pounds per Square Inch | | |
| 4 | 204.31 | 41,742 | .032784 | .4012 | 27,000 | .4288 | 33,000 | .4605 | 40,000 | 200.90 | 336.0 |
| 5 | 181.94 | 33,102 | .025998 | .5058 | 27,500 | .5408 | 34,000 | .5818 | 42,000 | 159.30 | 266.4 |
| 6 | 162.02 | 26,250 | .020617 | .6380 | 28,000 | .6820 | 35,000 | .7325 | 44,000 | 126.35 | 211.4 |
| 7 | 144.28 | 20,816 | .016349 | .8044 | 29,000 | .8600 | 36,000 | .9235 | 46,000 | 100.21 | 167.6 |
| 8 | 128.49 | 16,509 | .012966 | 1.634 | 30,000 | 1.105 | 37,000 | 1.187 | 48,000 | 79.46 | 133.2 |
| 9 | 114.43 | 13,094 | .010284 | 1.278 | 32,000 | 1.367 | 39,000 | 1.468 | 50,000 | 62.99 | 105.4 |
| 10 | 101.89 | 10,381 | .0081532 | 1.613 | 33,000 | 1.724 | 40,000 | 1.852 | 51,000 | 48.71 | 83.6 |
| 11 | 90.74 | 8,234.0 | .0064670 | 2.033 | 35,000 | 2.173 | 41,000 | 2.335 | 53,000 | 39.63 | 66.3 |
| 12 | 80.81 | 6,529.9 | .0051286 | 2.565 | 39,000 | 2.741 | 42,000 | 3.084 | 55,000 | 31.43 | 52.6 |
| 13 | 71.96 | 5,178.4 | .0040671 | 3.233 | | 3.456 | | 3.712 | | 24.83 | |
| 14 | 64.08 | 4,106.8 | .0031469 | 4.179 | | 4.467 | | 4.798 | | 19.76 | |

*Silicon-* and *aluminum-bronze wires* have high tensile strength and are free from corrosion, thus rendering them especially suitable for guy wires; they resist corrosion fully as well as hard-drawn copper. Some silicon-bronze wires have a tensile strength of 80,000 lb. per sq. in. and are capable of standing 80 twists in a length of 6 in. before breaking. An aluminum-bronze wire showed a strength of 110,000 lb. per sq. in., but its ductility was less than that of the silicon-bronze wire. The low conductivity of bronze wires (not much over 35% that of pure copper, and much lower for some of the alloys) excludes them from use for line wires.

Bronze wires cost about six times as much as either iron or steel. On account of their cost, they are used but very little, if at all, in the United States; on some long lines in Europe, it is quite customary to use bronze wires of some kind.

## GERMAN-SILVER WIRE

German silver is an alloy consisting of 18% to 30% nickel and the balance about 4 parts copper to 1 part zinc. Weight of the alloy per cubic foot about 530 lb.; specific gravity, 8.5. Resistance of the 18% alloy at 25° C. 18 times that of copper, and of the 30% alloy about 28 times that of copper. Temperature coefficient from 0° to 100° C. .044% increase of resistance for 1° C. increased temperature. The maximum safe carrying capacity of German-silver wire in spirals in open air for continuous duty is such that the circular mils per ampere varies from about 1,500 in No. 10 wire to about 475 in No. 30. For intermittent duty, the capacity is twice as great.

## SIZES OF WIRE FOR TELEPHONE AND TELEGRAPH LINES

**Telephone Lines.**—No definite rules can be given for choosing the proper wire to be used for overhead telephone lines, but the following wires and sizes will ordinarily answer for the purposes mentioned. For telephone lines in the country and small towns for distances not exceeding 8 mi., No. 14 B. W. G., B. B. galvanized-iron wire may be used; for distances not to exceed 25 mi., No. 12 B. W. G., B. B.

## RESISTANCE OF GERMAN-SILVER WIRE

| B. & S. G. No. | Resistance per 1,000 Feet International Ohms | | B. & S. G. No. | Resistance per 1,000 Feet International Ohms | |
|---|---|---|---|---|---|
| | 18% Wire | 30% Wire | | 18% Wire | 30% Wire |
| 6 | 7.20 | 11.21 | 21 | 232.92 | 362.32 |
| 7 | 9.12 | 14.18 | 22 | 295.38 | 459.48 |
| 8 | 11.54 | 17.95 | 23 | 370.26 | 575.96 |
| 9 | 14.55 | 22.63 | 24 | 468.18 | 728.28 |
| 10 | 18.18 | 28.28 | 25 | 590.22 | 918.12 |
| 11 | 22.84 | 35.53 | 26 | 748.08 | 1,163.68 |
| 12 | 28.81 | 44.82 | 27 | 937.98 | 1,459.08 |
| 13 | 36.48 | 56.75 | 28 | 1,191.24 | 1,853.04 |
| 14 | 46.17 | 71.82 | 29 | 1,481.22 | 2,304.12 |
| 15 | 58.21 | 90.55 | 30 | 1,891.8 | 2,942.8 |
| 16 | 72.72 | 113.12 | 31 | 2,388.6 | 3,715.6 |
| 17 | 93.40 | 145.29 | 32 | 2,955.6 | 4,597.6 |
| 18 | 118.20 | 183.87 | 33 | 3,751.2 | 5,835.2 |
| 19 | 145.94 | 227.02 | 34 | 4,764.6 | 7,411.6 |
| 20 | 184.68 | 287.28 | 35 | 6,031.8 | 9,382.8 |

galvanized-iron wire may be used; for distances from 25 to 100 mi., No. 10 B. W. G., B. B. galvanized-iron wire may be used; for distances of 100 mi. and over, hard-drawn copper wire should be used, not smaller than No. 10 B. & S. for 150 mi. and over. The size most generally used on farmers' lines is No. 12 B. B. galvanized-iron wire, weighing about 165 lb. per mi. although No 14 will answer up to about 8 mi.

For small city or town lines, No. 14 B. W. G., B. B. galvanized-iron wire is extensively used; although in towns where cable forms part of the line, steel wire may be used. For lines connected with large city exchanges, hard-drawn copper wire (usually No. 12 B. & S.) is almost always used.

For toll lines not exceeding 75 mi., B. B. galvanized-iron wire, generally No. 10 B. W. G. (but in a few cases No. 8 B. W. G.) is used; from 75 to 150 mi., the E. B. B. grade or hard-drawn copper should be used. For good toll lines of any length, the best practice calls for complete metallic circuits of hard-drawn copper, No. 10 B. & S. up to about 500 mi., and No. 8 B. & S. up to about 1,000 mi.

For interior wiring for telephones, No. 16 or No. 18 B. & S. copper wire should be used—in dry places weather-proof office wire, and in damp places rubber-covered wire.

**Telegraph Lines.**—The following sizes are those in use for telegraph lines:

No. 10 B. & S. hard-drawn copper and No. 4 B. W. G. galvanized-iron wires are now used on important quadruplex circuits. Formerly, No. 6 B. W. G. galvanized-iron wire was used for this purpose.

No. 6 B. W. G. galvanized-iron wire is used for important circuits between cities.

No. 8 B. W. G. galvanized-iron wire, or No. 12 B. & S. hard-drawn copper wire, is much used for circuits of 400 mi., or less, in length. No. 9 B. W. G. galvanized-iron wire was formerly used for this purpose.

No. 9 B. W. G. galvanized-iron wire was, until recently, the size generally used in the United States. It is now used on short circuits where No. 8 is not considered necessary.

Nos. 10 and 11 B. W. G. galvanized-iron wires are used for still shorter circuits and for railway telegraph, police, fire-alarm, and private lines. No. 12 B. W. G. galvanized-iron wire is also used for these purposes.

Nos. 13 and 14 B. W. G. steel wires are used for short private lines and where strength is especially necessary.

. No. 8 B. & S. copper wire should be used for permanent ground wires in terminal telegraph offices

## OHM'S LAW

The law governing the flow of current in an electric circuit is known as *Ohm's law*, and may be stated as follows: *The strength of the continuous current in any circuit is directly proportional to the electromotive force in the circuit, and inversely proportional to the resistance of the circuit;* that is, $I=\frac{E}{R}$, from which $R=\frac{E}{I}$, and $E=IR$, where $I$ = current in amperes, $E$ = E. M. F. in volts, and $R$ = resistance in ohms. If there is in the circuit more than one source of electromotive force, the value of the resultant electromotive force must be used in these formulas. Furthermore, if the circuit contains inductance or capacity, the formulas are not applicable to variable- or alternating-currents.

## CAPACITY

Capacity ($C$) is comparable to the capacity of a bottle containing air. The addition of a given amount of air will raise the pressure more or less, and the amount of air required to produce a certain pressure in the bottle may be taken as the measure of the capacity of the bottle. This capacity is analogous to the electrostatic capacity of a condenser, which is measured by the quantity of electricity with which it must be charged in order to raise its electrical potential from zero to unity. The *unit of capacity* is the *farad*. A condenser has a capacity of 1 farad when 1 coulomb is required to raise its potential from zero to 1 volt. Since the farad is very large, its millionth part, or the *microfarad*, is generally used as the practical unit. The microfarad $=\frac{1}{1,000,000}$, or $10^{-6}$ farads. Condensers from $\frac{1}{10}$ to 6 micro-

farads capacity are the sizes most commonly used in the United States.

## CAPACITY OF CONDENSERS

If a difference of potential of $E$ volts exists across the terminals of a condenser of $C$ farads capacity, then the charge of $Q$ coulombs in the condenser may be calculated from the formula

$$Q = CE$$

from which $$C = \frac{Q}{E}$$

and $$E = \frac{Q}{C}$$

The capacity of a condenser is given by the formula

$$C \text{ (microfarads)} = \frac{885\ Ka}{d10^{10}},$$

in which $K$ is the inductivity of the dielectric between the tin-foil or metal plates; $a$ is the area in square centimeters of all the dielectric sheets actually between and separating the condenser plates; and $d$ is the average thickness in centimeters of the dielectric sheets. If there are $n$ insulating sheets, each of area $s$, then $a = ns$.

When $a$ and $d$ are given in square inches and inches, respectively, the formula becomes

$$C \text{ (microfarads)} = \frac{2{,}248\ Ka}{d \times 10^{10}}.$$

**Condensers in Parallel.**—When two or more condensers are connected in parallel, the joint capacity $C$ is equal to the sum of their capacities, that is, $C = C_1 + C_2 + C_3 + \text{etc.}$

**Condensers in Series.**—When two or more condensers $C_1$, $C_2$, $C_3$, etc. are joined in series, their joint capacity $C$ is equal to the reciprocal of the sum of their reciprocals, that is,

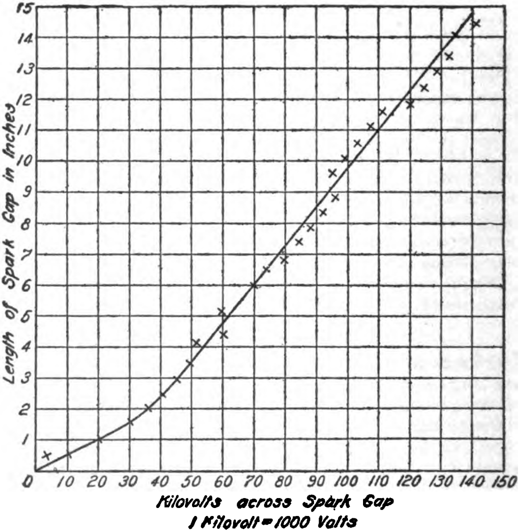
$$C = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \text{etc.}}$$

There are as many terms in the denominator as there are condensers connected in series. For example, the capacity of four condensers of 2, 4, 5, and 8 microfarads capacity connected in series is calculated as follows: $\frac{1}{2} + \frac{1}{4} + \frac{1}{5} + \frac{1}{8} = 1.075$, and $\frac{1}{1.075} = .93$ microfarad.

## INDUCTIVITY

The *inductivity*, or *specific inductive capacity*, of a substance is its *dielectric power*, or ability to convey the influence of an electrified body. Calling the inductivity of dry air at standard atmospheric pressure 1, the inductivity of any other substance is measured by the ratio of the capacity

DIELECTRIC STRENGTH OF AIR




of a condenser when its plates are separated by that substance to the capacity of the same condenser when its plates are separated by the same thickness of dry air. Various methods are used to determine the inductivities of substances and the capacities of condensers, and these methods do not all give the same results. Values obtained by the so-

## INDUCTIVITIES OF VARIOUS SUBSTANCES

| Material | Inductivity $K$ |
|---|---|
| Air, vacuum at about .001 mm. pressure | .9400 |
| Air, vacuum at about 5 mm. pressure | .9990 |
| Hydrogen, at ordinary pressure | .9997 to 1.00026 |
| Air, at ordinary pressure, standard | 1.0000 |
| Carbon dioxide, at ordinary pressure | 1.00036 to 1.00095 |
| Olefiant gas, at ordinary pressure | 1.0007 |
| Methane | 1.0009 |
| Sulphur dioxide, at ordinary pressure | 1.0037 |
| Manila paper | 1.50 |
| Carbon bisulphide | 1.60 to 1.81 |
| Paraffin, clear | 1.68 to 2.32 |
| Beeswax | 1.86 |
| Paraffin, solid | 1.9936* to 2.32 |
| Resin | 1.77 to 2.55 |
| Ozokerite | 2.00 |
| Petroleum | 2.03 to 2.42 |
| Ebonite | 2.05* to 3.15 |
| Turpentine | 2.15 to 2.43 |
| India rubber, pure | 2.22 to 2.497 |
| Sulphur | 2.24 to 3.84 |
| Gutta percha | 2.46* to 4.20 |
| Shellac | 2.74* to 3.60 |
| Olive and neat's-foot oils | 3.00 to 3.16 |
| Sperm oil | 3.02 to 3.09 |
| Glass | 3.013* to 3.258* |
| Mica | 4.00 to 8 |
| Porcelain | 4.38 |
| Quartz | 4.50 |
| Flint glass, very light | 6.57 |
| Flint glass, light | 6.85 |
| Flint glass, very dense | 7.40 |
| Flint glass, double extra dense | 10.10 |

*Results obtained by instantaneous methods.

called instantaneous methods are invariably lower than values obtained by the slower charge and discharge methods.

## DIELECTRIC STRENGTH

The *dielectric strength* of an insulating substance is the maximum difference of potential that it will stand without being punctured. It is determined by placing a thin layer of the substance between two metal electrodes, and increasing the difference of potential between the electrodes by small steps until a spark passes through the dielectric; the difference of potential in volts preceding that which punctures the insulation is the maximum strength of the dielectric.

The curve on page 122 shows the dielectric strength of air, as determined by C. P. Steinmetz, with a frequency of 125 cycles per sec., using needle points $2\frac{1}{2}$ in. long.

## INDUCTANCE

*Inductance*, or the *coefficient of self-induction* $L$, is the ratio between the total induction through a circuit to the current producing it. The unit of inductance is the *henry*. An inductance of 1 henry exists in a circuit when a current changing at a rate of 1 ampere per sec. induces an electromotive force of 1 volt in the circuit. As the henry is quite large, the one-thousandth part of it, or the millihenry, is frequently used. The millihenry $= \frac{1}{1000}$ or $10^{-3}$ henry.

## WORK AND POWER

*Work*, or *energy*, is expended in a circuit or conductor when a current of electricity flows through it. The unit of electrical work or energy is called the *joule*, after an eminent English scientist. If $E$ is the electromotive force, or difference of potential, in volts that causes $Q$ coulombs of electricity to flow through a circuit, the work expended in joules is

$$J = E \times Q$$

If an electromotive force, or difference of potential, of $E$ volts causes a current of $I$ amperes to flow for $t$ seconds through a resistance of $R$ ohms, then

$$J = EIt$$
$$J = \frac{E^2t}{R}$$
$$J = I^2Rt$$

The joule may be defined as the work done when 1 ampere flows for 1 second through a resistance of 1 ohm.

The energy used in forcing current through a resistance is converted into heat as follows:

4.2 joules = 1 small calorie
1 joule = .24 small calorie

The *watt-hour* is an extensively used unit of work. Watt-hours equal the product of the average number of watts and the number of hours during which they are expended. One *kilowatt-hour* = 1,000 watt-hours, or the product of the average number of kilowatts and the number of hours. Although five figures are given in most of the values in the accompanying table, it is rarely necessary to use more than three figures, and in very many cases two figures are sufficient. For instance, it is usually sufficient to use 1 calorie (gram-degree-C.) = 4.2 joules, or, to be a little more exact, 1 calorie = 4.19 joules. This table was calculated on the basis of 1 B. T. U. being equal to 778 ft.-lb., and the acceleration of gravity $g$ was taken as equal to 981 cm. per sec. per sec.

**Power** ($P$), which is the rate at which work is done, is equal to the work divided by the time, and may be calculated by any one of the following formulas.

$$P = IE = I^2R = \frac{E^2}{R} = \frac{J}{t}$$

If $I$ is in amperes, $R$ in ohms, $E$ in volts, $J$ in joules, and $t$ in seconds, $P$ is in watts.

The *watt*, or unit of electric power, is equal to 1 joule per sec. It is the rate at which work is expended when 1 ampere flows through a resistance of 1 ohm. The watt is too small a unit for convenient use in many cases, so that the kilowatt (K. W.), or 1,000 watts, is frequently used. 1 H. P. equals 746 watts; therefore, $\text{H. P.} = \frac{P \text{ (in watts)}}{746}$,

or $$\text{H. P.} = \frac{P \text{ (in kilowatts)}}{.746}$$

## NUMBER OF VOLTS REQUIRED TO PRODUCE A SPARK BETWEEN BALLS IN AIR

| Length of Spark Gap in | | Diameter of the Balls | | |
|---|---|---|---|---|
| Centi-meters | Inches | 1 Cm. =.3937 In. | 2 Cm. =.787 In. | 6 Cm. =2.36 In. |
| | | Volts | Volts | Volts |
| .02 | .0079 | 1,560 | 1,530 | |
| .04 | .0157 | 2,460 | 2,430 | |
| .06 | .0236 | 3,300 | 3,240 | |
| .08 | .0315 | 4,050 | 3,990 | |
| .10 | .0394 | 4,800 | 4,800 | 4,500 |
| .20 | .0787 | 8,400 | 8,400 | 7,800 |
| .30 | .1181 | 11,400 | 11,400 | 10,800 |
| .40 | .1575 | 14,400 | 14,400 | 13,500 |
| .50 | .1969 | 17,100 | 17,100 | 16,500 |
| .60 | .2362 | 19,500 | 19,800 | 19,500 |
| .70 | .2756 | 21,600 | 22,500 | 22,500 |
| .80 | .3150 | 23,400 | 24,900 | 26,100 |
| .90 | .3543 | 24,600 | 27,300 | 29,000 |
| 1.00 | .3937 | 25,500 | 29,100 | 32,700 |

## DIELECTRIC STRENGTH OF VARIOUS SUBSTANCES

(*Macfarlane and Pierce*)

| Substance | Strength in Volts per Centimeter |
|---|---|
| Oil of turpentine | 94,000 |
| Paraffin oil | 87,000 |
| Olive oil | 82,000 |
| Paraffin (melted) | 56,000 |
| Kerosene oil | 50,000 |
| Paraffin (solid) | 130,000 |
| Beeswaxed paper | 540,000 |
| Air (thickness 5 cm.) | 23,800 |
| $CO_2$ (thickness 5 cm.) | 22,700 |
| Oxygen (thickness 5 cm.) | 22,200 |
| Hydrogen (thickness 5 cm.) | 15,100 |
| Coal gas (thickness 5 cm.) | 22,300 |

## DIELECTRIC STRENGTH OF VARIOUS SUBSTANCES

(*Parshall and Hobart*)

| Substance | Thickness in Inches | Puncturing Voltage | Volts per $\frac{1}{1000}$ In. |
|---|---|---|---|
| Composite sheets of mica and paper prepared so as to be moisture-proof | .005 | 3,600 to 5,860 | |
| | .007 | 7,800 to 10,800 | |
| | .009 | 8,800 to 11,400 | |
| | .011 | 11,600 to 14,600 | |
| Leatheroid | $\frac{1}{64}$ or .0156 | 5,000 | 320 |
| | $\frac{1}{32}$ or .0313 | 8,000 | 256 |
| | $\frac{3}{64}$ or .0469 | 12,000 | 256 |
| | $\frac{1}{16}$ or .0625 | 15,000 | 240 |
| | $\frac{1}{8}$ or .125 | 15,000 | 120 |
| | $\frac{3}{16}$ or .188 | 6,000 | 32 |
| | $\frac{1}{4}$ or .25 | 6,000 | 24 |
| Vulcanized fiber | $\frac{1}{8}$ or .125 to 1 | about 10,000 | |
| Hard rubber | | | 500 |
| Kiln-dried maple and other similar woods | 1 | 10,000 to 20,000 | |
| Vulcabeston | $\frac{1}{2}$ | 10,000 | |
| Red pressboard | .03 | 10,000 | |
| Red rope paper | .01 | 1,000 | |
| Manila paper | .003 | 400 | |
| Oiled cambric | .007 | 2,500 to 4,500 | |
| Oiled cotton | .003 | 6,300 to 7,000 | |
| Oiled paper | .004 | 3,400 to 4,800 | |
| | .010 | 5,000 | |
| Mica | | 1 to $2.1 \times 10^6$ per mm. | |

## RELATION BETWEEN UNITS OF WORK

| Name of Unit | Ergs | Joules | Kilowatt-Hours | Calories | Foot-Pounds | B. T. U. |
|---|---|---|---|---|---|---|
| 1 erg | 1 | $\frac{1}{10^7}$ | $\frac{2{,}778}{10^{17}}$ | $\frac{23{,}882}{10^{12}}$ | $\frac{73{,}734}{10^{12}}$ | $\frac{94{,}774}{10^{15}}$ |
| 1 joule | $10^7$ | 1 | $\frac{2{,}778}{10^{10}}$ | .23882 | .73734 | $\frac{94{,}774}{10^8}$ |
| 1 kilowatt-hour | $36 \times 10^{12}$ | $36 \times 10^5$ | 1 | 859,770 | 2,654,400 | 3,411.8 |
| 1 calorie (gram-deg. C.) | 41,872,000 | 4.1872 | $\frac{11{,}631}{10^{10}}$ | 1 | 3.0873 | $\frac{39{,}683}{10^7}$ |
| 1 foot-pound | 13,562,000 | 1.3562 | $\frac{37{,}673}{10^{11}}$ | .32390 | 1 | .001285 |
| 1 British thermal unit (lb.-deg. F.) | $10{,}551 \times 10^6$ | 1,055.1 | $\frac{29{,}310}{10^8}$ | 252.00 | 778 | 1 |

# MAGNETISM

## MAGNETIC QUANTITIES

**Strength of Pole** ($m$).—A magnetic pole of unit strength is one that repels with a force of 1 dyne another similar and equal pole when placed 1 cm. from it.

**Magnetic Moment** ($\mathfrak{M}$).—The magnetic moment of a magnet is equal to the product of the strength $m$ of one of its poles and the distance $l$ between the poles. That is $\mathfrak{M} = m \times l$.

**Intensity of Magnetization** ($\mathfrak{I}$).—The intensity of magnetization is equal to the strength $m$ of a magnetic pole divided by its area $A$; that is, $\mathfrak{I} = \frac{m}{A}$.

The *intensity of magnetic field, field density*, or *magnetizing force* $\mathcal{H}$ at any point is measured by the force with which the field acts on a unit pole placed at that point. A unit field, called a *gauss*, acts with a force of 1 dyne on a unit pole, and is represented by 1 line of force, or 1 *maxwell*, per sq. cm. A field having an intensity of 5 lines of force per sq. cm. may be called a field of 5 maxwells per sq. cm., or simply a field of 5 gausses. The number of lines of force, or maxwells per unit area of a magnetic substance is variously called its *magnetic induction, flux density, magnetic density*, or simply *magnetism*, and is represented by $\mathcal{B}$ when the unit area is 1 sq. cm. or by **B** when the unit area is 1 sq. in.

*Magnetic flux*, or *total induction*, usually designated by the Greek letter $\Phi$ (phî), is the total number of lines of force threading a magnetic circuit, and is equal to the product of the magnetic density and the cross-sectional area; that is,

$$\Phi = \mathcal{B}A$$

If $\mathcal{B}$ is expressed in lines of force per square centimeter, or gausses, then $A$ must be in square centimeters; and if **B** is in lines of force per square inch, $A$ must be in square inches.

**Magnetic Permeability** ($\mu$).—Magnetic permeability is the ratio between the flux density $\mathcal{B}$ and the field intensity $\mathcal{H}$; that is, if the flux density through a solenoid is $\mathcal{H}$

## MAGNETIC QUALITIES OF ANNEALED SHEET IRON

| Magnetic Density per | | Magnetizing Force per | | Ampere-Turns per | | Permeability |
|---|---|---|---|---|---|---|
| Square Centimeter $\mathfrak{B}$ | Square Inch **B** | Square Centimeter $\mathcal{H}$ | Square Inch **H** | Centimeter Length $\frac{IT}{l}$ | Inch Length $\frac{IT}{1}$ | $\mu$ |
| 1,550 | 10,000 | 2.480 | 16 | 1.973 | 5.011 | 625.0 |
| 3,100 | 20,000 | 3.565 | 23 | 2.836 | 7.204 | 869.6 |
| 4,650 | 30,000 | 4.340 | 28 | 3.452 | 8.770 | 1,071.4 |
| 6,200 | 40,000 | 5.115 | 33 | 4.069 | 10.34 | 1,212.1 |
| 7,750 | 50,000 | 6.510 | 42 | 5.179 | 13.15 | 1,190.4 |
| 9,300 | 60,000 | 8.215 | 53 | 6.535 | 16.60 | 1,132.0 |
| 10,075 | 65,000 | | | | | |
| 10,850 | 70,000 | 10.54 | 68 | 8.384 | 21.30 | 1,029.4 |
| 12,400 | 80,000 | 14.57 | 94 | 11.59 | 29.44 | 851.0 |
| 13,950 | 90,000 | 21.39 | 138 | 17.02 | 43.22 | 652.2 |
| 15,500 | 100,000 | 33.17 | 214 | 26.39 | 67.02 | 467.3 |
| 16,275 | 105,000 | | | | | |
| 17,050 | 110,000 | 57.97 | 374 | 46.11 | 117.14 | 294.1 |
| 17,825 | 115,000 | | | | | |
| 18,600 | 120,000 | 112.38 | 725 | 89.39 | 227.07 | 165.5 |
| 19,375 | 125,000 | 166.63 | 1,057 | 132.55 | 346.69 | 116.3 |

## MAGNETIC QUALITIES OF UNANNEALED CAST STEEL

| Magnetic Density per | | Magnetizing Force per | | Ampere-Turns per | | Permeability |
|---|---|---|---|---|---|---|
| Square Centimeter $\mathfrak{B}$ | Square Inch **B** | Square Centimeter $\mathcal{H}$ | Square Inch **H** | Centimeter Length $\frac{IT}{l}$ | Inch Length $\frac{IT}{1}$ | $\mu$ |
| 1,550 | 10,000 | 2.790 | 18 | 2.219 | 5.638 | 555.5 |
| 3,100 | 20,000 | 4.340 | 28 | 3.452 | 8.770 | 714.3 |
| 4,650 | 30,000 | 5.425 | 35 | 4.312 | 10.96 | 857.1 |
| 6,200 | 40,000 | 6.665 | 43 | 5.302 | 13.47 | 930.2 |
| 7,750 | 50,000 | 8.370 | 54 | 6.658 | 16.91 | 925.9 |
| 9,300 | 60,000 | 11.16 | 72 | 8.878 | 22.55 | 833.3 |
| 10,075 | 65,000 | | | | | |
| 10,850 | 70,000 | 15.35 | 99 | 10.85 | 31.01 | 707.1 |
| 12,400 | 80,000 | 22.63 | 146 | 18.00 | 45.73 | 547.3 |
| 13,950 | 90,000 | 34.88 | 225 | 27.74 | 70.47 | 400.0 |
| 15,500 | 100,000 | 58.13 | 375 | 46.24 | 117.45 | 266.6 |
| 16,275 | 105,000 | | | | | |
| 17,050 | 110,000 | 113.15 | 730 | 90.01 | 228.64 | 150.7 |
| 17,825 | 115,000 | 157.33 | 1,015 | 125.15 | 317.90 | 113.3 |

## MAGNETIC QUALITIES OF WROUGHT-IRON FORGINGS

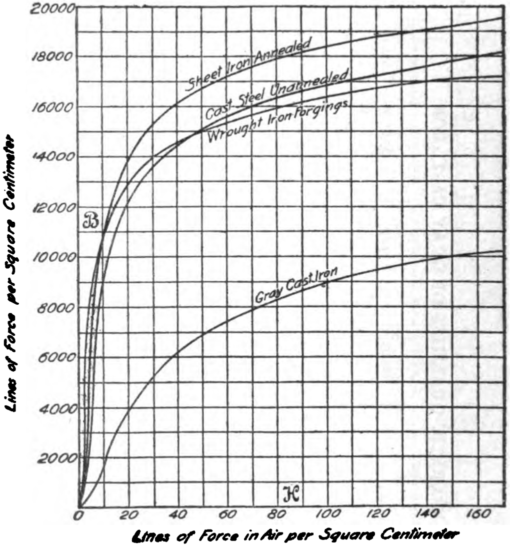
| Magnetic Density per | | Magnetizing Force per | | Ampere-Turns per | | Permeability |
|---|---|---|---|---|---|---|
| Square Centimeter $\mathcal{B}$ | Square Inch **B** | Square Centimeter $\mathcal{H}$ | Square Inch **H** | Centimeter Length $\frac{IT}{l}$ | Inch Length $\frac{IT}{1}$ | $\mu$ |
| 1,550 | 10,000 | 1.860 | 12 | 1.480 | 3.758 | 833.3 |
| 3,100 | 20,000 | 2.325 | 15 | 1.850 | 4.698 | 1,333.3 |
| 4,650 | 30,000 | 2.790 | 18 | 2.219 | 5.638 | 1,595.7 |
| 6,200 | 40,000 | 3.565 | 23 | 2.836 | 7.204 | 1,739.1 |
| 7,750 | 50,000 | 4.650 | 30 | 3.699 | 9.396 | 1,666.6 |
| 9,300 | 60,000 | 6.820 | 44 | 5.425 | 13.78 | 1,363.6 |
| 10,075 | 65,000 | | | | | |
| 10,850 | 70,000 | 10.08 | 65 | 8.015 | 20.36 | 1,076.9 |
| 12,400 | 80,000 | 16.12 | 104 | 12.82 | 32.57 | 769.2 |
| 13,950 | 90,000 | 31.00 | 200 | 24.66 | 62.64 | 450.0 |
| 15,500 | 100,000 | 66.05 | 430 | 53.02 | 134.68 | 232.6 |
| 16,275 | 105,000 | 97.65 | 630 | 77.68 | 197.32 | 166.6 |
| 17,050 | 110,000 | 160.43 | 1,035 | 127.62 | 324.16 | 106.3 |

## MAGNETIC QUALITIES OF GRAY CAST IRON

| Magnetic Density per | | Magnetizing Force per | | Ampere-Turns per | | Permeability |
|---|---|---|---|---|---|---|
| Square Centimeter $\mathfrak{B}$ | Square Inch **B** | Square Centimeter $\mathfrak{H}$ | Square Inch **H** | Centimeter Length $\frac{IT}{l}$ | Inch Length $\frac{IT}{1}$ | $\mu$ |
| 1,550 | 10,000 | 9.92 | 64 | 7.891 | 20.04 | 156.3 |
| 3,100 | 20,000 | 16.28 | 105 | 12.95 | 32.89 | 190.5 |
| 4,650 | 30,000 | 25.42 | 164 | 20.22 | 51.36 | 182.9 |
| 6,200 | 40,000 | 40.61 | 262 | 32.30 | 82.06 | 152.9 |
| 7,750 | 50,000 | 66.65 | 430 | 53.02 | 134.68 | 116.3 |
| 9,300 | 60,000 | 112.29 | 718 | 88.53 | 224.49 | 83.6 |
| 10,075 | 65,000 | 159.65 | 1,030 | 127.0 | 322.60 | 63.1 |

when the core consists of air, and is $\mathcal{B}$ when the core consists of iron, the permeability of the iron is

$$\mu = \frac{\mathcal{B}}{\mathcal{H}} = \frac{\mathbf{B}}{\mathbf{H}}$$

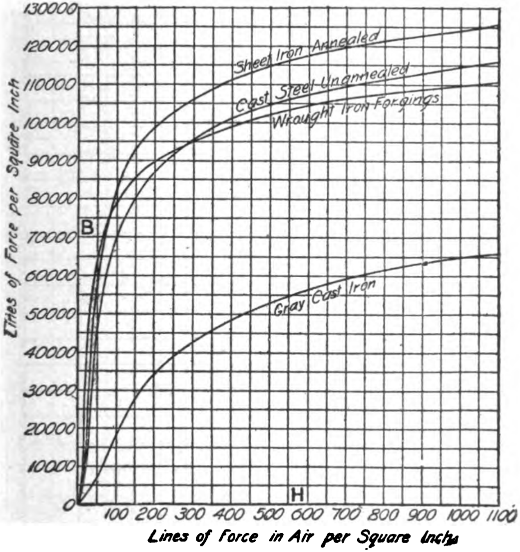



$\mathcal{B}$–$\mathcal{H}$ CURVES

**Magnetomotive force** $\mathfrak{F}$ (sometimes written M. M. F.), for the unit of which the name *Gilbert* has been proposed, is the total magnetizing force produced by a coil of $T$ turns through which a current of $I$ amperes is flowing. The magnetomotive force

$$\mathfrak{F} = \frac{4\pi I T}{10} = 1.257\, IT$$

**Reluctance** $\mathcal{R}$, for the unit of which the name *oersted* has been proposed, is the magnetic resistance, or opposition, offered by a substance to the passage of magnetic flux. Unit magnetomotive force will produce unit flux through



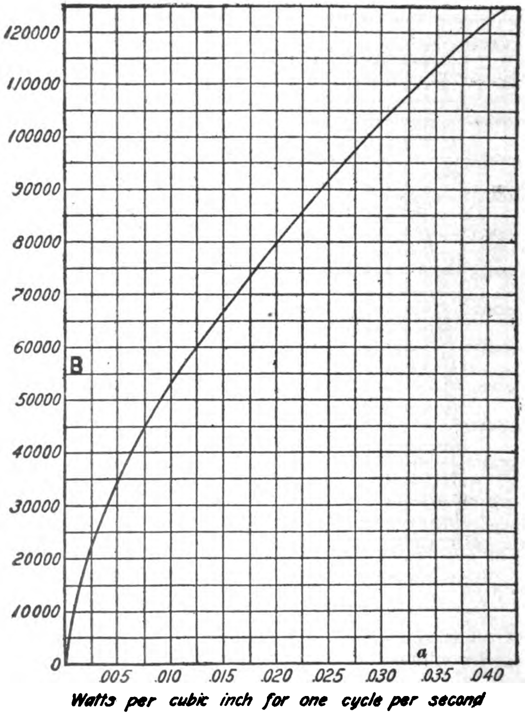

**B-H** CURVES

unit reluctance. A cubic centimeter of a perfectly non-magnetic substance, such as air, has unit reluctance.

## HYSTERESIS

**Hysteresis** may be defined as the tendency of a magnetic substance to persist in any magnetic state that it may have acquired. When an alternating or variable current flows

**HYSTERESIS LOSS AT ONE CYCLE PER SECOND AT VARIOUS FREQUENCIES**




in a coil around iron, some work is expended due to the hysteresis of the iron; this work appears as heat in the iron.

If $a$ is the power in watts expended in 1 cu. in. of iron for 1 cycle per sec.; $\mathbf{V}$, the volume of iron in cubic inches; $n$, the number of cycles per second; and $P$, the total watts expended in hysteresis; then,

$$P = a\mathbf{V}n$$

Obtain the value of $a$ from the curve given on page 136 for any given density $\mathbf{B}$.

The Steinmetz formula for the power in watts lost in hysteresis is

$$P = \frac{kV\mathcal{B}^{1.6}n}{10^7}$$

where $V$ is the volume in cubic centimeters and $\mathcal{B}$ is the induction per square centimeter.

The constant $k$ will vary a great deal, depending on the quality of the iron. A fair value for $k$ for annealed sheet iron and steel, such as used in dynamo and motor armatures, is .0035; for gray cast iron .013; and for cast steel, .003.

The total hysteresis loss in watts in iron, where the dimensions are given in inches, is very nearly

$$P = \frac{.83k\mathbf{B}^{1.6}\mathbf{V}n}{10^7}$$

## EDDY-CURRENT LOSS

*(From Parshall and Hobart)*

In sheet iron not over .025 in. thick, the eddy-current loss should theoretically conform to the formula

$$W = 1.5t^2n^2\mathbf{B}^2 \times 10^{-10}$$

where $W$ = watts per pound of iron at 0° C.;
$t$ = thickness of iron, in inches;
$n$ = number of cycles per second;
$\mathbf{B}$ = number of lines of force per square inch.

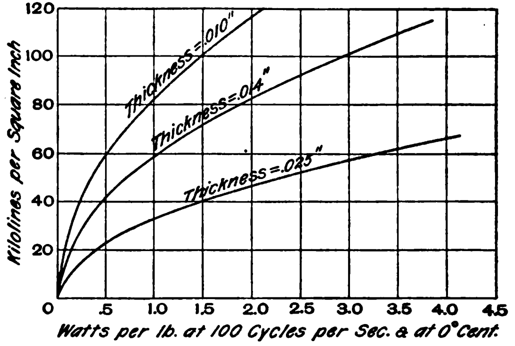
The loss decreases .5% per degree C. increase of temperature. The formula holds for iron whose specific resistance is 10 microhms per cm. cube at 0° C. and whose specific weight is .282 lb. per cu. in. For thicknesses greater than .025 in., the results given by the above formula are greatly modified. The curves in the accompanying figure show eddy-current losses in various thicknesses of sheet iron.

## LAWS OF MAGNETIC CIRCUIT

The total magnetic flux in a circuit is directly proportional to the magnetomotive force acting in the circuit and inversely proportional to the reluctance of the circuit; or

$$\Phi = \frac{\mathfrak{F}}{\mathcal{R}}$$

If $\Phi$ is the flux in maxwells, then $\mathfrak{F}$ will be the magnetomotive force in C. G. S. units, or gilberts, and $\mathcal{R}$ will be the reluctance in C. G. S. units, or oersteds.




The reluctance of a magnetic circuit is directly proportional to the length of the circuit, and inversely proportional to the product of the area of the cross-section of the circuit and the permeability, or

$$\mathcal{R} = \frac{l}{A\mu}$$

If $l$ and $A$ are in centimeters and square centimeters, respectively, $\mathcal{R}$ will be in C. G. S units; if in inches and square inches, the reluctance will be in units to which no name has been given. Since for air and all other non-magnetic substances $\mu = 1$, the reluctance $\mathcal{R} = \frac{1}{A}$. In a

complex magnetic circuit, the total reluctance is equal to the sum of the reluctances of all the parts.

The magnetomotive force due to an electromagnetic solenoid is directly proportional to the current and to the number of turns in the solenoid; that is,

$$\mathcal{F} = \mathcal{H}l = 1.257IT$$

$$\mathbf{H}\,\mathrm{l} = 3.192IT$$

in which $l$ must be expressed in centimeters and l in inches. The field density (in air) produced inside a long solenoid, and approximately inside any coil, whose length is large compared with its diameter, can be determined by the preceding formulas. From the same formulas can be determined the ampere-turns $IT$ required to produce a given field density $\mathcal{H}$ or $\mathbf{H}$ inside a coil whose length is known. The field density multiplied by the average area of the coil gives the total number of lines threading the coil when it contains no iron. If iron is introduced, it is necessary to multiply the field density by the permeability of the iron for that particular field density, and then by the sectional area of the iron, in order to get the total flux threading the iron.

Since $\mathcal{H}l = 1.257IT.$, $IT = .796\mathcal{H}l$, where $l$ is in centimeters; and for a given magnetizing force in a complex magnetic circuit, the number of ampere-turns is

$$IT = .796\mathcal{H}_1 l_1 + .796\mathcal{H}_2 l_2 + .796\mathcal{H}_3 l_3 + \text{etc.},$$

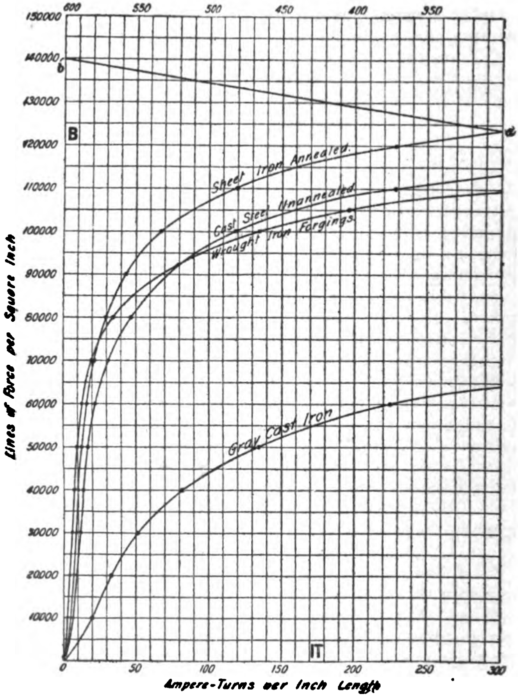
or, when the dimensions are in inches,

$$IT = .313\mathbf{H}_1 \mathrm{l}_1 + .313\mathbf{H}_2 \mathrm{l}_2 + .313\mathbf{H}_3 \mathrm{l}_3 + \text{etc.}$$

The following ampere-turn curves are plotted respectively with $\mathcal{B}$ and $\mathbf{B}$ as ordinates and $.796\mathcal{H}$ and $.313\mathbf{H}$ as abscissas. For a given density, find from the curve the corresponding abscissa which multiplied by the length will give the ampere-turns required for that part of the circuit. The sum of the ampere-turns for each part will give the total number of ampere-turns required.

## AMPERE-TURN CURVE—ENGLISH MEASURES

To reduce the length of the curve for sheet iron, the portion *a b* for densities greater than 123,000 per sq. in. is plotted backwards; for example, a density of 125,000 lines per sq. in. requires 325 ampere-turns per in.



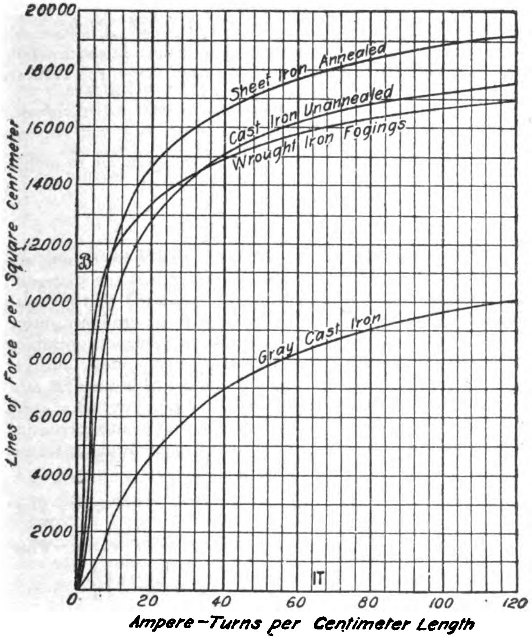

## INDUCTION

The electromotive force $E$ in volts generated in a conductor cutting $\Phi$ lines of force in $t$ seconds may be computed by the formula

$$E = \frac{\Phi}{10^8 t}$$

**Self-Induction.**—A coil has 1 C G. S. unit of inductance when 1 C. G. S. unit of current flowing through 1 turn pro-

AMPERE-TURN CURVE—METRIC MEASURES




duces 1 line of force. If $I$ is the current in amperes, $T$ the number of turns in a coil, $\Phi$ the number of lines of force due to the coil, then the inductance of the coil in henrys is

$$L = \frac{\Phi T}{10^8 I}$$

The inductance in henrys of a coil containing no iron may be computed by the formula

$$L=\frac{4\pi T^2A}{10^9l}$$

in which $T$ is the number of turns in the coil, $A$ is its mean area in square centimeters, and $l$ is its length in centimeters. For a cylindrical coil whose mean area is $\pi r^2$, the formula reduces to

$$L=\frac{3{,}948r^2T^2}{10^{11}l}$$

If the radius and the length of the coil are given in inches, then the inductance in henrys is similar to $r$

$$L=\frac{10{,}028r^2T^2}{10^{11}l}$$

These two formulas are strictly true only for a long coil in which the length is twenty or more times the diameter, and the depth of winding is small compared to the mean radius. However, they may be used to determine approximately the inductance of any ordinary solenoid containing no magnetic material. A formula for the inductance in C. G. S. units of coils having any number of layers and said by L. Cohen to be exact to $\frac{1}{4}$ of 1 per cent., even for short solenoids whose length is twice the diameter and increasing in accuracy as the ratio of length to diameter increases, is as follows: $L=$

$$4\pi^2n^2m\left\{\frac{2a_0^4+a_0^2l^2}{\sqrt{a_0^2+l^2}}-\frac{8a_0^3}{3\pi}\right\}+8\pi^2n^2\Big\{[(m-1)a_1^2+(m-2)a_2^2$$
$$+\cdots][\sqrt{a_1^2+l^2}-\tfrac{3}{4}a_1]+\tfrac{1}{2}[m(m-1)a_1^2+(m-1)(m-2)a_2^2$$
$$+(m-2)(m-3)a_3^2+\cdots]\left[\frac{a_1r_1}{\sqrt{a_1^2+l^2}}-r\right]\Big\}$$

in which $m$ is the number of layers; $a_0$, the mean radius of the solenoid; $a_1$, $a_2$, $a_3$, etc., the mean radii of the various layers; $l$, the length of solenoid; $r$, the radial distance between two consecutive layers; $n$, the number of turns per unit length; all dimensions are in centimeters. For a solenoid whose length is at least four times its diameter, the last formula reduces to

$$L = 4\pi^2n^2m^2\left[\frac{2a_0^4+a_0^2l^2}{\sqrt{4a_0^2+l^2}}-\frac{8a_0^3}{3\pi}\right]+8\pi^2n^2\Big\{[(m-1)a_1^2$$
$$+(m-2)a_2^2+\cdots][\sqrt{a_1^2+l^2}-\tfrac{7}{8}a_1]\Big\}$$

For a single layer the first formula reduces to

$$L=4\pi^2n^2\left[\frac{2a^4+a^2l^2}{\sqrt{4a^2+l^2}}-\frac{8a^3}{3\pi}\right]$$

If the solenoid contains magnetic material the inductance given by these formulas must be multiplied by the permeability $\mu$ of the magnetic material at the density to which the coil magnetizes the iron.

The mutual inductance between two coils in henrys is

$$M=\frac{\Phi T}{10^8 I}$$

## RULES FOR DIRECTION OF CURRENT AND MOTION

**Rule.**—*If the current in a conductor is flowing from south to north, and a compass is placed under the conductor, the north end of the needle will be deflected to the west; if the compass is placed over the conductor, the north end of the needle will be deflected to the east.*

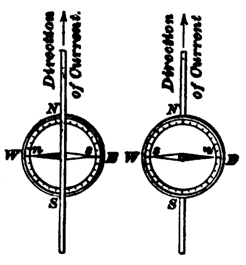


FIG. 1

To determine the polarity of an electromagnetic solenoid: In looking at the end of a solenoid, if an electric current flows in it clockwise, the end next to the observer is a south pole and the other end is a north pole; if counter-clockwise, the position of the poles is reversed.

To determine the direction of the lines of force set up around a conductor: If the current in a conductor is flowing away from the observer, then the direction of the lines of force will be clockwise around the conductor.

To determine the direction of motion of a conductor carrying a current when placed in a magnetic field: Place

**INDUCTION-COIL DATA SUITABLE FOR WIRELESS TELEGRAPHY AND ROENTGEN-RAY WORK**

(*From "Scientific American Supplement"*)

| Length of Spark Gap Inches | Length of Core Inches | Diameter of Core Inches | Primary Wire B. & S. Gauge | Number of Layers in Primary Coil | Secondary Wire B. & S. Gauge | Secondary Wire Pounds | Condenser | | Voltage of Battery |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | Number of Sheet | Area of Each Sheet Square Inches | |
| [illegible] | 3 | 1/4 | 23 | 2 | 36 | [illegible] | 25 | 2×1 | 2 |
| [illegible] | 3[illegible] | [illegible] | 23 | 2 | 36 | [illegible] | 40 | 2×1½ | 2 |
| [illegible] | 4[illegible] | 1/2 | 22 | 2 | 36 | [illegible] | 45 | 4×2 | 4 |
| [illegible] | 6 | 1/2 | 19 | 2 | 36 | [illegible] | 50 | 4×2 | 4 |
| [illegible] | 7 | 3/4 | 16 | 2 | 36 | [illegible] | 60 | 4×4 | 6 |
| 1 | 8 | 1 | 16 | 2 | 34 | 1 | 100 | 7×5 | 12 |
| 2 | 10 | 1[illegible] | 14 | 2 | 34 | 3 | 100 | 9×7 | 12 |
| 3 | 11[illegible] | 1¼ | 14 | 2 | 33 | 5 | 150 | 9×7 | 12 |
| 6 | 14[illegible] | 1½ | 12 | 2 | 33 | 10 | 200 | 9×9 | 16 |
| 12 | 19 | 1[illegible] | 10 | 3 | 33 | 12 | 60 | 12×8 | 10 |

thumb, forefinger, and middle finger of the *left hand* each at right angles to the other two; if the forefinger shows the direction of the lines of force and the middle finger shows the direction of the current, then the thumb will show the direction of the motion given to the conductor.

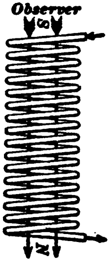



FIG. 2

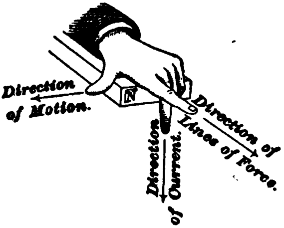



FIG. 3

To determine the direction of an induced current in a conductor that is moving in a magnetic field: Place thumb, forefinger, and middle finger of the *right hand* each at right angles to the other two; if the forefinger shows the direction of the lines of force and the thumb shows the direction of motion of the conductor, then the middle finger will show the direction of the induced current.

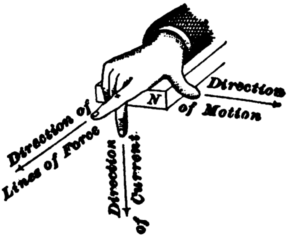



FIG. 4

A rule that is sometimes useful is the following: If the effect of the movement of a closed coil is to diminish the number of lines of force that pass through it, the current will flow in the conductor in a clockwise direction, when viewed by a person looking along the magnetic field in the direction of the lines of force; but if the effect is to increase

## COTTON-COVERED ANNEALED COPPER WIRE

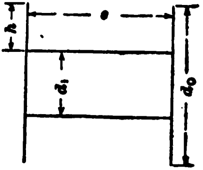
| B. & S. Gauge | Bare | | Single Cotton-Covered | | |
|---|---|---|---|---|---|
| | Dia. Mils $d$ | Area Cir. Mils $d^2$ | Dia. Over Ins. Mils $d_x$ | Wires per In. $\frac{1,000}{d_x}$ | Wires per Sq. In. $\left(\frac{1,000}{d_x}\right)^2$ |
| 0000 | 460 | 212,000 | | | |
| 000 | 410 | 168,000 | | | |
| 00 | 365 | 133,000 | | | |
| 0 | 325 | 106,000 | | | |
| 1 | 289 | 83,700 | | | |
| 2 | 258 | 66,400 | | | |
| 3 | 229 | 52,600 | | | |
| 4 | 204 | 41,700 | 211 | 4.73 | 22.3 |
| 5 | 182 | 33,100 | 189 | 5.29 | 27.9 |
| 6 | 162 | 26,300 | 169 | 5.91 | 34.9 |
| 7 | 144 | 20,800 | 151 | 6.62 | 43.8 |
| 8 | 128 | 16,500 | 136 | 7.35 | 54.0 |
| 9 | 114 | 13,100 | 121 | 8.26 | 68.2 |
| 10 | 102 | 10,400 | 108 | 9.25 | 85.5 |
| 11 | 90.7 | 8,230 | 97 | 10.3 | 106 |
| 12 | 80.8 | 6,530 | 87 | 11.4 | 129 |
| 13 | 71.9 | 5,180 | 78 | 12.8 | 163 |
| 14 | 64.1 | 4,110 | 70 | 14.2 | 201 |
| 15 | 57.1 | 3,260 | 63 | 15.8 | 249 |
| 16 | 50.8 | 2,580 | 56 | 17.8 | 316 |
| 17 | 45.3 | 2,050 | 50 | 20.0 | 400 |
| 18 | 40.3 | 1,620 | 45 | 22.2 | 492 |
| 19 | 35.9 | 1,290 | 39 | 25.6 | 655 |
| 20 | 32.0 | 1,020 | 36 | 27.7 | 767 |
| 21 | 28.5 | 810 | 32.5 | 30.7 | 942 |
| 22 | 25.3 | 642 | 29.0 | 34.4 | 1,180 |
| 23 | 22.6 | 510 | 26.6 | 37.5 | 1,400 |
| 24 | 20.1 | 404 | 24.1 | 41.4 | 1,710 |

TABLE—(*Continued*)

| B. & S. Gauge | Bare | | Single Cotton-Covered | | |
|---|---|---|---|---|---|
| | Dia Mils $d$ | Area Cir. Mils $d^2$ | Dia. Over Ins. Mils $d_x$ | Wires per In. $\frac{1,000}{d_x}$ | Wires per Sq. In. $\left(\frac{1,000}{d_x}\right)^2$ |
| 25 | 17.9 | 320 | 21.9 | 45.6 | 2,070 |
| 26 | 15.9 | 254 | 19.9 | 50.2 | 2,520 |
| 27 | 14.2 | 202 | 18.2 | 54.9 | 3,010 |
| 28 | 12.6 | 160 | 16.6 | 60.2 | 3,620 |
| 29 | 11.3 | 127 | 15.3 | 65.3 | 4,260 |
| 30 | 10.0 | 101 | 14.0 | 71.4 | 5,090 |
| 31 | 8.93 | 79.7 | 12.9 | 77.5 | 6,000 |
| 32 | 7.95 | 63.2 | 11.9 | 84.0 | 7,050 |
| 33 | 7.08 | 50.1 | 11.1 | 90.0 | 8,100 |
| 34 | 6.31 | 39.8 | 10.3 | 97.0 | 9,400 |
| 35 | 5.62 | 31.5 | 9.6 | 104 | 10,800 |
| 36 | 5.00 | 25.0 | 8.5 | 117 | 13,600 |
| 37 | 4.45 | 19.8 | | | |
| 38 | 3.97 | 15.7 | | | |
| 39 | 3.53 | 12.5 | | | |
| 40 | 3.15 | 9.89 | | | |

the number of lines of force that pass through the coil, the current will flow in the opposite direction.

## MAGNET-WINDING CALCULATIONS



Suppose a winding space, having the length $s$ and the depth $h=\frac{d_0-d_1}{2}$, is to be filled with wire wound in layers as closely as possible. The diameter of the bare wire is $d$, and over the insulation $d_x$. The space will hold $\frac{s}{d_x}$ turns per layer and

## COTTON-COVERED ANNEALED COPPER WIRE

| B. & S. Gauge | Double Cotton-Covered Wire | | | Triple Cotton-Covered Wire | | |
|---|---|---|---|---|---|---|
| | Dia. Over Ins. Mils $d_x$ | Wires per In. $\frac{1,000}{d_x}$ | Wires per Sq. In. $\left(\frac{1,000}{d_x}\right)^2$ | Dia. Over Ins. Mils $d_x$ | Wires per In. $\frac{1,000}{d_x}$ | Wires per Sq. In. $\left(\frac{1,000}{d_x}\right)^2$ |
| 0000 | | | | 478 | 2.09 | 4.36 |
| 000 | | | | 428 | 2.33 | 5.42 |
| 00 | | | | 383 | 2.61 | 6.81 |
| 0 | 339 | 2.94 | 8.64 | 343 | 2.91 | 8.46 |
| 1 | 303 | 3.30 | 10.8 | 307 | 3.25 | 10.5 |
| 2 | 272 | 3.67 | 13.4 | 276 | 3.62 | 13.1 |
| 3 | 242 | 4.13 | 17.0 | 247 | 4.04 | 16.3 |
| 4 | 216 | 4.62 | 21.3 | 220 | 4.54 | 20.6 |
| 5 | 194 | 5.15 | 26.5 | 198 | 5.05 | 25.5 |
| 6 | 174 | 5.74 | 32.9 | 178 | 5.61 | 31.4 |
| 7 | 156 | 6.41 | 41.0 | 160 | 6.25 | 39.0 |
| 8 | 141 | 7.09 | 50.2 | 145 | 6.89 | 47.4 |
| 9 | 126 | 7.93 | 62.8 | 130 | 7.69 | 59.1 |
| 10 | 112 | 8.92 | 79.5 | 116 | 8.02 | 64.3 |
| 11 | 101 | 9.90 | 98.0 | 105 | 9.52 | 90.6 |
| 12 | 91 | 10.9 | 118 | 95 | 10.5 | 110 |
| 13 | 82 | 12.1 | 146 | 86 | 11.6 | 134 |
| 14 | 74 | 13.5 | 182 | 78 | 12.8 | 163 |
| 15 | 67 | 14.9 | 222 | 71 | 14.0 | 196 |
| 16 | 59 | 16.9 | 285 | 63 | 15.8 | 249 |
| 17 | 53 | 18.8 | 353 | 57 | 17.5 | 306 |
| 18 | 48 | 20.8 | 432 | 52 | 19.2 | 368 |
| 19 | 43 | 23.2 | 538 | 47 | 21.2 | 449 |
| 20 | 40 | 25.0 | 625 | 44 | 22.7 | 515 |
| 21 | 36.5 | 27.3 | 745 | 40.5 | 24.6 | 605 |
| 22 | 33.0 | 30.3 | 918 | 37 | 27.0 | 729 |
| 23 | 30.6 | 32.6 | 1,060 | 34.6 | 28.9 | 835 |
| 24 | 28.1 | 35.5 | 1,260 | 32.1 | 31.1 | 967 |

TABLE—(*Continued*)

| B. & S. Gauge | Double Cotton-Covered Wire | | | Triple Cotton-Covered Wire | | |
|---|---|---|---|---|---|---|
| | Dia. Over Ins. Mils $d_x$ | Wires per In. $\frac{1,000}{d_x}$ | Wires per Sq. In. $\left(\frac{1,000}{d_x}\right)^2$ | Dia. Over Ins. Mils $d_x$ | Wires per In. $\frac{1,000}{d_x}$ | Wires per Sq. In. $\left(\frac{1,000}{d_x}\right)^2$ |
| 25 | 25.9 | 38.6 | 1,480 | These small-size wires are seldom, if ever, covered with three layers of cotton. | | |
| 26 | 23.9 | 41.8 | 1,740 | | | |
| 27 | 22.2 | 45.0 | 2,020 | | | |
| 28 | 20.6 | 48.5 | 2,350 | | | |
| 29 | 19.3 | 51.8 | 2,680 | | | |
| 30 | 18.0 | 55.5 | 3,080 | | | |
| 31 | 16.9 | 59.1 | 3,490 | | | |
| 32 | 15.9 | 62.8 | 3,940 | | | |
| 33 | 15.1 | 66.2 | 4,380 | | | |
| 34 | 14.3 | 69.9 | 4,880 | | | |
| 35 | 13.6 | 73.5 | 5,400 | | | |
| 36 | 12.0 | 83.3 | 6,930 | | | |

$\frac{h}{d_x}$ layers, and the total turns

$$T=\frac{sh}{(d_x)^2}=\frac{s(d_0-d_1)}{2(d_x)^2} \qquad (1)$$

The mean diameter of all the turns is

$$d_0-h=d_0-\frac{d_0-d_1}{2}=\frac{d_0+d_1}{2},$$

the length $l$ of a mean turn is

$$\frac{\pi(d_0+d_1)}{2},$$

and the total length of wire is

$$L=\frac{\pi T(d_0+d_1)}{2}=\frac{\pi sh(d_0+d_1)}{2(d_x)^2}. \qquad (2)$$

The resistance is

$$R=\frac{\pi\rho sh(d_0+d_1)}{2(d_x)^2} \qquad (3)$$

where $\rho$ is the resistance per unit length of wire. Since $\frac{\pi sh(d_0+d_1)}{2}=V$, the total volume of winding space, then

$$R=\frac{\rho V}{(d_x)^2} \qquad (4)$$

In general, if $m$ is the resistance per mil-foot, $a$ the circular mils cross-section of the wire, $l$ the mean length in inches of one turn, and $T$ the total number of turns, the resistance is

$$R=\frac{mlT}{12a} \qquad (5)$$

If the coil is to be used on a fixed voltage $E$, the current $I=\frac{E}{R}=\frac{12Ea}{mlT}$, and the ampere-turns $IT=\frac{12Ea}{ml}$ from which the circular mils

$$a=\frac{mlIT}{12E} \qquad (6)$$

For copper wire at 75° F. $m=10.5$ ohms, but if the wire is heated until the resistance is increased about 14%, the constant becomes 12 ohms, a value frequently used. If $m=12$, the formula becomes

$$a=\frac{lIT}{E} \qquad (7)$$

which gives the cross-section of wire needed for a given number of ampere-turns when the temperature of the wire is about 135° F.

By making no allowance for the thickness of insulation, except that each wire occupies a space of $d_x{}^2$ square inches, the diameter of a wire required to fill a winding space of outside diameter $d_0$, inside diameter $d_1$, and length $s$ all in inches, and offer a given resistance of $R$ ohms, is given approximately by the formula

$$d=.0288\sqrt[4]{\frac{s(d_0{}^2-d_1{}^2)}{R}} \qquad (8)$$

where $d$ is the diameter of the bare wire in inches.

A more exact formula for determining the diameter is

$$d=\sqrt{i^2+\sqrt{\frac{.7854ns(d_0{}^2-d_1{}^2)}{R}}}-i \qquad (9)$$

where $i$ is the radial thickness of the insulation, and $n$ is the resistance of a wire 1 inch long and 1 inch in diameter.

The bare diameter $d$ of a wire for a coil that will produce $IT$ ampere-turns with a given voltage $E$ may be determined from the formula

$$d=\sqrt{\frac{.000001374(d_0+d_1)IT}{E}} \qquad (10)$$

The length of insulated wire on a spool or bobbin is given by the formula

$$L=\frac{.7854s(d_0^2-d_1^2)}{(2i+d)^2} \qquad (11)$$

When the volume $V$ of the winding space in cubic inches and the ohms per cubic inch $o$ of the sized wire used is known, the resistance of a coil can be determined from the formula

$$R=Vo=.7854so(d_0^2-d_1^2) \qquad (12)$$

The heating effect of the energy lost in a magnet coil depends on the shape of the coil and on the conditions of ventilation. If $d_0$ is the outside diameter of a coil and $s$ is its length, both in inches, and $W$ is the total watts lost in the coil, the watts per square inch of cylindrical surface is

$$w=\frac{W}{\pi d_0 s}$$

The safe value for $w$ varies generally from .25 to 1.5, a fair value, if the ventilating conditions are good, being .75 to 1 for coils at 75° F. above the temperature of the surrounding air. Higher values of $w$ can be used only for exceptionally good ventilating conditions or for intermittent-service conditions.

Since $W=I^2R=\frac{E^2}{R}$, $R=\frac{E^2}{W}=\frac{E^2}{\pi d_0 s w}$, which gives the resistance of a coil when dissipating $w$ watts per square inch.

The following formula gives the diameter $d$ of a wire that will produce the greatest number of ampere-turns with a rise in temperature of $t°$ F.:

$$d=\sqrt{\sqrt{.000002159\times(1+.00223t°)\times d_0\times s^2\times(d_0^2-d_1^2)W_s}+i^2}-i$$

in which $i$ is the radial thickness of the insulation on the wire, and $W_s$ is the watts radiated per square inch of cylindrical surface of the coil.

The greatest number of ampere-turns $IT$ that can be obtained in a coil of given size for a given voltage $E$, a given rise in temperature $t°$F., and $W_s$ watts radiated per square

inch of cylindrical surface, can be calculated from the formula

$$IT=\frac{E\left[\sqrt{\sqrt{\dfrac{.000002159\times(1+.00223\,t^{\circ})\times d_0\times s^2\times(d_0^2-d_1^2)\times W_s}{E^2}+i^2}-i}\,\right]^2}{.000001374\times(d_0+d_1)}$$

## DRY-CORE PAPER-INSULATED TELEPHONE CABLES

| No. of Pairs | No. 19 B. & S. Wire | | | No. 20 B. & S. Wire | No. 22 B. & S. Wire | | |
|---|---|---|---|---|---|---|---|
| | Diameter of Cable Inches | Splicing Sleeve | | Diameter of Cable | Diameter of Cable Inches | Splicing Sleeve | |
| | | Diameter Inches | Length Inches | | | Diameter Inches | Length Inches |
| 15 | | 2 | 20 | | | $1\frac{1}{2}$ | 18 |
| 20 | 1.134 | 2 | 20 | 1.032 | .860 | $1\frac{1}{2}$ | 18 |
| 25 | 1.169 | 2 | 25 | 1.100 | .928 | 2 | 20 |
| 50 | 1.534 | 2 | 28 | 1.427 | 1.251 | 2 | 20 |
| 75 | 1.822 | $2\frac{1}{4}$ | 28 | 1.648 | 1.461 | $2\frac{1}{4}$ | 22 |
| 100 | 2.063 | $2\frac{1}{2}$ | 28 | 1.907 | 1.632 | $2\frac{1}{2}$ | 24 |
| 125 | 2.268 | 3 | 28 | 2.102 | 1.805 | $2\frac{3}{4}$ | 28 |
| 150 | 2.457 | 3 | 30 | 2.269 | 1.943 | 3 | 28 |
| 175 | 2.630 | $3\frac{1}{4}$ | 30 | 2.423 | 2.080 | 3 | 28 |
| 200 | 2.784 | $3\frac{1}{2}$ | 32 | 2.578 | 2.200 | 3 | 28 |
| 300 | | 4 | 36 | | 2.630 | $3\frac{1}{2}$ | 28 |
| 400 | | 4 | 40 | | | 4 | 28 |

Each conductor has a capacity of .08 microfarad per mile. The 20-pair No. 20 B. & S. conductor cable and the 25-pair No. 22 B. & S. conductor cable have lead sheaths $\frac{7}{64}$ in. thick, the 20-pair No. 22 B. & S. conductor cable has a lead sheath $\frac{3}{32}$ in. thick; all others have lead sheaths $\frac{1}{8}$ in. thick. For Y and loop splices, use a sleeve one size larger than given for straight splices in this table.

**Concrete.**—For manholes and around conduits good concrete may be made of 1 part of Portland cement or 2 parts of Rosendale or native cement, 3 parts of sand, and 5 or 6 parts of broken stone, or good cinders or furnace slag, that will pass through a ring 1½ inches in diameter, but not through a ring ¼ inch in diameter. For good results, concrete should be mixed as follows First, mix the sand and cement, turning them together at least three times dry, then add the stone, which should previously have been thoroughly wetted, and turn the mixture at least once over, finally add enough water to make the concrete tamp nicely, but not so moist as to have water run from it, and turn over at least three times. The water should not be supplied from a hose giving a strong stream that will cause the finely divided cement to be washed away; use buckets or a weak stream of water.

**Poles.**—Telephone and telegraph poles 25-ft. long should be set in 5-ft. holes; 30-ft. poles in 5½-ft. holes; 35- and 40-ft. poles in 6-ft. holes; 45-ft. poles in 6½-ft. holes; 50-ft. poles in 7-ft. holes; 55-ft. poles in 7½-ft. holes; 60-ft. poles in 8-ft. holes; 65-ft. poles in 8½-ft. holes; 70-ft. poles in 9-ft. holes, 75-ft. poles in 9½-ft. holes; 80-ft. poles in 10-ft. holes; 85-ft. poles in 10½-ft. holes, 90-ft. poles in 11-ft. holes. Poles on corners should be set about ½ ft. deeper.

Cross-arms should be placed at such a height on poles that the lowest wire will be, in hot weather, at least 27 ft. and preferably 30 ft. above railroad rails over which the wire crosses. Double cross-arms should be used on each side of a railroad track. Poles along a railroad should be at least 7 feet from the nearest rail with lowest cross-arm at least 22 ft. above rail. Lowest wire crossing a public road should be at least 19 ft. above crown of road. Standard telephone and telegraph cross-arms are 3¼ in.×4¼ in., and so-called telephone cross-arms are 2¾ in.×3¾ in.

For telephone drop lines, extending from line or cable to the house, many companies use a rubber-covered and braided copper, and occasionally iron, wire of No. 14 or 16 B. & S. gauge. Usually this wire comes twisted in pairs, but occasionally two single wires are used.

## SILK- AND COTTON-COVERED ANNEALED COPPER WIRE

(*S. G. McMeen in "Telephony"*)

| B. & S. Gauge | Diameter in Mils | | | | | Ohms per Cubic Inch | | |
|---|---|---|---|---|---|---|---|---|
| | Bare | Single Cotton | Double Cotton | Single Silk | Double Silk | Single Cotton | Double Cotton | Single Silk |
| 20 | 31.961 | 37.861 | 42.161 | 34.261 | 36.161 | .646 | .533 | .801 |
| 21 | 28.462 | 34.362 | 38.662 | 30.762 | 32.662 | .981 | .795 | 1.261 |
| 22 | 25.347 | 31.247 | 35.547 | 27.647 | 29.547 | 1.502 | 1.188 | 1.956 |
| 23 | 22.571 | 28.471 | 32.771 | 24.871 | 26.771 | 2.359 | 1.772 | 3.049 |
| 24 | 20.100 | 26.000 | 30.300 | 22.401 | 24.300 | 3.582 | 2.595 | 4.739 |
| 25 | 17.900 | 23.800 | 28.100 | 20.200 | 22.100 | 5.831 | 3.802 | 7.489 |
| 26 | 15.940 | 21.840 | 26.140 | 18.240 | 20.140 | 6.941 | 5.552 | 9.031 |
| 27 | 14.195 | 20.095 | 24.395 | 16.495 | 18.395 | 10.814 | 8.078 | 13.92 |
| 28 | 12.641 | 18.541 | 22.841 | 14.941 | 16.841 | 17.617 | 11.54 | 26.86 |
| 29 | 11.257 | 17.157 | 21.457 | 13.557 | 15.457 | 25.500 | 16.47 | 41.29 |
| 30 | 10.025 | 15.925 | 20.225 | 12.325 | 14.225 | 34.800 | 23.43 | 62.98 |
| 31 | 8.928 | 14.828 | 19.128 | 11.228 | 13.128 | 48.5 | 32.83 | 95.70 |
| 32 | 7.950 | 13.850 | 18.150 | 10.250 | 12.150 | 73.8 | 46.19 | 144.70 |
| 33 | 7.080 | 12.980 | 17.280 | 9.380 | 11.280 | 104.5 | 64.30 | 217.8 |
| 34 | 6.304 | 12.204 | 16.504 | 8.504 | 10.504 | 151.4 | 70.38 | 342.1 |
| 35 | 5.614 | 11.514 | 15.841 | 7.914 | 9.814 | 202.0 | 125.9 | 489.0 |
| 36 | 5.000 | 10.900 | 15.200 | 7.300 | 9.200 | 298.8 | 166.3 | 721.1 |
| 37 | 4.453 | 10.353 | 14.653 | 6.753 | 8.653 | 418 | 225.6 | 1062 |
| 38 | 3.965 | 9.865 | 14.165 | 6.265 | 8.165 | 567 | 305.5 | 1557 |
| 39 | 3.531 | 9.431 | 13.731 | 5.831 | 7.731 | 811 | 409.8 | 2266 |
| 40 | 3.144 | 9.044 | 13.344 | 5.344 | 7.344 | 1113 | 545.5 | 3400 |

## SILK- AND COTTON-COVERED ANNEALED COPPER WIRE

| B. & S. Gauge | Turns per Linear Inch | | | | Turns per Square Inch | | | |
|---|---|---|---|---|---|---|---|---|
| | Single Cotton | Double Cotton | Single Silk | Double Silk | Single Cotton | Double Cotton | Single Silk | Double Silk |
| 20 | 25.7 | 22.5 | 27.70 | 26.22 | 660.5 | 506.3 | 767.3 | 687.5 |
| 21 | 28.3 | 24.5 | 30.97 | 29.07 | 800.9 | 600.2 | 959.1 | 845.0 |
| 22 | 31.0 | 26.7 | 34.39 | 32.11 | 961.0 | 712.9 | 1.182.7 | 1,031.0 |
| 23 | 34.4 | 28.97 | 38.19 | 35.53 | 1,183.0 | 839.2 | 1,458.5 | 1,262.4 |
| 24 | 36.9 | 31.35 | 42.37 | 39.14 | 1,321.6 | 982.8 | 1,795.2 | 1,532.0 |
| 25 | 38.0 | 33.92 | 47.02 | 42.94 | 1,444.0 | 1150.8 | 2,210.9 | 1,843.8 |
| 26 | 42.0 | 36.29 | 52.06 | 46.81 | 1,764.0 | 1317.0 | 2,710.3 | 2,191.2 |
| 27 | 48.0 | 38.95 | 57.67 | 51.59 | 2,304.0 | 1517.2 | 3,326.0 | 2,661.6 |
| 28 | 53.0 | 41.61 | 63.36 | 56.43 | 2,809.0 | 1731.0 | 4,014.5 | 3,184.5 |
| 29 | 56.5 | 44.27 | 70.11 | 61.56 | 3,192.3 | 1959.9 | 4,915.5 | 3,789.8 |
| 30 | 59.66 | 46.93 | 77.14 | 66.79 | 3,559.2 | 2202.5 | 5,950.2 | 4,461.0 |
| 31 | 64.12 | 49.78 | 84.64 | 72.39 | 4,112.2 | 2478.0 | 7,164.0 | 5,240.0 |
| 32 | 68.60 | 52.34 | 92.72 | 78.19 | 4,692.5 | 2739.5 | 8,597.5 | 6,114.0 |
| 33 | 73.05 | 55.10 | 101.65 | 84.17 | 5,333.5 | 3036.1 | 10,332 | 7,085.0 |
| 34 | 77.90 | 57.57 | 112.11 | 90.44 | 6,068.5 | 3314.2 | 12,570 | 8,179.5 |
| 35 | 82.60 | 60.04 | 119.7 | 96.90 | 6,773.3 | 3605.0 | 14,328 | 9,389.5 |
| 36 | 87.10 | 62.51 | 130.15 | 103.55 | 7,586.5 | 3907.5 | 16,940 | 10,722 |
| 37 | 91.87 | 64.70 | 140.60 | 110.20 | 8,440.0 | 4186.1 | 19,770 | 12,145 |
| 38 | 95.0 | 66.80 | 151.05 | 116.85 | 9,025.0 | 4462.2 | 22,820 | 13,655 |
| 39 | 100.7 | 68.80 | 163.04 | 122.55 | 10,140.5 | 4733.6 | 26,700 | 15,018 |
| 40 | 106.0 | 71.20 | 177.65 | 129.20 | 11,236.0 | 5069.8 | 31,559 | 16,692 |

**DIAMETERS OF WIRES OF VARIOUS MATERIALS THAT WILL BE FUSED BY A CURRENT OF GIVEN STRENGTH**

(*W. H. Preece, F. R. S.*)

| Current Amperes | Diameters in Inches | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | Copper | Aluminum | Platinum | German Silver | Platinoid | Iron | Tin | Tin-Lead Alloy | Lead |
| 1 | .0021 | .0026 | .0033 | .0033 | .0035 | .0047 | .0072 | .0083 | .0081 |
| 2 | .0034 | .0041 | .0053 | .0053 | .0056 | .0074 | .0113 | .0132 | .0128 |
| 3 | .0044 | .0054 | .007 | .0069 | .0074 | .0097 | .0149 | .0173 | .0168 |
| 4 | .0053 | .0065 | .0084 | .0084 | .0089 | .0117 | .0181 | .021 | .0203 |
| 5 | .0062 | .0076 | .0098 | .0097 | .0104 | .0136 | .021 | .0243 | .0236 |
| 10 | .0098 | .012 | .0155 | .0154 | .0164 | .0216 | .0334 | .0386 | .0375 |
| 15 | .0129 | .0158 | .0203 | .0202 | .0215 | .0283 | .0437 | .0506 | .0491 |
| 20 | .0156 | .0191 | .0246 | .0245 | .0261 | .0343 | .0529 | .0613 | .0595 |
| 25 | .0181 | .0222 | .0286 | .0284 | .0303 | .0398 | .0614 | .0711 | .069 |
| 30 | .0205 | .025 | .0323 | .032 | .0342 | .045 | .0694 | .0803 | .0779 |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 35 | .0227 | .0277 | .0358 | .0356 | .0379 | .0498 | .0769 | .089 | .0864 |
| 40 | .0248 | .0303 | .0391 | .0388 | .0414 | .0545 | .084 | .0973 | .0944 |
| 45 | .0268 | .0328 | .0423 | .042 | .0448 | .0589 | .0909 | .1052 | .1021 |
| 50 | .0288 | .0352 | .0454 | .045 | .048 | .0632 | .0975 | .1129 | .1095 |
| 60 | .0325 | .0397 | .0513 | .0509 | .0542 | .0714 | .1101 | .1275 | .1237 |
| 70 | .036 | .044 | .0568 | .0564 | .0601 | .0791 | .122 | .1413 | .1371 |
| 80 | .0394 | .0481 | .0621 | .0616 | .0657 | .0864 | .1334 | .1544 | .1499 |
| 90 | .0426 | .052 | .0672 | .0667 | .0711 | .0935 | .1443 | .1671 | .1621 |
| 100 | .0457 | .0558 | .072 | .0715 | .0762 | .1003 | .1548 | .1792 | .1739 |
| 120 | .0516 | .063 | .0814 | .0808 | .0861 | .1133 | .1748 | .2024 | .1964 |
| 140 | .0572 | .0698 | .0902 | .0895 | .0954 | .1255 | .1937 | .2243 | .2176 |
| 160 | .0625 | .0763 | .0986 | .0978 | .1043 | .1372 | .2118 | .2452 | .2379 |
| 180 | .0676 | .0826 | .1066 | .1058 | .1128 | .1484 | .2291 | .2652 | .2573 |
| 200 | .0725 | .0886 | .1144 | .1135 | .121 | .1592 | .2457 | .2845 | .276 |
| 225 | .0784 | .0958 | .1237 | .1228 | .1309 | .1722 | .2658 | .3077 | .2986 |
| 250 | .0841 | .1028 | .1327 | .1317 | .1404 | .1848 | .2851 | .3301 | .3203 |
| 275 | .0897 | .1095 | .1414 | .1404 | .1497 | .1969 | .3038 | .3518 | .3417 |
| 300 | .095 | .1161 | .1498 | .1487 | .1586 | .2086 | .322 | .3728 | .3617 |

# DIRECT-CURRENT DYNAMOS AND MOTORS

A *dynamo-electric machine* is a device for converting mechanical energy into electric energy, or vice versa. The word *dynamo* is generally understood to mean a machine for converting mechanical energy into electric energy, that is, an *electric generator;* and the word *motor* means a machine for converting electric energy into mechanical energy. The essential parts of each are the same, namely: the *armature* and the *field magnet.*

Dynamos are divided into two general classes, according to the character of the current they deliver. A *direct-current dynamo* delivers a current that always flows in one direction; that is, the current never reverses, though it may change in value or pulsate. *Alternating-current dynamos*, or *alternators*, deliver a current that periodically reverses its direction of flow, the number of reversals per second depending on the number of poles in the dynamo and on the speed of rotation.

A direct-current dynamo or motor armature usually consists of a series of conductors arranged on the surface of a cylindrical iron core or in slots near the surface, the conductors, in most cases, being parallel with the axis of the core. The core is mounted on a shaft that is supported by bearings, so that the armature can be rotated near the pole faces of a field magnet. This magnet is excited by one or more field coils. Any even number of poles may be used, according to the size and type of the machine.

The torque, or turning moment, necessary to cause a dynamo armature to rotate and the torque produced by a motor armature are of precisely the same nature. Each depends on the strength of the magnetic field, the number of conductors on the armature, and the current in the conductors. Torque is usually expressed in *pounds-feet*, that is, the turning moment at the circumference of a circle with a radius of 1 ft. The circumference $= 2\pi$ ft., and the torque $T$ of a motor in pounds-feet is given by the formula

$$T = \frac{\text{H. P.} \times 33{,}000}{2\pi \times \text{R. P. M.}} = \frac{5.252 \text{ H. P.}}{\text{R. P. M.}}$$

Since horsepower (H. P.) = watts ($W$) ÷ 746,

$$T = \frac{5{,}252\ W}{746 \times \text{R. P. M.}} = \frac{7.04\ W}{\text{R. P. M.}},$$

where $W$ is the watts output, or the product of the watts input and the efficiency.

The movement of the conductors through the magnetic field sets up in them E. M. F.'s, and if the conductors are part of a complete circuit, current will flow through them.

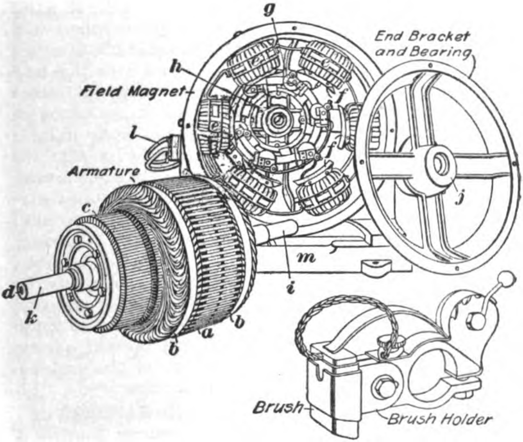



Fig. 1. Parts of a Dynamo

In the case of a dynamo armature, the flow of current is caused by the E. M. F. set up in the conductors themselves, while in a motor armature the E. M. F. set up in the conductors opposes the flow of current and limits it to just the strength necessary to produce the required torque; hence, the expression *counter E. M. F. of a motor*.

Fig. 1 shows the principal parts of a modern dynamo or motor, as follows:

*a*, armature core;
*b*, bands on armature heads;
*c*, commutator;
*d*, shaft;
*e*, field coils on poles;
*f*, pole faces;
*g*, brushes;
*h*, rear-end bearing;
*i*, rear-end journal;
*j*, front-end bearing;
*k*, front-end journal;
*l*, terminal block;
*m*, bedplate

The *commutator* of a dynamo or motor consists of a group of copper bars or segments arranged side by side, so as to form the outside of a cylinder. Sheets of mica insulate each bar from its neighbors and from the iron shell and clamping rings that hold the bars in place. The poles of the field magnet are arranged alternately, so that no two adjacent poles have the same polarity. The current in the armature conductors reverses as the conductor passes from under a pole of one polarity to one of opposite polarity, so that the current in the conductors is alternating. The commutator is mounted on the shaft near the armature and rotates with it. The armature conductors are so connected to the commutator bars that the current gathered by the stationary brushes rubbing on the surface of the commutator is direct. The brushes are usually blocks of carbon or graphite, which are retained in brush holders and pressed against the surface of the commutator by springs.

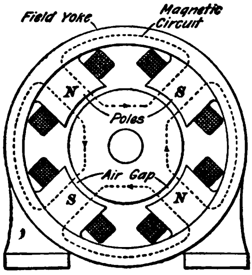



FIG. 2
FOUR-POLE FIELD MAGNET

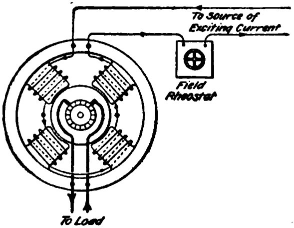
## FIELD MAGNETS

A *field magnet* consists of a suitable frame that, in conjunction with the iron armature core, forms a magnetic circuit for the field flux. The path thus provided for the magnetic flux is wholly through iron or steel, with the exception of the air gaps between the face of the armature core and pole faces. On each pole of the frame is a magnetizing coil. Fig. 2 shows a four-pole field magnet.

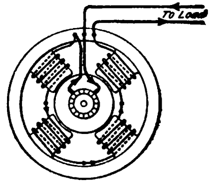
## METHODS OF EXCITING FIELD MAGNETS

Direct-current dynamos and motors are classified according to the method of field excitation as *separately excited*, *series-wound*, *shunt-wound*, and *compound-wound*, all of which are illustrated in Fig. 3.

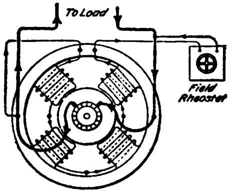
**Separately Excited Machines.**—In a separately excited dynamo the current is supplied to the magnetizing coils



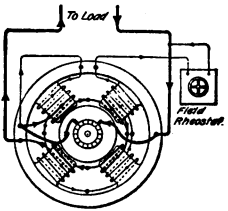

SEPARATELY EXCITED




SERIES WOUND




SHUNT WOUND




COMPOUND WOUND

FIG. 3. METHODS OF FIELD EXCITATION

from an outside source, such as a storage battery or another dynamo, an adjusting rheostat being included in the field circuit. The coils can be wound for any current or voltage desired, because the field winding has no connection with the armature of the dynamo. A separately excited dynamo.

with constant field excitation, will drop its voltage slightly from no load to full load, because of armature resistance and the demagnetizing effects of the armature current on the field. All direct-current motors may be called separately excited, because they do not generate their own field current. They are, however, series wound or shunt wound.

**Series-Wound Machines.**—In a series-wound dynamo the field winding is in series with the armature and the number of ampere-turns therefore varies with every change in the current output. The field is wound with a comparatively small number of turns of coarse wire capable of carrying the full current output. The voltage of a series-wound dynamo is low at light loads and increases rapidly as the load increases, until the field magnet approaches the saturation point. Series dynamos are little used except for boosters and for the operation of constant-current arc lamps.

A series motor varies in speed according to the current flowing through its armature and field, that is, according to its load; it will run fast and with little torque at light load and slow with greatly increased torque at heavy load.

**Shunt-Wound Machines.**—In a shunt-wound machine, a small portion of the current is shunted through the field winding made of a large number of turns of fine wire. The exciting current varies from about 5% of the total current in small machines to not over 1% in large ones; it depends only on the E. M. F. at the terminals of the field winding and on the resistance of the field. In a shunt dynamo, owing both to increased drop in the armature and to decreased excitation, the E. M. F. decreases considerably as the load increases. The decrease of E. M. F. can be prevented to some extent by adjusting the field rheostat. If supplied with constant E. M. F., as is usually the case, a shunt motor runs at nearly a constant speed at all loads, since the field then has constant excitation.

**Compound-Wound Machines.**—In a compound-wound machine, the field has two sets of windings—a *shunt winding* and a *series winding*. The shunt winding may be connected to the brushes (short shunt) or to the machine terminals (long shunt).

In a compound-wound dynamo, the shunt winding provides the initial excitation sufficient to generate full voltage at no load. This excitation is approximately constant. The series coils provide an excitation that increases as the load increases and strengthens the field so as to prevent the falling off in voltage that would otherwise occur. The series coils may be made sufficiently powerful to make the voltage rise as the load increases, in which case the machine is said to be *over-compounded.*

Compound dynamos are usually provided with a field rheostat, to permit an initial adjustment of the voltage and also to compensate for changes in the resistance of the shunt winding due to heating. This rheostat, however, is not intended for voltage-regulating purposes in the sense that it is used with the shunt machine, because after the voltage has been once adjusted, the strengthening of the field to suit the various loads is accomplished automatically by means of the series coils. The compound dynamo is the type almost universally used for direct-current power and lighting work.

Compound motors are used where the load varies considerably, where the extreme speed variations of series motors would be objectionable, and where the sudden increases in the load require increased torque that shunt motors could not give.

## RELATION BETWEEN ARMATURE AND FIELD CONNECTIONS

**Building Up a Dynamo Field.**—Any iron after being magnetized retains a certain amount of *residual magnetism.* Owing to the residual magnetism in a dynamo, a small E. M. F. is generated in the armature winding, when the armature is rotated, even if the field circuit is open. When a dynamo is started and the shunt field circuit closed, the small E. M. F. generated in the armature by the residual magnetism sends a small current through the magnetizing coils, producing a small magnetizing force. If this magnetizing force tends to send lines of force through the magnetic circuit in the same direction as the residual magnetism, there

will be a further increase of E. M. F. and field current, this action continuing until the magnetism in the field magnet approaches *saturation*, when the increase in the E. M. F. becomes slower and slower and ceases when the E. M. F. is just sufficient to send the required exciting current through the field coils. The field of a shunt dynamo will build up more slowly with the external circuit closed than if it is open, and, in fact, will not build up at all if the external resistance is very low.

A series-wound machine, on the contrary, must have its external circuit closed in order that any current may flow through the magnetizing coils, and the lower the resistance of the external circuit, the more quickly will the machine build up. A compound-wound dynamo may be started with its external circuit either open or closed, since it has both series- and shunt-wound coils. Usually, however, such machines are started and brought to their full E. M. F. with the external circuit open.

**Failure of Dynamo Field to Build Up.**—If the minute field current due to residual magnetism does not excite the field in the direction of the residual magnetism, the field will not build up. The disagreement, caused by wrong field connections, may be ascertained by connecting a voltmeter across the brushes and noting the reading, first with the field circuit open and then with the field circuit closed. The first reading gives the voltage due to residual magnetism and may be only a volt or two. If the second reading shows decreased voltage, the connections are wrong; if stationary or slightly increased voltage, the failure to generate is due to some other cause than faulty connections. Dirty brushes or commutator, a loose connection anywhere in the shunt field circuit or field rheostat, or a loss of residual magnetism from any cause will sometimes prevent a machine from picking up.

**Reversal of Residual Magnetism.**—A backward flow of current through the field coils or the demagnetizing action of a heavy short-circuited current through the armature may reverse the residual magnetism of the fields. Reversed residual magnetism causes reversed polarity of the dynamo

brushes, thus reversing the direction of the field current and making it agree with the new direction of the residual magnetism; that is, the field will build up, but the polarity of the dynamo will be reversed. The residual magnetism can be again reversed so that the dynamo will have the same polarity as before by sending a current from another dynamo or a battery through the field in the proper direction.

## DIRECTION OF ROTATION

If, when facing the commutator end of the armature of a dynamo or a motor, the rotation is in the direction of the

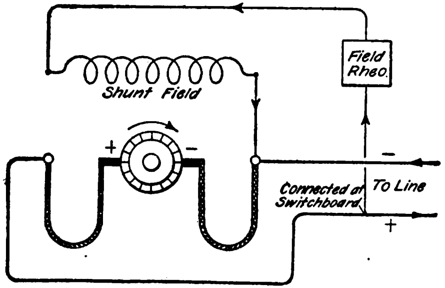



FIG. 4

hands of a clock, it is said to be *clockwise*, while the reverse direction is called *counter-clockwise*. If it is desired to reverse the direction of rotation of a dynamo, the connections between field and armature must be changed so that the current will flow through the field as before, otherwise the machine will not pick up. Fig. 4 shows the connections of a shunt-wound machine for clockwise rotation; Fig. 5 shows the same machine arranged for counter-clockwise rotation, with reversed polarity of the line wires; and Fig. 6 shows the connections for counter-clockwise rotation, retaining the same polarity of line wires as in Fig. 4. In each

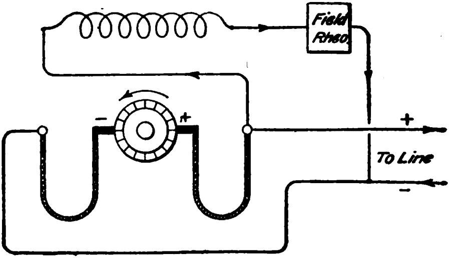



FIG. 5

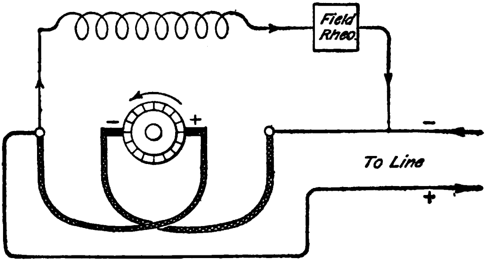



FIG. 6

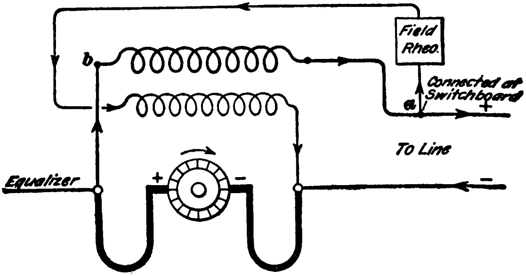



FIG. 7

case, the direction of the current through the field coil remains unchanged, so that there is no disagreement between

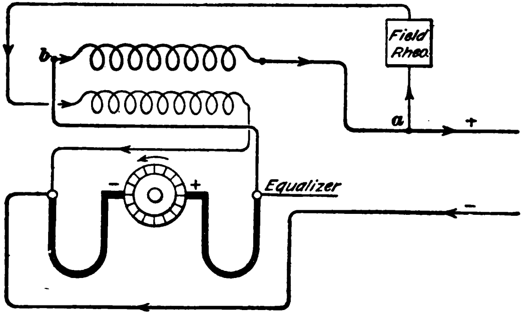



FIG. 8

the direction of the field magnetism and that of the residual magnetism. By reversing the residual magnetism, the con-

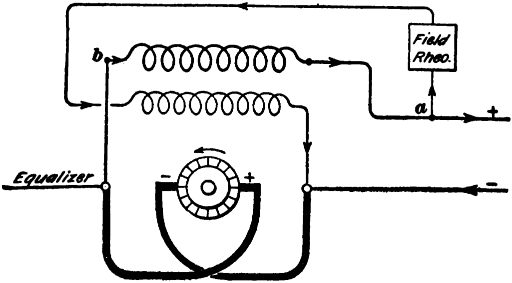



FIG. 9

nections could remain as in Fig. 4 and the rotation be reversed without reversing the polarity of the line wires.

Fig. 7 shows the connections of a compound-wound machine arranged for clockwise rotation. Figs. 8 and 9 show changes necessary for counter-clockwise rotation—Fig. 8 by

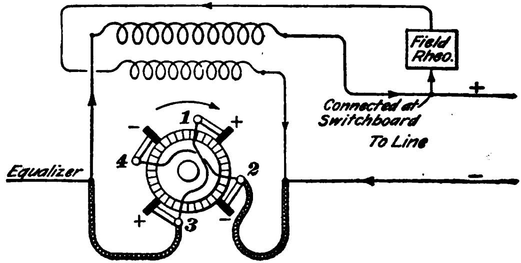



FIG. 10

changing the field connections and Fig. 9 by interchanging the brush leads. The direction of the current through both series and shunt fields remains the same in all three methods,

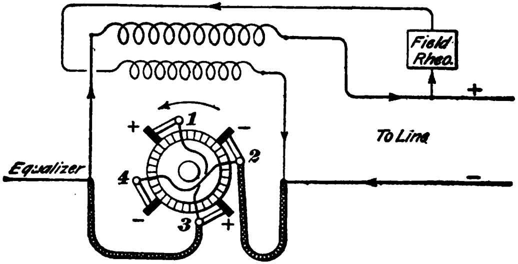



FIG. 11

and in each case the equalizer and the series field are connected to the same brush. The shunt field in each case is connected outside the series field, that is, *long shunt*. By

changing the shunt-field connection from *a* to *b* in each case, the connections would be made *short shunt;* it makes little difference in the operation of the machine which method is used.

Fig. 10 shows the connections of a multipolar compound-wound dynamo for clockwise rotation. By reversing the brushes on the studs, as shown in Fig. 11, and shifting them until they are on the neutral points, the dynamo can be run counter-clockwise without changing the field connections or the line polarity.

# DIRECT-CURRENT ARMATURE WINDINGS

## TYPES AND CLASSES

*Direct-current armatures* are (1) of the *constant-potential* or the *constant-current type*, according to the kind of energy they deliver; (2) of the *ring type* or the *drum type*, according to the arrangement of the coils on the core; and (3) of the *open-coil* or the *closed-coil type*, according to the connections of the coils to the commutator.

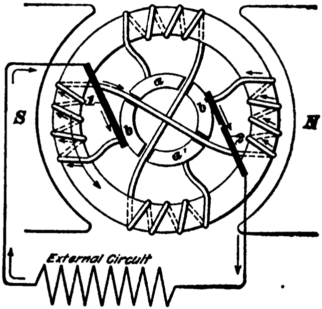



FIG. 12
RING-WOUND, OPEN-COIL ARMATURE

**Ring-Wound Open-Coil Armature.**—Fig. 12 shows a ring-wound, open-coil, or open-circuit, type armature; it is ring wound because the coils are wound around the rim of a ring-shaped iron core, and is open-coil type because the coils, as a whole, do not form a closed circuit, but each coil is in circuit only when the commutator bars *a*, *a'* or *b*, *b'* to which it is

connected are in contact with the brushes *1*, *2*. For example, at the instant shown, the coil connected to bars *a*, *a'* is open-circuited.

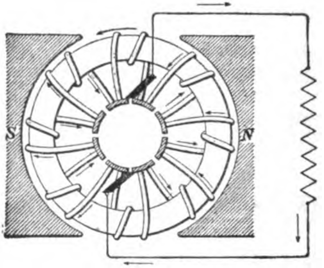



FIG. 13. RING-WOUND, CLOSED-COIL ARMATURE

**Ring-Wound, Closed-Coil Armature.**—Fig. 13 shows a ring-wound, closed-coil, or closed-circuit, winding. The coils as a whole, together with the commutator segments to which they are connected, form a closed circuit upon themselves, and each coil is in circuit all the time.

**Drum-Wound, Open-Coil Armature.**—A drum-wound, open-coil armature for a bipolar machine is suggested by the single coil in Fig. 14. On a multipolar machine, the coils,

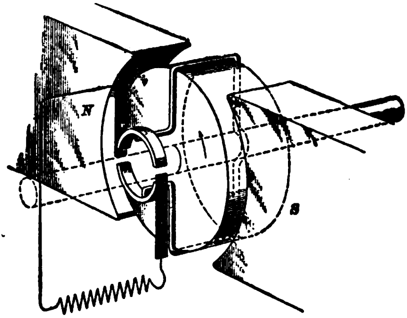



FIG. 14. DRUM-WOUND, OPEN-COIL ARMATURE

instead of passing completely around the armature core, span a chord. The core may be solid to the shaft, as shown, or

may be in the form of a cylindrical ring, but the coils do not pass through the interior of the ring.

**Drum Armature Partly Wound.**—Fig. 15 shows a partly wound, closed-circuit, drum-type iron-clad, or slot-wound, armature, the type and style of winding in most general use for direct-current machines. When the armature is complete, all the slots are full and the coils are held in place by bands over the core or the ends of the coils near the core, or by wooden or fiber wedges in the tops of the slots. The

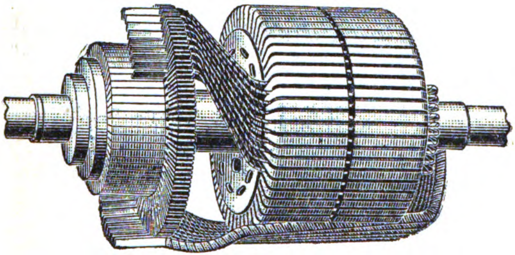


FIG. 15. DRUM ARMATURE PARTLY WOUND

coil leads are then soldered into the necks of the commutator bars and the projecting ends are cut off.

## ARMATURE COILS

An *armature coil* may be defined as that portion of the winding passed over in tracing a path from one commutator segment to the next one in the circuit. Those portions of an armature coil in the slots on the face of the core where they pass under the pole faces are the *face*, or *active*, *conductors*. An armature coil may consist of one or several turns. A ring winding has one active conductor per turn, while a drum winding has two. In Fig. 12, two coils are shown, each in two equal sections, with four turns and four face conductors in each section. Fig. 14 shows one single-turn coil with two face conductors.

**Pitch, or Spread, of Coils.**—Fig. 16 shows a single drum-wound coil in a multipolar field. The sides of the coil *a* and *b* should be separated by about the same angle *c* as the angle *d* between centers of adjacent poles. Angle *c* is often referred to as the angular pitch, or *spread*, of the coils, or the *winding pitch*, and angle *d* as the angular pitch of the poles. The spread of the coils may differ a little from the angular pole pitch without affecting the action of the machine, but if the difference is large there is liable to be sparking at the brushes.

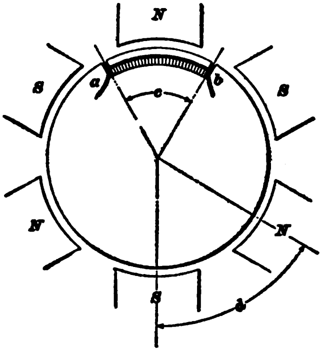
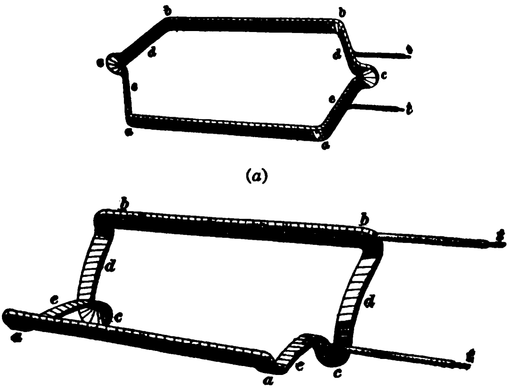



Fig. 16

**Shape of Coils.**—Fig. 17 shows two common styles of *form-wound armature coils*. In each, *aa* and *bb*, the sides of the coils that lie in the slots, constitute the active conductors; *d*, *e* are the end connections; and *t*, *t* the terminals, or leads, for connection to the commutator. The end connection *dce* across the front, or lead, end of the coils indicates that each coil has more than one turn, for coils with one turn, or loop, have no front end connection.

Fig. 15 shows an armature core with a few of the coils in place. Each coil is so formed, as shown by the turn at *c*, Fig. 17, that when in place on the core, one side *aa* lies in the bottom half of a slot, and the other *bb* lies in the top half of another slot. The coils lie in the slots in two layers and constitute a *two-layer winding;* they are arranged in this way so as to allow of a symmetrical arrangement of all of the coils and so that the end connections may cross each other without interfering.



(*a*)

(*b*)

FIG. 17. ARMATURE COILS

## PARALLEL WINDINGS

**Single-Parallel Windings.**—Fig. 18 illustrates the simplest method of connecting armature coil leads to the commutator, namely, by connecting the two leads of each coil to adjacent commutator bars, thus forming a *parallel-*, or *multiple-*, or *lap-wound armature.* In this figure, the coils are not shown, but are represented by the loops with ends connected to commutator bars $AB$, $BC$, $CD$, etc. To each bar is connected two leads; hence, for all closed-coil windings, there are as many bars as there are coils.

**Commutation.**—For convenience, in Fig. 18, the brushes are represented as rubbing on the inside of the commutator. In a dynamo, a current $I$ enters by the negative terminal and passes by way of the *negative bus-ring* $r$ to the negative brushes $a$, $e$, $c$, of which there are as many as there are pairs of poles—three in the winding shown. If the windings are symmetrical and the machine has $p$ poles, each

negative brush will receive $I+\frac{p}{2}$ amperes, which will divide so that $I+p$ amperes flows each way through the armature winding toward the next adjacent positive brush. The brushes must be so placed that the bars to which any coil is connected are in contact with a brush at the time the current in the coil reverses. For example, when coil $OP$

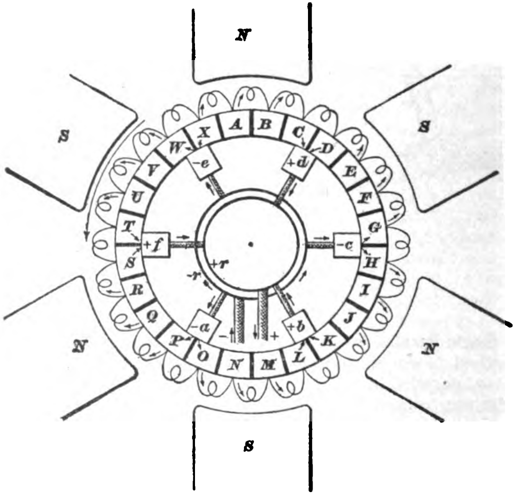



Fig. 18. Single-Parallel Armature Connections

was in the position $PQ$, $I+p$ amperes was flowing from $O$ to $P$; at the instant shown, only a short-circuit current flows in coil $OP$, but as soon as the coil moves to the right so that bar $O$ leaves the brush $a$, $I+p$ amperes will flow from $P$ to $O$. The E. M. F. generated by a coil reverses as the coil passes from under one pole to the pole of opposite

polarity, and the current in the coil should reverse at same time. The space between the poles where no lines of force pass into the armature is called the *neutral region*, and the brushes should be so placed that the current in the coils is reversed, or commutated, while the coils are in the neutral region. After the current has passed from the negative brushes through the winding, it is collected by the positive brushes $b$, $d$, $f$ and passes by way of the *positive bus-ring* $r$ out at the positive terminal of the machine.

Fig. 18 shows what is properly called a single-parallel winding, because, starting at any point, say at bar $A$, and following the circuit through the various coils, $AB$, $BC$, $CD$, etc. in succession, all the coils are traced in one circuit before returning to bar $A$. There are as many paths for current as there are poles $p$; hence, there must be as many brushes as there are poles, or else some of the paths will not be supplied with current and the armature will be electrically unbalanced. This type of winding is much used for low-potential machines, or those with large current output. The numerous paths make the resistance low and the current capacity large. There may be any number $N$ of coils on such an armature and each will generate $E \div \frac{N}{p}$ volts, where $E$ is the total voltage of the machine and $\frac{N}{p}$ is the number of coils per pole or per path.

**Double-Parallel Windings.**—There is also in some use the *double-parallel winding*, in which the coil terminals, instead of being connected to adjacent segments $AB$, $BC$, $CD$, etc., are connected to alternate segments; that is, in the winding shown in Fig. 18, one group, or series, of coils would be connected to segments $AC$, $CE$, $EG$, $GI$, etc., and the other series to segments $BD$, $DF$, $FH$, etc. When this winding is used, each brush must be thick enough to touch at least two segments at all times, otherwise the proper division of current between the two windings would not be maintained.

## SERIES WINDING

**Single-Series Windings.**—In the *series type of winding*, sometimes called *wave winding*, the coils are connected to segments removed from one another by approximately the angle of two poles. Fig. 19 represents a *single-series winding* for a six-pole machine. The connections of the thirty-two armature coils are represented by the curved

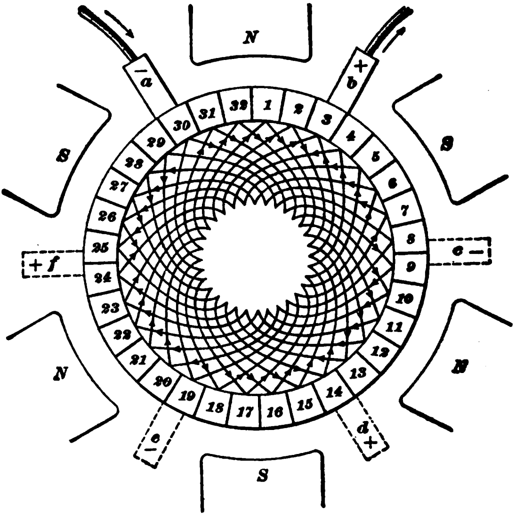



FIG. 19. SINGLE-SERIES ARMATURE CONNECTIONS

lines inside the commutator. For any number of poles there are two paths, or circuits, by which the current can pass from a negative brush *a* to a positive brush *b*. These paths can be traced by giving the numbers of the bars successively passed, a coil being included between each

two bars designated, as follows: (1) Brush *a*–bars *29–18–7–28–17–6–27–16–5–26–15–4*–brush *b*; (2) brush *a*–bars *30–9–20–31–10–21–32–11–22–1–12–23–2–13–24–3*–brush *b*. In tracing these circuits, the armature is encircled once for each $\frac{p}{2}$ coils; for example, from bar *30* to bar *9* is one coil, from bar *9* to bar *20* another, and from bar *20* to bar *31* a third, and in this case, $\frac{p}{2}=3$.

In Fig. 19, only two brushes are shown in full lines—one positive and one negative—while at the other neutral points they are shown dotted. The series of coils from segment *29* to *8*, *8* to *19*, *19* to *30* has segments *29* and *30* under brush *a*. segment *8* under brush *c*, and segment *19* under brush *e*. Likewise, the series of coils *4* to *25*, *25* to *14*, *14* to *3* has segments *4* and *3* under brush *b*, segment *25* under brush *f*, and segment *14* under brush *d*. If the current is not too large, two brushes *a* and *b* can collect it; but brushes are generally used in each neutral region.

**Winding Requirements.**—On a single-series armature, the number of coils and commutator bars must always be such that a series of $\frac{p}{2}$ coils connected to bars equidistant from each other will encircle the armature and terminate in a bar adjacent to the one from which the series started; that is,

$$N=\frac{p}{2}\times y\pm 1$$

in which $N$ is the number of commutator bars, or coils; $p$, the number of poles; and $y$ a number called the *pitch of the connections*, or the *connecting pitch*, it being the number of bars passed over between the terminals of a coil. In the machine shown in Fig. 19, $N=32$ and $p=6$; hence, $32=3\,y\pm 1$, from which equation, $3\,y=31$ or $33$ and $y=10\frac{1}{3}$, or 11. Since $y$ must be a whole number, the connecting pitch must be 11; that is, one terminal of a coil is connected to bar *1* and the other to bar *12*, the terminals of another coil to bars *12* and *23*, another to bars *23* and *2*, etc., the difference between the two numbers being 11 in each case.

**Double-Series Winding.**—In a *double-series winding*, the connecting pitch is determined by the formula

$$N = \frac{p}{2} \times y \pm 2$$

For example, if $N=32$ and $p=6$, then $3y=30$ or 34 and $y=10$; one series of coils will be connected to bars *1–11–21–31–9*, etc., and the other to bars *2–12–22–32–10*, etc. The brushes must be thick enough so that at least one brush of each sign will touch a segment of each winding, in order that the current in each series of coils may not be broken.

Double windings, either series or parallel, are not common. Machines having such windings very frequently commutate poorly, the sparking being severe and the commutator soon becoming rough and blackened.

**Triple windings** are very rarely used. The terminals of triple-parallel windings are connected to every third segment, and those of triple-series windings to segments separated from each other by the pitch $y$ determined by the formula

$$N = \frac{p}{2} \times y \pm 3$$

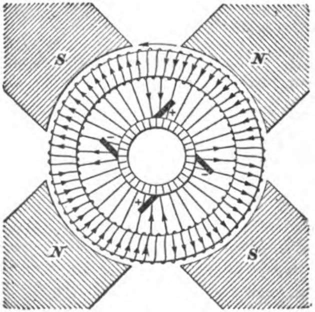



FIG. 20. RING-WOUND ARMATURE

## DIAGRAMS OF ARMATURE WINDINGS

**Ring windings** are not much used in modern machines, except for some types of generators producing high voltages, or very large currents, or constant current. Fig. 20 shows a ring-wound armature in a four-pole field magnet. The winding is represented as continuous, every second turn being connected to a commutator bar, thus making two turns per coil. The bus-rings are not shown; the two positive brushes are connected with the positive ring and

## WINDING DATA

| | Parallel Windings* | | | Series Windings† | | |
|---|---|---|---|---|---|---|
| | Single | Double | Triple | Single | Double | Triple |
| Coils terminate in | Adjacent segments | Adjacent but one segment | Adjacent but two segments | | | |
| A series of $\frac{p}{2}$ coils terminate in | | | | Adjacent segments | Adjacent but one segment | Adjacent but two segments |
| Number of paths | As many as poles | Twice as many as poles | Thrice as many as poles | Two | Four | Six |
| Brushes must always touch at least | One segment | Two segments | Three segments | At least one brush of each sign must always touch a segment of each winding | | |
| Number of coils possible in winding | Any | Any | Any | $N=\frac{p}{2}\times y\pm 1$ | $N=\frac{p}{2}\times y\pm 2$ | $N=\frac{p}{2}\times y\pm 3$ |

***Require as many brushes as poles.** **†Require but two brushes.**

the negative brushes with the negative ring, while the rings are connected with the external circuit. The arrowheads indicate the direction of the flow of current; one coil in each neutral region carries no current, except, perhaps, a short-circuit current. This type of armature can be used in a field frame having any number of poles, provided there is a brush in each neutral space. The E. M. F. between any two coils is never more than that generated in one coil, so that the insulation on the coils need not be heavy.

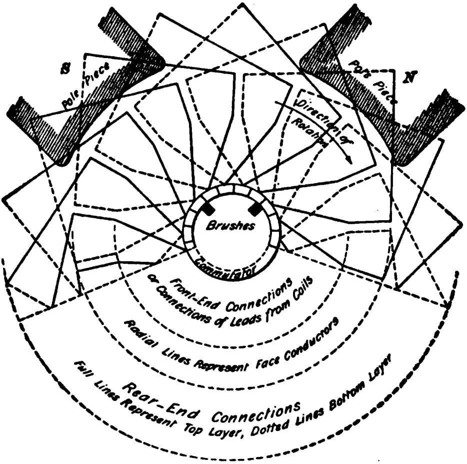
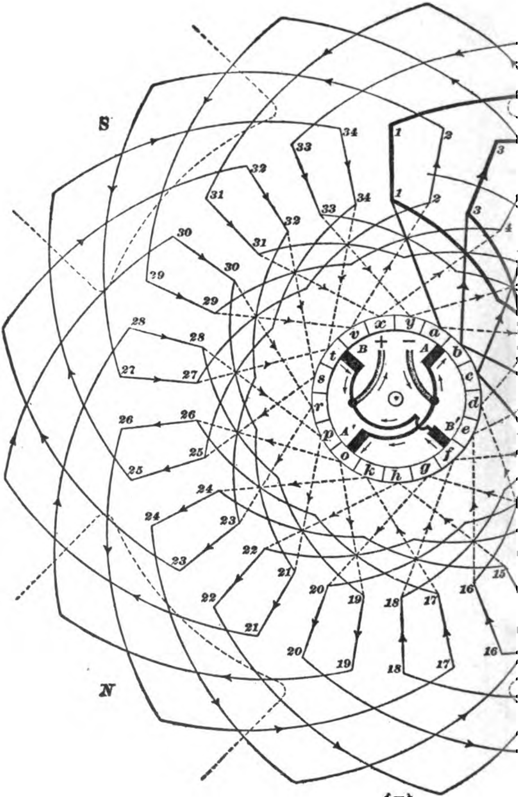



Fig. 21

**Drum windings** are usually represented by diagrams as in Fig. 21. Short radial lines represent face conductors, or the portion of the conductor lying in the slots on the cylindrical surface of the core. Inside the face conductors are shown the front-end connections and leads to the commutator, and outside the face conductors the rear-end connections.

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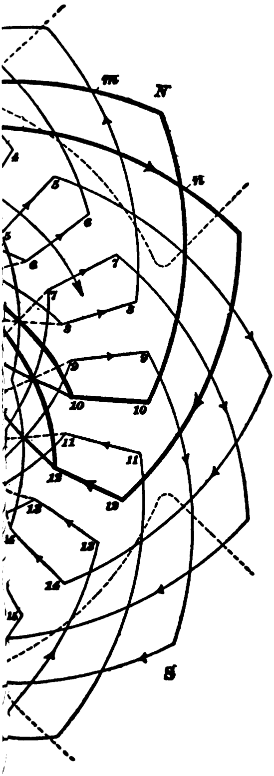
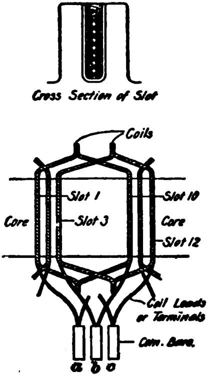
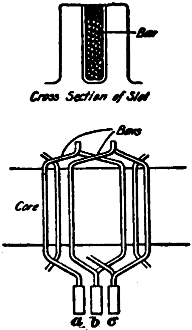
(a)

Fig. 22. Four-Pole, Single-Parallel, Sing




(b)




(c)

...LE-LAYER, DRUM-ARMATURE WINDING DIAGRAM 14

For convenience, the brushes are shown rubbing on the inside of the commutator instead of on the outer surface where they belong. Nearly all direct-current generators of more than 50 K. W. output have only one turn per coil, and rectangular bars are generally used instead of round wire.

## PARALLEL WINDINGS

Fig. 22 (*a*) shows a complete diagram of a four-pole, single-parallel drum winding with seventeen coils, seventeen commutator segments, and thirty-four slots; (*b*) shows a cross-section of a slot and a sketch of a wire-wound coil, and (*c*) the same for a bar-wound coil. The current returns from the outside circuit and enters the armature winding through brushes $A, A'$ and segments $a, b$, and $o$. To $a$ is connected one lead, or terminal, of coil *1–10*, the other end of which is connected to segment $b$. The sides of the coil are midway between the poles, or in the neutral region, while the bars to which its terminals are connected are short-circuited by the brush $A$. The four paths through the other coils, from negative to positive, may be traced as follows:

$$-\left\{\begin{array}{llllll} A, & b\text{–}3\text{–}12, & c\text{–}5\text{–}14, & d\text{–}7\text{–}16, & e\text{–}9\text{–}18, & f\text{–}B' \\ A, & a\text{–}8\text{–}33, & y\text{–}6\text{–}31, & x\text{–}4\text{–}29, & v\text{–}2\text{–}27, & t\text{–}B \\ A', & o\text{–}19\text{–}28, & p\text{–}21\text{–}30, & r\text{–}23\text{–}32, & s\text{–}25\text{–}34, & t\text{–}B \\ A', & o\text{–}26\text{–}17, & k\text{–}24\text{–}15, & h\text{–}22\text{–}13, & g\text{–}20\text{–}11, & f\text{–}B' \end{array}\right\}+$$

In Fig. 22 the commutator end connections, for example, the cross-connection shown from *1* to *10* inside the face conductors, indicate that the coils have more than one turn each, for with only one turn per coil, the coil would appear as an open loop. It is not necessary to show the front-end connections in order to trace a diagram, and since they make the diagram confusing they are usually omitted. The main point to consider is the manner of connecting the coil ends to the commutator.

The position the brushes occupy on the commutator with regard to the pole pieces depends on the way in which the terminals of the coils are connected. In Fig. 22, the leads for connection to the commutator are of nearly equal length, so that when the face conductors of a coil are in the neutral

regions, the bars to which the leads are connected, and on which the brushes should rest, are nearly opposite the center of the coil and the center of a pole. For example, leads *1–a* and *10–b* are of about equal length, and each is brought around toward the center of the coil, that is, given a *throw*, or *lead*, of about one-half a coil pitch or one-eighth of a circumference. If each coil had one short lead coming straight out to a commutator segment, that is, with no throw, and one long lead having a throw equal to the coil pitch or, on a four-pole machine, one-fourth a circumference, the neutral spaces on the commutator would be opposite the spaces between the poles. For example, if lead *1–a* came straight down to a commutator bar and lead 10–*b* was brought over to the next adjacent bar, and all other coils connected accordingly, the brushes should be opposite the interpolar spaces. This is done on some kinds of railway motors, but with most generators the neutral position of the brushes at no load is nearly opposite the center of the pole pieces.

In Fig. 22, the leads of a coil do not cross each other on the way from the slots to the commutator bars, but if face conductor *1* were connected to *b*, *10* to *a*, *3* to *c*, *12* to *b*, etc., that is, if the connections of the leads of each coil were interchanged, the winding would still be single parallel; the coils would terminate in adjacent segments, and the E. M. F. generated in each coil would be the same in direction and amount as before the change. However, the E. M. F. of the completed armature would be reversed and the brushes *A* and *A′* would become positive brushes, while *B* and *B′* would become negative brushes. Each lead would have to be a little longer than before, thus requiring a little more copper.

At each end of any drum-wound armature, part of the connections extend to the right and part to the left. In order to permit this crossing without the leads interfering with each other, two-layer windings are generally used; that is, each slot contains one side of each of two coils, so that all the leads swinging one way issue from the bottom of slots and all the leads swinging the other way, from the

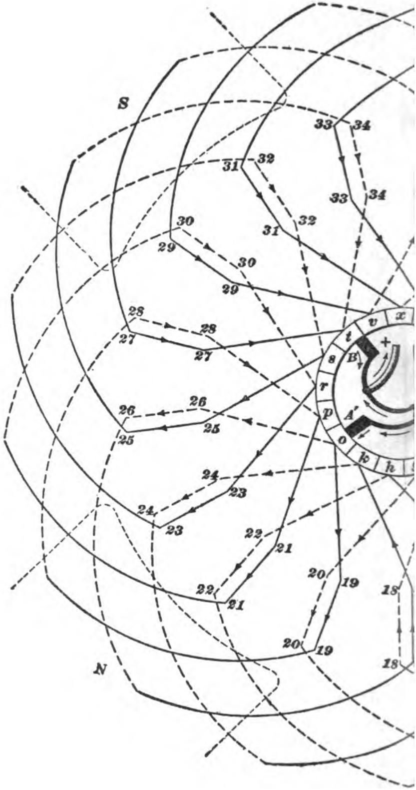



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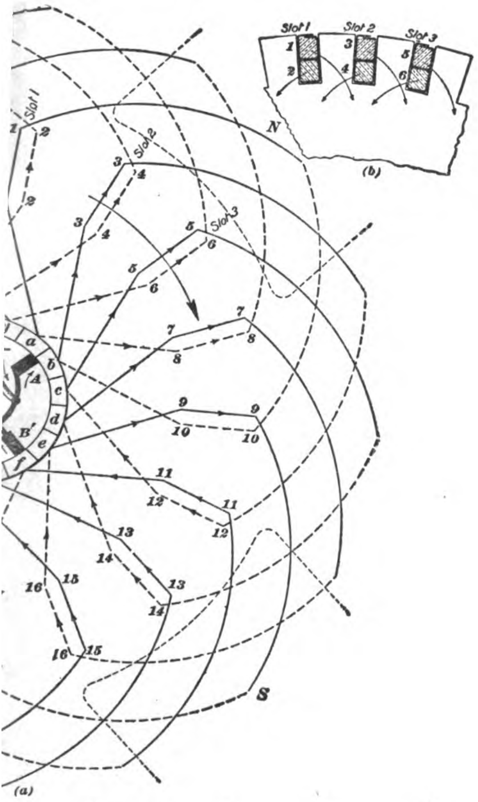
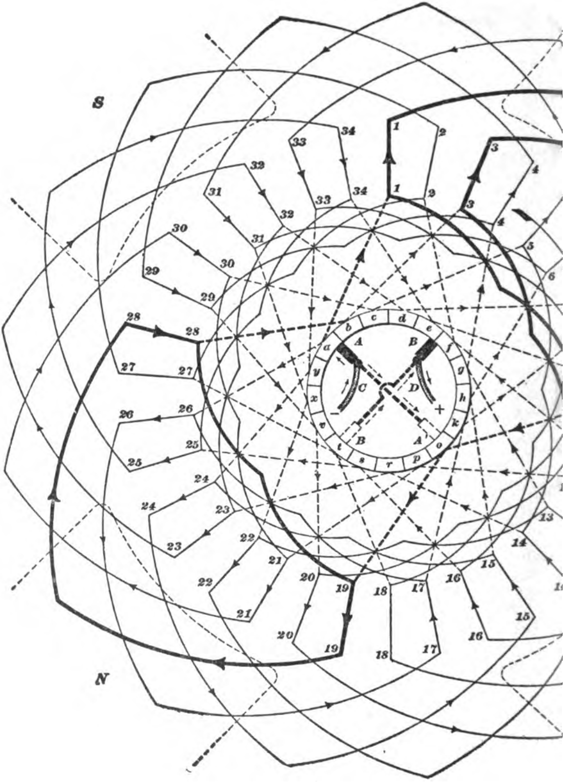
FIG. 23. FOUR-POLE, SINGLE-PARALL




, DOUBLE-LAYER, DRUM-ARMATURE WINDING 14

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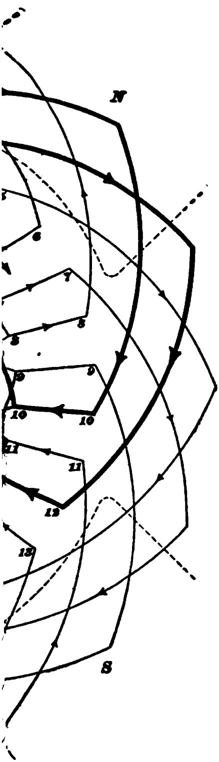
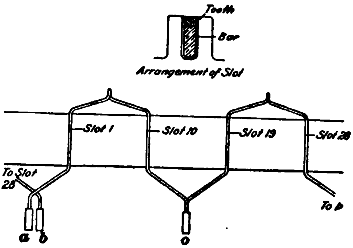
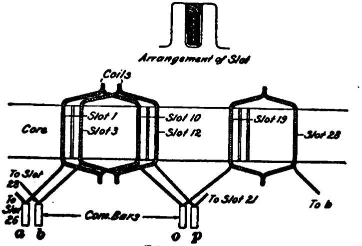
(a)

FIG. 24. FOUR-POLE, SINGLE-SERIES,




(c)




(b)

NGLE-LAYER, DRUM-ARMATURE WINDING 14

tops of slots. Fig. 23 (*a*) shows a two-layer winding of one turn per coil having the same number of coils and commutator bars as that shown in Fig. 22, but with only seventeen slots instead of thirty-four, making one coil (two half-coils) per slot. The four paths from negative to positive brushes may be traced precisely the same as for Fig. 22. Fig. 23 (*b*) shows how the sides of the coils lie in the slots.

## SERIES WINDINGS

A diagram of a *single-series winding* of the single-layer type is shown in Fig. 24. This winding is exactly like that of Fig. 22, except in regard to the connections to the commutator. In the winding shown in Fig. 22, each coil forms nearly a closed loop, its two terminals being connected to adjacent bars as at (*c*); in Fig. 24 (*c*), the coils appear as open loops and the winding progresses in a series of zigzag lines, or waves. The winding shown in Fig. 22, therefore, is sometimes called a *lap winding* and that shown in Fig. 24, a *wave winding;* but the terms *parallel* and *series* are preferred, since they are applicable to both ring- and drum-type windings, while the terms lap and wave are applicable only to the drum type.

When only one pair of brushes is used, the two paths through the winding, Fig. 24, are as follows:

$$-\left\{\begin{matrix} A,\ a\text{–}1\text{–}10,\ o\text{–}19\text{–}28,\ b\text{–}3\text{–}12,\ p\text{–}21\text{–}30, \\ c\text{–}5\text{–}14,\ r\text{–}23\text{–}32,\ d\text{–}7\text{–}16,\ s\text{–}25\text{–}34,\ e\text{–}B \\ A,\ a\text{–}26\text{–}17,\ k\text{–}8\text{–}33,\ y\text{–}24\text{–}15,\ h\text{–}6\text{–}31, \\ x\text{–}22\text{–}13,\ g\text{–}4\text{–}29,\ v\text{–}20\text{–}11,\ f\text{–}B \end{matrix}\right\}+$$

If the second pair of brushes, shown by dotted lines, were also in use, the foregoing circuits would be somewhat modified.

At the instant shown, there are eight coils in one path and seven in the other, there being a series of $\frac{p}{2}$ coils *e–9–18* and *t–27–2–f* short-circuited by the brush *B*. Since there are seven coils in one path and eight in the other, it would appear that the winding is not symmetrical and that one path would take more current than the other; but when the armature turns through a small fraction of a revolution,

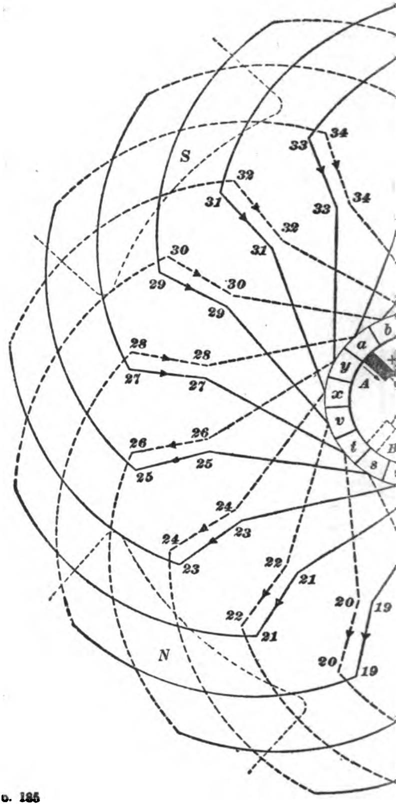
the path that now has eight coils will then have seven and that with seven will then have eight, so that during a complete revolution, the two paths will have the same average number of coils even though they may not be alike at a given instant. Since there are from twenty to fifty, or more, coils per path in most machines, the differences caused bv having one coil more or less in a path is insignificant.

In Fig. 22 each path has only four coils in series; but in Fig. 24, with the same total number of coils, each path has eight coils in series; hence, with the same number of turns per coil and the same speed and field strength, the series winding will generate twice, or $\frac{p}{2}$ times, the E. M. F. of the parallel winding. Since the series winding has two paths and the parallel four, or $\frac{p}{2}$ times as many, the series winding has only $\frac{2}{p}$ times as much current capacity as the parallel winding. If the watts $W=E\,I$ for the parallel-wound armature, for the series armature $W=\frac{p}{2}E\times\frac{2}{p}I=EI$; that is, the output in watts is the same for both windings.

When the number of commutator bars $N=17$ and $p=4$, the connecting pitch as determined by the formula $N=\frac{p}{2}\times y\pm 1$ could be either 9 or 8. Fig. 24 shows a connecting pitch of 9, that is, the terminals of coil *1–10* are 9 segments removed from each other.

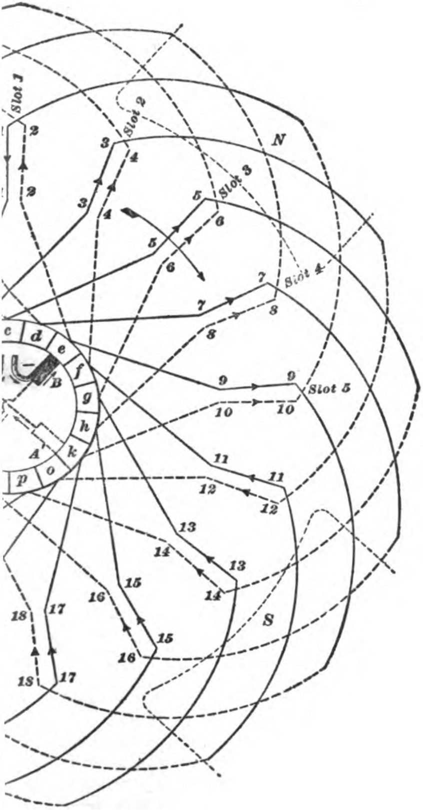
Fig. 25 shows a two-layer winding with a connecting pitch of 8. Figs. 24 and 25 are electrically the same, except that in Fig. 24 a series of $\frac{p}{2}$ coils spans a circumference plus 1 segment on the commutator, while in Fig. 25 the series lacks 1 segment of spanning a circumference. The effect of this is the same as though the terminals of each coil were reversed; the direction of the current in the face conductors is not changed, but the polarity of the machine terminals is reversed.

**Front and Back Pitches.**—In Fig. 25, conductor *1* on the armature is connected across the back of the armature to




o. 185

**Fig. 25. Four-Pole. Single-Series,**




ouble-Layer, Drum-Armature Winding 14

conductor *10*, making the *back pitch* on the armature 9. Conductor *10* is connected to conductor *17* across the front of the armature by way of commutator bar *k*. The *front pitch* is therefore 7, and the average armature pitch is $\frac{9+7}{2}=8$, which is equal to the pitch on the commutator, since the commutator pitch must always be equal to the average armature pitch. In Fig. 24, both front and back pitches are 9, hence the average pitch is also 9. The terms front pitch and back pitch are seldom used.

**Equalizer Rings.**—Many different windings have been proposed and used successfully, but the majority of direct-current armatures are wound with either the single-parallel ring winding, the single-parallel drum winding, or the single-series drum winding. The single-parallel drum winding is probably more used than any other, and, on large multipolar machines, any unbalanced condition in the magnetic circuits is likely to cause the current to divide unequally among the several paths through the armature. For example, if the air gap on one side of the armature becomes slightly shorter than that on the opposite side, which may easily occur due to wear of the bearings, the flux in the short air gap will become unduly dense, thus causing the generation of higher E. M. F. in the armature conductors on that side than in those on the other side, and the path that develops the highest E. M. F. takes the greatest share of the current. In some cases this unbalanced condition may not be bad enough to cause trouble other than some slight sparking, but in extreme cases, the E. M. F. of one path or of the paths on one side of the armature may become so excessive as to reverse the current in some of the other paths, making part of the armature act as a generator and part of it as a motor at the same time. This condition is usually accompanied by severe vibration of the whole machine, due to excessive mechanical strains, with more or less violent sparking or flashing at the brushes, and the machine is said to be *bucking*. On account of the effects of armature reactions, bucking is somewhat more liable to occur in motors than in generators. The bad effects due to unbalancing can be eliminated by

providing the armature with *equalizer rings*, as shown at *E, E*, Fig. 26. By means of leads *S, S*, these rings connect points of equal potential in the winding and allow an equalization of current between the various paths in the armature.

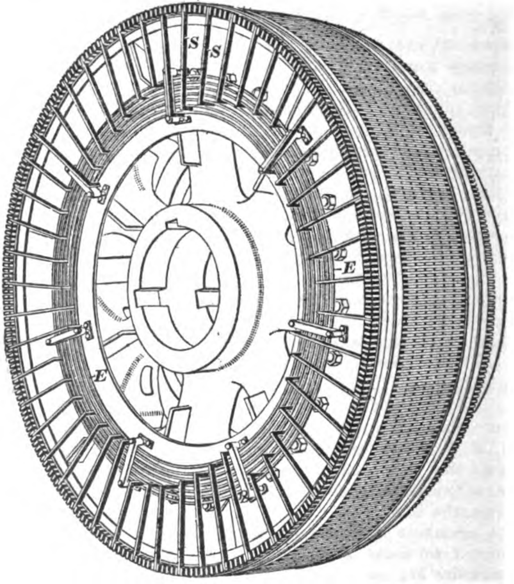



FIG. 26. PARALLEL-WOUND ARMATURE WITH EQUALIZER RINGS

**Appearance of Complete Series- and Parallel-Wound Armature.**—Whether a completed armature is parallel- or series-connected can usually be told by noting the direction taken by the front- and rear-end connections of the face conductors. If both end connections of a conductor swing the same way,

right or left, the armature is parallel-connected; but if the connections on the two ends swing in opposite directions, the armature is series-connected. For example, in Figs. 22 and 23 both the front- and the rear-end connections of face conductor *1* swing to the right, thus showing parallel connection; but in Figs. 24 and 25 the rear-end connection of face conductor *1* swings to the right and the front-end connection to the left, thus showing series-connection.

**Winding Pitch.**—The number of slots spanned by the sides of a coil, or the *winding pitch*, is usually the whole number nearest to, or next smaller than, the quotient of the number of slots in the armature divided by the number of poles. Thus, with seventeen slots and four poles, the quotient is 4¼ and the nearest whole number is 4; this is the winding pitch used in Figs. 23 and 25, one side of a coil being in the top of a slot and the other in the bottom of the fifth slot away.

## RAILWAY-MOTOR ARMATURE WINDINGS

*Street-railway motors* are almost invariably four-pole type with series, or two-circuit, armatures. Among the advantages of the series winding for this work are the absence of any unbalancing due to unequal air gaps or other causes, and the necessity for only two brushes. Most railway-motor commutators are accessible from only one side—top or bottom—and if four brushes were used, two of them would be hard to reach.

**Fig. 27** illustrates some of the features to which it will be necessary to refer in describing railway-motor armature windings. The diagram is for an armature with 99 slots and 99 coils and commutator bars, or 1 coil per slot. Only a few of the slots and bars are represented. Some motors are installed so that lines connecting opposite pole centers will be horizontal and vertical, as $mn$ and $op$, and others so that the pole centers are on diagonal lines, as $m' n'$ and $o' p'$. If the opening in the frame for making inspections is directly over the commutator, the brushes must rest on top, as indicated at $x, y$; if the center of the opening is 45° to the right of the vertical, the brushes must be in the position suggested by $x' y'$.

On four-pole railway motors, the ***pitch of the coils***, or the ***winding pitch***, is usually a little less than one-fourth the number of slots. In Fig. 27, this pitch is twenty-four slots; that is, one side of a coil lies in slot *1* and the other in slot *25*, etc. The ***pitch of the leads***, or the ***connecting pitch***, as determined by the formula $N = \frac{p}{2} \times y \pm 1$, could be 49 or 50. In this case, 49 is used, lead *a* connecting to bar *1* and lead *b* to bar *50*. The connections *c*, *d* of another coil, with one side in slot *25* and the other in slot *49*, are also shown to bars *25* and *74*.

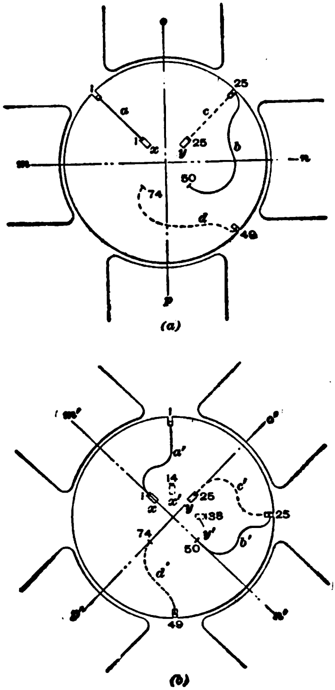



Fig. 27

The *throw of the leads* must be such that the brushes will rest on bars connected to face conductors in the neutral region. For example, with poles and brushes placed as in Fig. 27 (*a*), one lead *a* of each coil has no throw, but comes from the slot straight out to a commutator bar; the other lead *b* has a throw sufficient to give the correct pitch of the

leads, or in this case, twenty-five bars (bar *25* to bar *50*). With poles diagonal as in Fig. 27 (*b*) and brushes as at $x$, $y$, each lead of a coil has a throw away from the coil of about one-fourth the total pitch of the leads; in this case, lead $a'$ has a throw of thirteen bars to the left (*14–1*), and lead $b'$, of twelve bars to the right (*50–38*). If the brushes were at positions $x'$, $y'$, lead $a'$ would have no throw and lead $b'$ a throw of twenty-five bars to the right. The expressions right and left used in this connection are assumed to mean when facing the commutator end of the armature.

## ARMATURE-WINDING DATA

In both of the following tables, the winding pitch is given by the figures indicating the slots in which the sides of a

### WESTINGHOUSE DIRECT-CURRENT RAILWAY MOTORS

| Motor No. | Number of Slots | Coils per Slot | Number of Commutator Bars | Coils Lie in Slots | Leads Connect to Bars | Throw of Leads |
|---|---|---|---|---|---|---|
| 3 | 95 | 1 | 95 | 1–25 | 1–49 | 12 |
| 12A | 47 | 2 | 93* | 1–12 | 1–48 | 13 |
| 49 | 59 | 2 | 117* | 1–14 | 1–60 | 17 |
| 68 and 68C | 55 | 2 | 109* | 1–14 | 1–56 | 15 |
| 56 and 81 | 39 | 3 | 117 | 1–10 | 1–60 | 16 |
| 38 and 38B | 45 | 3 | 135 | 1–11 | 1–69 | 19 |
| 50L | 55 | 3 | 165 | 1–13 | 1–84 | 24 |
| 101 | 37 | 3 | 111 | 1–10 | 1–56 | 14 |
| 69 | 35 | 3 | 105 | 1–10 | 1–54 | 13 |
| 76 | 39 | 3 | 117 | 1–10 | 1–60 | 16 |
| 85 | 39 | 3 | 117 | 1–10 | 1–60 | 16 |
| 89 | 45 | 3 | 135 | 1–11 | 1–69 | 19 |
| 92A | 41 | 3 | 123 | 1–10 | 1–62 | 17 |
| 93A | 45 | 3 | 135 | 1–11 | 1–69 | 19 |
| 112 | 45 | 5 | 225 | 1–11 | 1–113 | 31 |

*Armature has one idle coil.

## GENERAL ELECTRIC AND MISCELLANEOUS DIRECT-CURRENT RAILWAY MOTORS

| Motor | | Armature Data | | | | | |
|---|---|---|---|---|---|---|---|
| | | Number of Slots | Coils per Slot | Number of Commutator Bars | Coils Lie in Slots | Leads Connect to Bars | Throw of Leads |
| General Electric Co. | 800 | 105 | 1 | 105 | 1–27 | 1–53 | 26 |
| | 1200 | 105 | 1 | 105 | 1–27 | 1–53 | 26 |
| | 50 | 105 | 1 | 105 | 1–27 | 1–53 | 26 |
| | 1000 | 93 | 1 | 93 | 1–24 | 1–47 | 12 |
| | 51A | 37 | 3 | 111 | 1–10 | 1–56 | 14 |
| | 52 | 29 | 3 | 87 | 1– 8 | 1–44 | 11 |
| | 53 | 37 | 3 | 111 | 1–10 | 1–56 | 0 |
| | | 33 | 3 | 99 | 1– 9 | 1–50 | 0 |
| | 54 | 29 | 4 | 115* | 1– 8 | 1–58 | 14 |
| | 55 | 47 | 3 | 141 | 1–12 | 1–71 | 13 |
| | 57 | 37 | 3 | 111 | 1–10 | 1–56 | 14 |
| | | 33 | 3 | 99 | 1– 9 | 1–50 | 12 |
| | 58 | 33 | 3 | 99 | 1– 9 | 1–50 | 12 |
| | 60 | 37 | 3 | 111 | 1–10 | 1–56 | 14 |
| | 61 | 41 | 3 | 123 | 1–11 | 1–62 | 15 |
| | 65 | 55 | 3 | 165 | 1–14 | 1–83 | 18 |
| | 66 | 39 | 5 | 195 | 1–10 | 1–98 | 16 |
| | 67 | 37 | 3 | 111 | 1–10 | 1–56 | 14 |
| | 70 | 37 | 3 | 111 | 1–10 | 1–56 | 15 |
| | 73 | 39 | 3 | 117 | 1–10 | 1–59 | 11 |
| | 74 | 39 | 3 | 117 | 1–10 | 1–59 | 15 |
| | 80 | 37 | 3 | 111 | 1–10 | 1–56 | 15 |
| Steel Motor Co. | C and C3 | 99 | 1 | 99 | 1–26 | 1–50 | 0 |
| Lorain Steel Co. | 22 and 28 | 29 | 3 | 87 | 1– 8 | 1–44 | 11 |
| | 20 and 34 | 33 | 3 | 99 | 1– 9 | 1–50 | 12 |
| | 18 | 37 | 3 | 111 | 1–10 | 1–56 | 14 |
| Christensen | AA1 | 29 | 3 | 87 | 1– 8 | 1–45 | 3 |
| | B2 and C3 | 47 | 2 | 93* | 1–12 | 1–48 | 3 |

*Armature has one idle coil.

coil lie, the connecting pitch by giving the bars to which the leads of a coil are connected, and the throw of leads by giving the number of bars between the bar directly opposite the slot from which the first lead to be connected issues and the bar to which the lead should connect. For example, in the G. E. 800 armature, the coils lie in slots 1–27, that is, the winding pitch is 26; the leads connect to bars 1–53, making the connecting pitch 52; and the throw of the first lead, or of all leads from corresponding sides of coils is 26.

# TESTS FOR LOCATING FAULTS

## FAULTS IN FIELD CIRCUITS

**Open-Circuited Field Coil.**—To find an *open-circuited field coil*, connect a source of E. M. F. across the terminals of the field circuit, as *a*, *e*, Fig. 28, and apply the leads of a voltmeter *V* successively to the field-coil terminals *ab*, *bc*, etc. No deflection of the voltmeter needle will occur until the leads are applied to the terminals of the defective coil, and then the deflection will show the voltage between *a* and *e*. Or, instead, one lead of the voltmeter can be left connected to one side of the circuit as at *a* and the other touched to successive coil terminals *b*, *c*, etc. No deflection will be obtained until the defect is bridged, and then the voltmeter will show the full voltage between *a* and *e*.

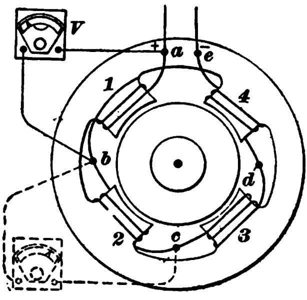


FIG. 28

**Short-Circuited Field Coil.**—To find a *short-circuited field coil*, proceed as just described, but limit the current through

the field circuit to a safe amount. The drop across the defective field coil, as shown by the voltmeter, will be much less than that across any other coil.

Fig. 29 shows a transformer convenient for quickly indicating short circuits in coils of any kind that can be placed over its laminated iron core. The size of core depends on the kind of coils to be tested; that shown has a cross-section 3 in. × 4 in., and is intended for testing railway-motor field coils. The upper part of the core is in the form of a hinged yoke, which can be thrown back so that the coil to be tested may be slipped over the outer leg. The hinged yoke is then replaced and alternating current is sent through the fixed primary coil, making an alternating magnetic flux thread the coil under test, as indicated by the dotted lines. A considerable E. M. F. is thereby induced in the field coil, and a heavy local current flows through any short-circuited turns, soon burning them out.

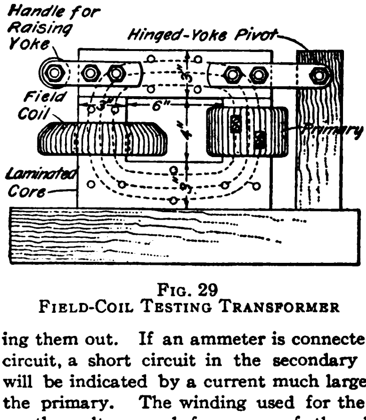



Fig. 29
Field-Coil Testing Transformer

If an ammeter is connected in the primary circuit, a short circuit in the secondary (coil under test) will be indicated by a current much larger than normal in the primary. The winding used for the primary depends on the voltage and frequency of the alternating-current supply. For current at 104 volts and 60 to 125 cycles, seventy-five turns of No. 5 or No. 6 B. & S. wire will answer; the larger the wire the better, especially if the transformer is to be used for burning out defects.

In the method shown in Fig. 30, a telephone receiver is connected in series with two symmetrically placed coils *a*, *b*.

Very little sound will be heard when the flux through the two coils *ab* is the same; but if a short-circuited coil is being tested, the fluxes through the coils *a*, *b* will not be equal and a noise can be heard in the receiver.

**Grounded Field Coil.**—A *grounded field coil* can be found by applying an E. M. F. to the machine terminals, connecting one lead of a voltmeter to the frame, and touching the other lead successively to exposed parts of the field circuit. The nearer the ground the exploring lead is touched, the less will be the deflection.

**Voltmeter Method of Measuring Insulation Resistance.** If the ground is only partial, the insulation resistance can be measured, provided the resistance of the voltmeter is

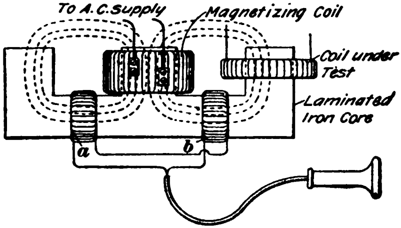



FIG. 30. FIELD-COIL TESTING WITH TELEPHONE RECEIVER

known. The deflection of a voltmeter is inversely proportional to the resistance in its circuit. If $E$ is the deflection measured directly across the circuit, $E'$ the deflection measured through the insulation resistance, $r$ the resistance of the voltmeter, and $R$ the resistance of the insulation, then

$$\frac{E'}{E} = \frac{r}{R+r}$$

or,

$$R = r\left(\frac{E}{E'} - 1\right)$$

As $E'$ and $E$ are in volts and $r$ in ohms, $R$ will be in ohms.

## FAULTS IN ARMATURES

In winding an armature, it is advisable to test the winding for grounds and short circuits before the coils are connected to the commutator; any defective coils can then be replaced. The armature can be given an insulation-breakdown test by joining all the free ends of the coils together by means of a

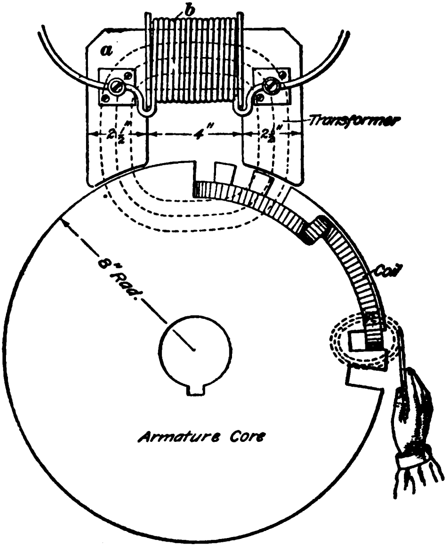



FIG. 31. ARMATURE-COIL TESTING TRANSFORMER

small wire and then connecting the series of coils to one terminal of a high-pressure transformer. The other terminal of the transformer is connected to the shaft. The alternating-current pressure to be applied for breakdown tests

## INSULATION TESTS

| Rated Voltage of Device Under Test | Capacity | Testing Voltage |
|---|---|---|
| Not exceeding 400 volts | Under 10 K. W. | 1,000 volts |
| Not exceeding 400 volts | 10 K. W. and over | 1,500 volts |
| 400 and over, but less than 800 | Under 10 K. W. | 1,500 volts |
| 400 and over, but less than 800 | 10 K. W. and over | 2,000 volts |
| 800 and over, but less than 1,200 | Any | 3,500 volts |
| 1,200 and over, but less than 2,500 | Any | 5,000 volts |
| 2,500 and over, but less than 10,000 | Any | Double the rated voltage |
| 10,000 and over, but less than 20,000 | Any | 10,000 volts above normal rated voltage |
| 20,000 and over | Any | 50% above normal rated voltage |
| Synchronous motor fields and fields of rotary converters started from the alternating-current side | | 5,000 volts |

on completed machines, according to recommendations of the American Institute of Electrical Engineers, is given in the table on page 195:

**Short-Circuited Armature Coils.**—A *short-circuited armature coil* can be found by sending through it an alternating flux, which will set up a current in the coil. Fig. 31 shows a special transformer with a laminated iron core *a* having a width approximately equal to the length of the armatures to be tested and pole-face curvature about the same as that of the armature cores. The dimensions shown in the figure are appropriate for a transformer used in testing many railway-motor armatures; the dimensions of a section of the transformer core are 8 in.$\times 2\frac{1}{2}$ in., that is, the cross-section is 20 sq. in. The magnetizing coil *b* has forty-five turns of No. 5 B. & S. wire.

When the transformer is placed over an armature core, as shown, and an alternating current is sent through the magnetizing coil, the alternating flux induces an E. M. F. in the armature coils under the transformer poles; and if any coil is short-circuited, the resulting current will heat it or will cause a piece of iron held near it to vibrate. By turning the armature through a complete revolution and holding a piece of iron over the coils being tested, any short-circuited coils are easily located. The transformer should not be put in position nor removed while current is flowing through the magnetizing coil.

**Bar-to-Bar Armature Test.**—The *bar-to-bar test*, Fig. 32, will reveal the existence and location of open circuits, short circuits, wrong connections, and grounds in a completed armature. A steady current, adjusted to suit the requirements, is sent through the armature by way of terminals *a*, *b* clamped to opposite sides of the commutator, and the leads of a galvanometer or low-reading voltmeter are connected to adjacent commutator segments by means of contact points *c*, *d* insulated from each other and held a fixed distance apart by a block of wood or fiber *i*. If there are no defects in the armature, the reading of the voltmeter will be the same for each pair of adjacent segments, because the drop of voltage is the same in all the coils. By trying

several pairs, a standard deflection can be obtained, the current being adjusted until this deflection is easily readable. If two bars are short-circuited, as at *e*, or a coil is short-circuited, as at *f*, the voltmeter will show almost no deflection when the contacts are on the bars *1–2* and *3–4*, respectively. A poor connection between a bar and the two coil leads belonging in it will be revealed by an unusually large drop between the bar and either adjacent bar. An open circuit in any coil, as at *g*, prevents any flow of current through that portion of the armature, and when the points *c*, *d* bridge any two bars in the open section, the voltmeter

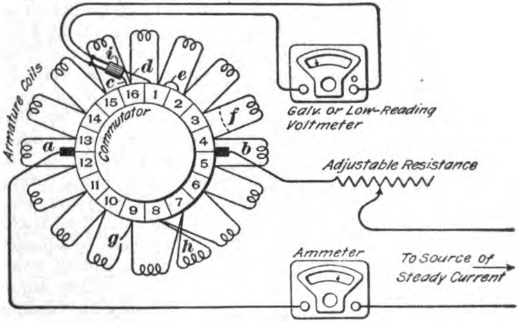



FIG. 32. BAR-TO-BAR ARMATURE TEST

will show no deflection until bars *9* and *10* are bridged, when the full voltage between the terminals *a* and *b* will be impressed on the instrument and the needle will be thrown violently. Crossed leads, as at *h*, will cause double deflection when the points are on bars *6* and *7* or *8* and *9*, because the drop then measured is that in two coils; between bars *7* and *8* the drop is normal in value but reversed in direction.

**Grounded Armature Coil.**— A *grounded armature coil* can be found by impressing full voltage on the test brushes, connecting one voltmeter lead to the shaft, and touching the

other successively to each commutator bar. The deflection will be least when a bar to which the grounded coil is connected is touched.

## RHEOSTATS FOR LOAD TESTS

When testing a dynamo, the load current, if small, may be run through a bank of incandescent lamps, the resistance of which can be adjusted by cutting lamps in or out; but for large currents, lamp banks are too expensive. For moderately large currents, a cheap resistance can be made by tacking strips of ordinary roofing tin to a wooden frame, as shown in Fig. 33. The strips should be from ½ in. to ¾ in. wide, and the sheet should be slit to within ½ in. of opposite edges, so that when pulled apart and tacked on the frame, it will form a continuous conductor. The resistance can be adjusted by a sliding crosspiece, which short-circuits any desired amount of resistance.

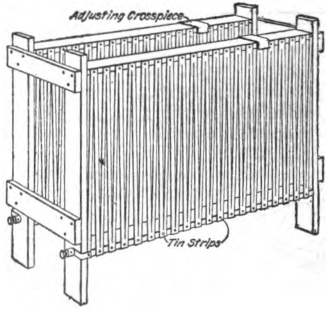



FIG. 33. LOAD RHEOSTAT

Frames of this kind can be made of different capacities, depending on the width of strip used, and by connecting a number of frames in series or in parallel a considerable output can be handled.

**Water Rheostats.**—For absorbing larger outputs where great accuracy is not essential, *plain water rheostats* are useful. A comparatively small water rheostat can be made of an ordinary oil barrel having an iron plate placed on the bottom, to which plate a heavy insulated wire is connected, as shown in Fig. 34. The upper electrode is an iron plate suspended by a heavy copper wire and a cord attached to a counter-

weight, so that the plate can easily be moved up or down and thus made to vary the resistance. Common salt, sal-ammoniac, or sal-soda is added to the water in order to increase its conductivity; this should be added in solution rather than in solid form, in order that the amount added may be easily controlled. An ordinary barrel rheostat can carry a current of 90 to 100 amperes without boiling excessively.

The resistance of the liquid in an ordinary water rheostat decreases with increase in temperature; a *submerged-wire rheostat*, a form of which is shown in Fig. 35, gives a more steady load and is capable of handling a larger current. Resistance coils, usually of galvanized-iron wire, are mounted on a wooden frame, and the whole frame is submerged in the water, which is kept cool by a continuous circulation of water. The coils are connected to terminals, so that they can be arranged in any desired combination to suit the voltage or current output of the machine under test. Wooden partitions between the coils prevent the current from passing across from coil to coil and thereby producing electrolytic action. A large amount of power can be absorbed in a rheostat of this kind, and it provides a load that is steady and easily controlled. The carrying capacity of galvanized-iron wire when

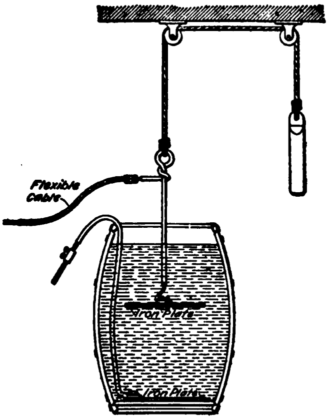


FIG. 34. PLAIN WATER RHEOSTAT

## SAFE CARRYING CAPACITY OF IRON WIRE

*(From American Electrician)*

| Size B. & S. Gauge No. | Area Circular Mils $d^2$ | Safe Capacity in Air $= .014d^{\frac{3}{2}}$ Amperes | Length for Safe Capacity in Air Across 110 Volts Feet | Safe Capacity in Water $= .2d^{\frac{3}{2}}$ Amperes | Length for Safe Capacity in Water Across 110 Volts Feet | Feet per Pound of Wire |
|---|---|---|---|---|---|---|
| 20 | 1.018 | 2.5 | 594 | 36 | 25 | 369.0 |
| 19 | 1.253 | 3.0 | 626 | 43 | 27 | 293.2 |
| 18 | 1.624 | 3.6 | 673 | 51 | 29 | 232.5 |
| 17 | 2.048 | 4.3 | 710 | 61 | 30 | 184.2 |
| 16 | 2,583 | 5.1 | 750 | 72 | 32 | 146.0 |
| 15 | 3,257 | 6.0 | 790 | 86 | 34 | 107.5 |
| 14 | 4,107 | 7.2 | 840 | 103 | 36 | 91.9 |
| 13 | 5,178 | 8.5 | 886 | 122 | 38 | 72.2 |
| 12 | 6,530 | 10.1 | 941 | 145 | 40 | 57.8 |
| 11 | 8,234 | 12.1 | 990 | 173 | 42 | 45.8 |
| 10 | 10,380 | 14.4 | 1,054 | 205 | 45 | 36.4 |
| 9 | 13,090 | 17.2 | 1,103 | 245 | 47 | 33.3 |
| 8 | 16,510 | 20.4 | 1,354 | 292 | 58 | 25.0 |

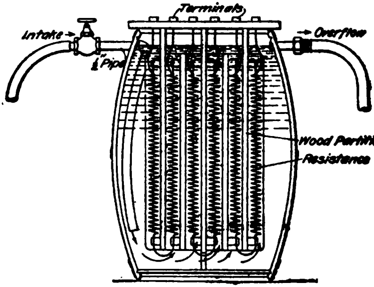



FIG. 35. SUBMERGED-WIRE RHEOSTAT

submerged in water is from twelve to fourteen times as much as when in open air, as shown in the table on page 200. The circulation of water through the barrel should be brisk enough to keep the temperature down to 200° F.

## PARALLEL RUNNING OF DIRECT-CURRENT DYNAMOS

**Shunt Dynamos in Parallel.**—If two shunt machines, *A*, *B*, Fig. 36, are operating in parallel, each supplying one-

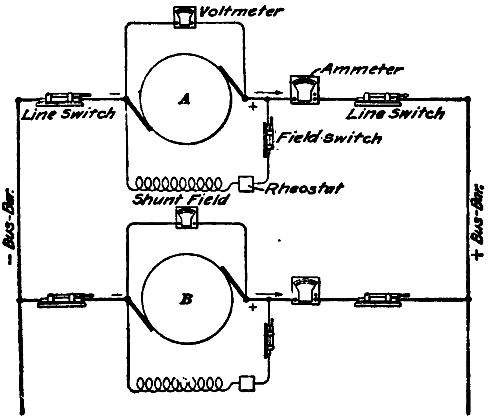



FIG. 36. SHUNT DYNAMOS IN PARALLEL

half the required current and one, say *A*, owing to a speed drop or any other cause, should reduce its voltage, and fail to supply its half of the current, more than one-half the load would then be thrown on *B*. As the load on a shunt dynamo decreases, its voltage rises; and as the load increases, the voltage falls; therefore, the tendency would be for the voltage of *A* to increase and for that of *B* to decrease until

the two were again equal. Shunt dynamos, therefore, are well adapted for parallel operation, but, owing to poor voltage regulation, they are not much used.

**Compound Dynamos in Parallel.**—Compound-wound dynamos may be adjusted to maintain very nearly a constant voltage or even to cause an increase of voltage with increasing load. When compound-wound dynamos are operated in parallel (Fig. 37), a conductor of low resistance, called an

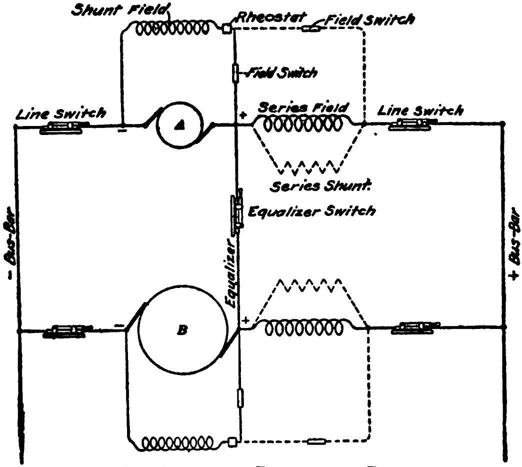



Fig. 37. Compound Dynamos in Parallel

equalizer, must connect the brushes to which the series fields are connected. If the positive brushes are connected, there are two paths for current to pass from the positive brush of either machine to the positive bus-bar—one through its own series field and one through the equalizer and the series field of the other machine. If the resistance of the equalizer were zero, it is evident that the division of the

current from either positive brush through the two paths would be inversely proportional to the resistance of the paths; that is, the path having the higher resistance would carry the smaller current. The division of the current between the two series fields may therefore be made as desired by adjusting the resistance of one or both paths.

In adjusting the division of load between two compound-wound dynamos $A$, $B$, Fig. 37, in parallel, the compounding of each machine is first made right by adjusting its series shunt so that the voltages of the two machines will agree

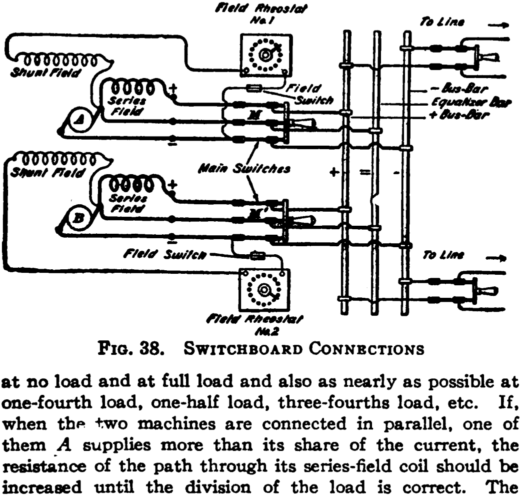



FIG. 38. SWITCHBOARD CONNECTIONS

at no load and at full load and also as nearly as possible at one-fourth load, one-half load, three-fourths load, etc. If, when the two machines are connected in parallel, one of them $A$ supplies more than its share of the current, the resistance of the path through its series-field coil should be increased until the division of the load is correct. The resistance of the series-field coil of a dynamo is usually very small, so that only a very slight addition will be needed in any case. The necessary resistance may be obtained by using a longer lead between the machine and the switch-board, or possibly by inserting iron or German-silver washers

under a terminal lug. It is useless to try to adjust the division of load by adjusting the series-field shunts, for when the machines are connected in parallel, adjusting either shunt affects both machines alike.

For the most successful parallel operation, dynamos should be of the same design and construction and should possess as nearly as possible the same characteristics; that is,

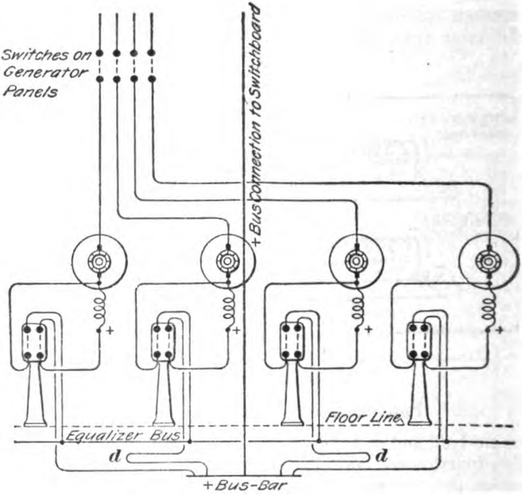



FIG. 39. EQUALIZER CONNECTIONS

each should respond with the same readiness, and to the same extent, to any change in its field excitation. Any number of such machines may be operated in parallel.

The usual practice is to connect the equalizer and the series field to the positive terminal, though if desired they may be connected to the negative terminal; both, however,

must be connected to the same terminal. The resistance of the equalizer should be as low as possible, and it must never be greater than the resistance of any of the leads from the dynamos to the bus-bar.

Sometimes, a third wire is run to the switchboard from each dynamo and there connected to an equalizer bar, as in Fig. 38; but the usual practice is to run the equalizer directly between the dynamos and to place the equalizer switches on pedestals near the machines. This shortens the connections and leads to better regulation. The positive and equalizer switches of each machine differ in potential only by the slight drop in the series coil, and in some large stations these two switches are placed side by side on a pedestal near the machine, as in Fig. 39. In such cases, the equalizer and positive bus-bars are often placed under the floor near the machines, so that all leads may be as short as possible. If all the dynamos are of equal capacity, all the leads to bus-bars should be of the same length, and it is sometimes necessary to loop some of them as shown at *d*, Fig. 39.

It is not practicable to run a compound dynamo and a shunt dynamo in parallel; for, unless the field rheostat of the shunt machine is adjusted continually, the compound dynamo will take more than its share of the load.

## MAIN CABLES FOR DYNAMOS

The *main cables* running from a dynamo should be carefully selected, so as to carry the current without overheating. The cables may have paper or rubber insulation provided with an outer lead sheath, or they may have rubber covering with an outer braiding.

In the table on page 206, the temperatures are based on continuous operation, with lead-armored cable. For 2 hours' overload, the temperature rise will be 90% of the values quoted for continuous operation at overload. For paper insulated cables, the temperatures will be increased 10%. If braided cables are used, the temperatures will be reduced 10%.

# RUBBER-INSULATED CABLES WITH STANDARD LEAD COVERING

*(Recommended by the General Electric Company)*

| Amperes | Number of Cables | Size of Main Cables | Kilowatts 575 Volts | Kilowatts 275 Volts | Degrees C. Rise Continuous Operation at | | Equalizer Cables Recommended |
|---|---|---|---|---|---|---|---|
| | | | | | Full Load | 50% Overload | |
| 10–24 | 1 | No. 8 B. & S. solid | | | 10 | 23 | Same as main cable |
| 25–36 | 1 | No. 6 B. & S. solid | | | 9 | 20 | Same as main cable |
| 37–56 | 1 | No. 4 B. & S. solid | | | 10 | 23 | Same as main cable |
| 57–84 | 1 | No. 2 B. & S. stranded | | | 10 | 23 | Same as main cable |
| 85–124 | 1 | No. 0 B. & S. stranded | | | 10 | 23 | Same as main cable |
| 125–169 | 1 | No. 000 B. & S. stranded | | | 12 | 27 | Same as main cable |
| 170–224 | 1 | 250,000 cir. mils | 100 | | 12 | 27 | Same as main cable |
| 225–264 | 1 | 300,000 cir. mils | 150 | | 14 | 31 | Same as main cable |
| 265–324 | 1 | 400,000 cir. mils | | | 13 | 29 | Same as main cable |
| 325–374 | 1 | 500,000 cir. mils | 200 | | 12 | 27 | Same as main cable |
| 375–449 | 1 | 600,000 cir. mils | | | 12 | 27 | Same as main cable |
| 450–549 | 1 | 800,000 cir. mils | 300 | | 12 | 26 | Same as main cable |
| 550–699 | 1 | 1,000,000 cir. mils | 400 | | 13 | 29 | Same as main cable |
| 700–899 | 1 | 1,500,000 cir. mils | 500 | | 11 | 25 | Same as main cable |
| 900–1,099 | 1 | 2,000,000 cir. mils | | | 12 | 27 | Same as main cable |
| 1,100–1,299 | 2 | 1,000,000 cir. mils | 650–800 | 300 | 13 | 29 | Same as main cable |
| 1,300–1,749 | 2 | 1,500,000 cir. mils | 1,000 | 400 | 12 | 27 | 1–1,500,000 cir. mils |
| 1,750–2,249 | 2 | 2,000,000 cir. mils | 1,200 | 550 | 12 | 27 | 1–2,000,000 cir. mils |
| 2,250–2,649 | 3 | 1,500,000 cir. mils | 1,600 | | 14 | 31 | 2–1,500,000 cir. mils |
| 2,650–3,349 | 3 | 2,000,000 cir. mils | | 800 | 12 | 27 | 2–2,000,000 cir. mils |
| 3,350–3,599 | 4 | 1,500,000 cir. mils | 2,000 | | 15 | 33 | 3–1,500,000 cir. mils |
| 3,600–4,499 | 4 | 2,000,000 cir. mils | 2,400 | 1,000 | 12 | 27 | 2–2,000,000 cir. mils |
| 4,500–5,599 | 5 | 2,000,000 cir. mils | | 1,600 | 12 | 27 | 3–2,000,000 cir. mils |
| 5,600–6,600 | 6 | 2,000,000 cir. mils | 2,800–3,200 | | 12 | 27 | Same as main line |

## DIRECT-CURRENT MOTORS

**Starting Devices.**—*Direct-current motors* are nearly always operated on constant-potential circuits, and all except very small motors have low armature resistance. When starting, it is necessary to limit the flow of current through the armature until the motor generates enough counter E. M. F. to regulate the current automatically.

*Motor starting boxes*, or *rheostats*, consist of resistances capable of carrying the starting current for the few seconds required by the motor to come up to speed. The resistance is arranged in steps, so that it can be cut out of circuit step by step as the counter E. M. F. increases.

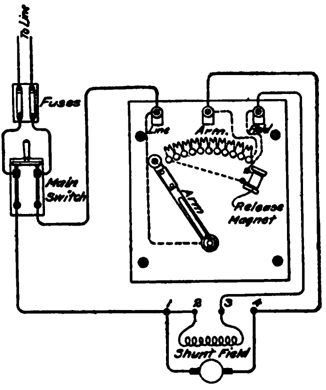



FIG. 40. STARTING RHEOSTAT FOR SHUNT MOTOR

Fig. 40 shows connections of a shunt motor and starting rheostat; the resistance steps are connected to a series of contact buttons or segments, over which the arm can be moved. The arm is normally held in the *off-position*, as shown, by a strong spring, so that closing the main switch does not close a circuit through the armature. When the arm is moved on to the first contact, current can flow through the shunt field, the release magnet, and the arm, to the other side of the circuit; also through the motor armature, all the resistance, and the arm. The shunt-field current therefore attains its full strength at once, but the armature current is limited by the resistance in the rheostat. The

armature should begin to rotate at once unless it has to start a considerable load, in which case it may not start until the arm is moved to the second or third contact; the arm is then moved on from contact to contact, stopping a few seconds on each, but moving quickly between them, until the last one—the full *on-position*—is reached, where the resistance is all cut out, and the motor attains full speed. Ordinarily, 15 sec. is sufficient time to start a motor and bring it up to full speed.

When the arm is on the last contact, it is held by the attraction of the release magnet; if, for any reason, the power should go off the line, the magnet releases the arm, which is then carried back to the off-position by the spring. The motor, in the absence of power, will soon stop, but when power returns to the line the motor is protected from the rush of current that would occur through its armature if the arm had remained at the on-position. The name, *automatic no-voltage release starting rheostat* applies to such a starter. To stop the motor, all that is necessary is to pull the main switch, and the starter will open automatically as soon as the armature slows down a little.

The front of the rheostat has three binding posts marked *Line*, *Arm.*; and *Field*, respectively, and care should be taken that the proper wires are connected to each. A mistake sometimes made is to interchange the line and the armature wires, in which case the motor will not start until the arm has passed over several contacts and may then start very suddenly with considerable sparking and flashing at the commutator.

Starting rheostats are also made with an overload-release device, so that the circuit through the motor is opened automatically when the motor current becomes too large for safety.

Rheostats for series and compound motors are the same in principle as those for shunt motors. When no shunt field is used, the no-voltage release coil is connected directly across the circuit, with or without a resistance in series—a method adopted by some manufacturers for all their rheostats.

**Direction of Rotation.**—To reverse the direction of any direct-current motor, the direction of the current in either the armature or the field must be reversed. If the directions in both are reversed, the direction of rotation will remain unchanged. The general practice is to reverse the current in the armature, rather than in the fields. Shunt motors are now usually provided with four terminals, so that the connections can be readily made for either direction of rotation. A double-pole, double-throw switch with opposite corner contacts connected (Fig. 41) affords a ready means for reversing the direction of the current through an armature.

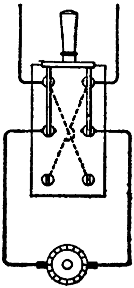


FIG. 41
REVERSING SWITCH

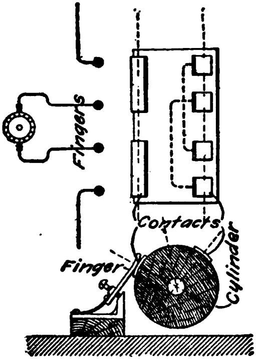



FIG. 42
CYLINDRICAL REVERSER

The arrangement of contacts on a cylinder that can be rotated so as to bring different contact pieces under stationary fingers (Fig. 42) is a convenient way of reversing a motor. Cylindrical reversers are used in street-car controllers.

## SPEED REGULATION OF MOTORS

There are two general methods for varying the speed of motors: (1) By varying the E. M. F. applied to the armature windings, and (2) by varying the field strength. Both

methods admit of many modifications and in some cases the two are combined.

**Speed Variation by Changing Applied E. M. F.**—The E. M. F. applied to a motor armature may be changed (1) by adjusting a resistance in series with the armature; (2) by series-parallel control; and (3) by multivoltage control.

The first method is much used because of its simplicity and low first cost, but it is very wasteful and gives poor speed regulation with varying load at reduced speed. An automatic starting and speed-regulating rheostat in much use has a notched segment on the end of the arm and a catch, or pawl, controlled by the release magnet. The connections are shown in Fig. 43. While the magnet is excited, the pawl is held firmly in a notch in the segment so that the arm will remain in any position; but, if the current goes off the line, the arm is released and is then carried back to the off-position by a spring.

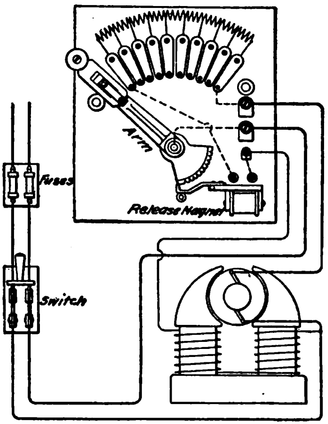



FIG. 43
SPEED-REGULATING RHEOSTAT

**Armature Speed Regulating Resistance.**—The resistance used for speed control must be of larger carrying capacity than is needed merely for starting purposes, and must be selected for the current actually required by the motor and not always according to the full capacity of the motor. For example, if a 5-H. P. motor is in use where only 3 H. P. is

required, a 3-H. P. regulator should be used. To reduce the speed of a 220-volt motor 50% requires a drop of 110 volts in the regulating resistance. A 5-H. P., 220-volt motor requires about 20 amperes at full load. If this current is also required at half speed, the regulating resistance must be $110 \div 20 = 5.5$ ohms; but if only 12 amperes is required at half speed, the resistance must be $110 \div 12 = 9.17$ ohms.

The speeds of fans and blowers are frequently regulated by means of armature-regulating resistances. Since the power required to drive a fan varies about as the third power of the speed, the power required at half speed is about one-eighth that at full speed; hence, if a fan requires 5 H. P.,

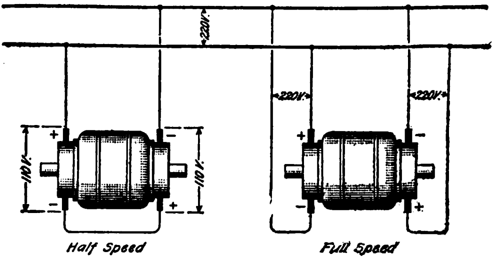



FIG. 44

or 3,730 watts, at full speed, it will require about 466 watts at half speed. If the motor efficiency is 85%, the input at full speed must be 4,400 watts, or 20 amperes at 220 volts. At half speed, the input will be about 5.25 amperes at 110 volts. The regulating resistance must be sufficient to give a drop of 110 volts at 5.25 amperes, or $110 \div 5.25 = 21$ ohms. Speed-regulating resistances for fan and blower motors, or for centrifugal-pump motors, must therefore be considerably higher than for most other classes of work, with correspondingly less carrying capacity.

**Series-Parallel Method.**—The *series-parallel method* of speed control may be applied to a single armature with two

separate windings, each connected to a commutator, as suggested by Fig. 44. When the two windings are connected in series, the armature will run at half speed, but when in parallel, at full speed. Intermediate speeds may be obtained by field regulation. This method of control is also used with two motors, as on street-railway cars. The two motors are connected in series for low speeds and in parallel for high speeds. The changes from series to parallel connection is effected by means of a controller.

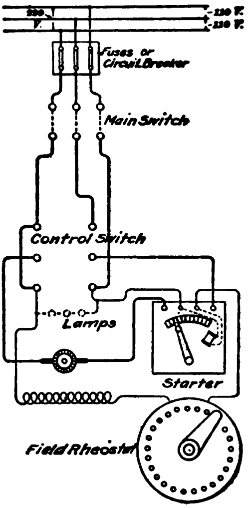



FIG. 45

**Multivoltage Systems.**—A considerable variation of speed can be obtained by operating a motor on a three-wire system, as shown in Fig. 45. The armature is connected to one side of the system to obtain half voltage for low speeds, and across the outside wires to obtain full voltage for high speeds. Intermediate speeds may be had by adjusting the field strength by means of a field rheostat. The changes in the connections can be made by means of a double-throw control switch, the shunt field remaining connected across the outside wires. A few lamps connected across the shunt-field terminals will serve to take the field discharge when the main switch is opened.

A *motor generator* consisting of a 110-volt motor and a double generator wound to give 60 volts at one end and 80 at the other may be run from a 110-volt circuit so that the system can be made to give 60, 80, 110, 140, 190, or 250 volts.

A similar set may be operated as a **balancer** across a 220-volt circuit (see Fig. 46), so as to obtain 53, 70, 97, 123, 167, or 220 volts. As a general rule, the balancer set should have a capacity of about 25% of the rated capacity of the variable-speed motors to be operated.

In the Bullock multivoltage system, three dynamos generating 60, 80, and 110 volts, respectively, are mounted on one base and their shafts coupled together. The set is used across 250-volt mains and is provided with a starting rheostat. Each armature has a field magnet and each field circuit a rheostat, so that the respective voltages can be adjusted. The variable-speed motors are provided with controllers, each of which has two cylinders—a main cylinder for changing the armature connections and a second cylinder for inserting resistance in the field circuit and securing intermediate speeds. The cylinders work together, so that while the operating cylinder is moved from position *1* to *2*, the field-resistance drum cuts in resistance in the field circuit and thus gradually increases the speed, so that when the main drum reaches position *2*, where the voltage is stepped up and the field current simultaneously returned to full value, there is no sudden jump in the speed. Standard 250-volt motors are used, and their shunt fields are excited from the 250-volt mains.

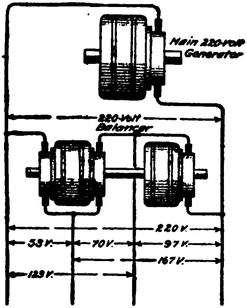



FIG. 46

**Speed Variation by Changing Field Strength.**—If $E$ is the voltage applied to the terminals of a shunt motor, $E'$ the counter E. M. F. generated in the motor, and $R$ the resistance of the motor armature and brushes, then $\frac{E-E'}{R}$ is the current, and the counter E. M. F. is automatically adjusted to keep the current just sufficient to produce the necessary

torque. If the motor is running free, that is, with no load, $E-E'$ is very small and the current is small. When the load on the motor is greatest, $E'$ is least and $E-E'$ is a maximum, thus causing maximum current to flow.

In any case, the armature will rotate fast enough to generate the counter E. M. F. $E'$, and the weaker the field the higher must be the speed; hence, the speed of a direct-

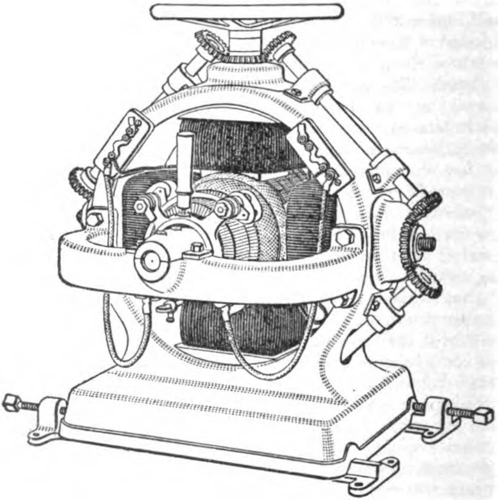


Fig. 47. Stow Variable-Speed Motor

current motor may be increased by weakening its field or decreased by strengthening its field. But the maximum current that a direct-current motor armature will take without sparking is less with a weak field than with a strong one; hence, the maximum safe output of a given motor is usually decreased by weakening the field. Ordinary standard motors

however, will usually operate at full rating on weakened field at from 25 to 50%, and sometimes 100%, increased speed without injurious sparking. Motors wound especially for variable-speed work may be made to produce full output at speed variations as high as 4 or 5 to 1. For example, a motor normally rated at 10 H. P. at 1,100 R. P. M. may be built with a specially strong field and a special armature winding, so that it will develop 3 H. P. at 300 R. P. M. with full field, and by weakening the field, the speed may be increased to 1,200 R. P. M. or possibly more without injurious sparking, the output remaining constant at all speeds.

**Stow Variable-Speed Motors.**—Sparking on a weakened field is caused by field distortion due to armature reaction. It is possible to maintain the full number of ampere-turns on the field and decrease the field magnetism by increasing the reluctance of the magnetic circuit, thus causing increased speed. The *Stow variable-speed motors*, Fig. 47, have field poles (Fig. 48), each consisting of an outer shell in which is a movable core. The distance of the cores from the armature can be adjusted by means of screws operated by a handwheel on the frame, thus varying the reluctance of the magnetic circuit. When the pole cores are drawn away from the armature, the reduced flux concentrates at the pole tips, and, owing to the high density, field distortion is largely prevented. These motors give wide speed variation with sparkless commutation and without the use of field rheostats, but they are somewhat complicated and expensive.

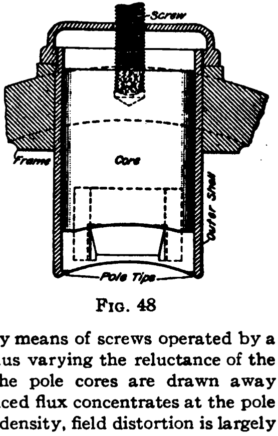


FIG. 48

**Interpole motors** are so-called because they are equipped with commutating poles between the main poles. The commutating poles are wound with a few turns of copper

strip or wire large enough to carry the armature current, and all are connected in series in the armature circuit. These poles cause a field in the neutral region always proportional to the armature reaction and opposed to it, so that commutation can occur with little or no sparking even with a very weak motor field.

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## ELECTRIC MOTOR DRIVING

The motor selected for a given service should be large enough to avoid overloading, but there should not be very much surplus capacity, for this would increase the first cost and cause the motors to work at an inefficient load. The power required to run a given machine tool depends on such variable conditions that it is impossible to give formulas that will prove correct in all cases. The following general formula, however, can be used:

$$\text{H. P.} = CW$$

in which H. P. is the horsepower, $C$ the H. P. required per pound of metal, and $W$ the number of pounds of metal removed per minute.

In general, the power $C$ per pound of metal cut away per minute may be taken as follows: Hard steel 3, wrought iron 2.5, soft steel 2, cast iron 1.5. To allow for friction of the machine itself and for unusually hard metal, it is well to add 25% to these values, making them 3.75, 3.13, 2.5, and 1.9, respectively. The value of $W$ is computed by multiplying together the width and depth (that is, the cross-section) of the cut and the speed of cutting in feet per minute.

For machines that demand a fluctuating supply of driving power, such as planers, shapers, slotters, and punch presses, compound-wound motors should be used; and the addition of heavy flywheels on the motor shafts will be an improvement.

### MACHINE-TOOL MOTORS

Calculations for the following list of motors for large railway repair shops were made according to the preceding formula and have proved very nearly correct in almost every instance.

| Tool | Builder | Horsepower of Motor | Field Winding of Motor |
|---|---|---|---|
| 18″×8′ lathe | Flather & Co. | 5 | Shunt |
| 18″×10′ lathe | Putnam Machine Co. | 7½ | Shunt |
| 20″×8′ lathe | F. E. Reed Co. | 5 | Shunt |
| 20″×11′ lathe | Putnam Machine Co. | 7½ | Shunt |
| 24″×12′ lathe | American Tool Works Co | 7½ | Shunt |
| 25″×6′ lathe | Putnam Machine Co. | 7½ | Shunt |
| 30″×15′ lathe | Putnam Machine Co. | 10 | Shunt |
| 36″×24′ lathe | Putnam Machine Co. | 10 | Shunt |
| 42″×8′ lathe | Niles Tool Works Co. | 15 | Shunt |
| Axle lathe | Pond Machine Tool Co. | 25 | Shunt |
| Axle lathe | Putnam Machine Tool Co | 35 | Shunt |
| 42-in. wheel lathe | Pond Machine Tool Co. | 20 | Shunt |
| 72-in. wheel lathe | Niles Tool Works Co. | 25 | Shunt |
| 90-in. wheel lathe | Putnam Machine Co. | 25 | Shunt |
| 34-in. turret lathe | Gisholt Machine Co. | 15 | Shunt |
| 30-in. drill press | J. E. Snyder | 5 | Shunt |
| 60-in. radial drill | Dreses Machine Tool Co. | 6½ | Shunt |
| 72-in. radial drill | Niles Tool Works Co. | 4 | Shunt |
| Heavy 2-spindle drill | Bement Miles & Co. (two) | 9 | Shunt |
| 42-in. car-wheel borer | Pond Machine Tool Co. | 10 | Shunt |
| 51-in. boring mill | Baush Machine Tool Co. | 15 | Shunt |
| 72-in. boring mill | Pond Machine Tool Co. | 25 | Shunt |
| 18″×36″ horizontal boring machine | Betts Machine Co | 15 | Shunt |
| Slab miller | Wm. Sellers & Co | 6½ | Compound |
| 3-in. pipe cutter | D. Saunders Sons | 3 | Shunt |
| 6-in. pipe cutter | D. Saunders Sons | 7½ | Shunt |

| Tool | Builder | Horsepower of Motor | Field Winding of Motor |
|---|---|---|---|
| 18-in. emery-wheel grinder | Bridgeport Emery Wheel Co. | 4 | Shunt |
| Bending rolls, No. 2 | Hilles & Jones Co | $7\frac{1}{2}$ | Compound |
| Bending rolls, No. 4 | Hilles & Jones Co | 25 | Compound |
| Straightening rolls, No. 2 | Hilles & Jones Co | 15 | Compound |
| Angle shear, No. 1 | Hilles & Jones Co | 10* | Compound |
| 84-in. quartering machine | Niles Tool Works Co. (two) | 5 | Shunt |
| 12-in. slotter | Betts Machine Co. | $6\frac{1}{4}$ | Compound |
| 19-in. slotter | Putnam Machine Co. | 13 | Compound |
| 12-in. shaper | Hughes & Phillips | 5 | Compound |
| 24-in. shaper | Gould & Eberhardt | $7\frac{1}{2}$ | Compound |
| 30″×30″×8′ planer | New Haven Manufacturing Co. | $7\frac{1}{2}$ | Compound |
| 42″×42″×12′ planer | Pond Machine Tool Co. | 15 | Compound |
| 60″×60″×20′ planer | Pond Machine Tool Co. | 20 | Compound |
| Punch, $\frac{11}{16}$-in. hole in $\frac{3}{4}$-in. plate | Cleveland Punch & Shear Works Co. | 5* | Compound |
| Punch, No. 2 | Hilles & Jones Co | 10 | Compound |
| Horizontal punch | Hilles & Jones Co | 10* | Compound |
| Punch and shear, No. 3 | Hilles & Jones Co | 10* | Compound |
| Punch, No. 5 | Hilles & Jones Co | 15* | Compound |
| Shear, No. 6 | Hilles & Jones Co | 15* | Compound |
| Punch and shears, No. 4 | Hilles & Jones Co | 15 | Compound |
| 100-in. hydraulic wheel press | Putnam Machine Co. | $7\frac{1}{2}$* | Compound |
| 200-ton hydraulic wheel press | Niles Tool Works Co. | $7\frac{1}{2}$* | Compound |

*Variable-speed motors.

## ECONOMY OF INDIVIDUAL MOTOR DRIVE

The power required to drive the line shafting and belts in a factory where the motive power is all in one source is about equal to the power required to drive all the machines in the shop at their maximum output; that is, if all the tools working simultaneously at maximum output require 100 H. P., the belts and shafting will require about 100 H. P. more, and this power, which is all lost in friction, must be supplied all the time, even though only a few of the machines are working. On the other hand, if, in the same shop, each tool is equipped with a motor, only about 43 H. P. will be lost in transmission from the motive power of the dynamo to the tools when they are all working at maximum output; that is, 143 H. P. is consumed and 100 H. P., or 70% of this is supplied to the tools, against 50% with belts and shafting. But, in machine-shop work, it is found that so many machines are always idle or working on light load that only about 30% of the total capacity of all the machines is in use at the same time; that is, the *load factor* is approximately 30%. With line shafting and belts, the total power supplied to the shop is then 130 H. P., of which 30 H. P., or 23%, is used by the tools; with individual motor drive, the losses decrease approximately as the load. At 30% load, the loss would be 30% of 43 H. P. = 13 H. P., and the total power is 43 H. P., of which 30 H. P., or 70%, is used. In other words, the individual motor drive, under ordinary working conditions, is more than three times as efficient as the line shaft and belt drive.

However, it would not usually be economical to install a small motor on each of a number of small machines; better results are often obtained by driving a number of small machines in a group from a comparatively short line shaft driven by a larger motor.

# DISEASES OF DIRECT-CURRENT DYNAMOS AND MOTORS

## SPARKING AT THE COMMUTATOR

A moderate amount of sparking at the commutator is not objectionable, but, if it becomes sufficient in amount or in duration to blacken or roughen the commutator bars, the cause should be located and removed if possible. Numerous small, white sparks, evenly distributed along the edge of the brush and producing no distinguishable noise, usually work little injury. Larger sparks, appearing at irregular intervals along the edge of the brush, usually with a greenish hue and accompanied by a hissing sound, are more serious. Such sparks usually cling tenaciously to one point on the brush edge; they are due to small particles of copper that are torn loose from the commutator by excessive local heat. On stopping the machine after running a few hours with this kind of sparking, a furrow, or strip, will be found cut into the commutator all around the circumference under the spot where the spark appeared, and a piece of copper will be found adhering to the surface of the brush. Sparks due to inductive voltage of commutation are generally accompanied by a vicious snapping sound, easily distinguished after having once been heard. A well-designed, modern, direct-current dynamo or motor, with the brushes in one position, should be sparkless from no load to full load and possibly to 25% overload. There should be no injurious sparking at 50% overload, and many manufacturers guarantee their machines to stand even 100% overload, momentarily, without injury.

The cause of sparking at the commutator may be any one or more of the following: (1) Overloads; (2) brushes defective in setting, in material, or in dimensions; (3) commutator in poor condition; (4) defective armature; (5) poor foundation; and (6) weak magnetic field.

An *overload* is usually easily detected by means of the switchboard instruments or by other surrounding conditions, and the remedy, if there is any, is also usually apparent.

Shifting dynamo brushes forwards or motor brushes backwards will generally give some relief. The overload should, of course, be removed as soon as possible.

The *brushes* may have the wrong lead, which can be corrected by moving them slowly backwards and forwards until the sparking is reduced to a minimum. They may not be properly spaced; the least number of commutator bars between corresponding points (centers or edges) of adjacent sets of brushes should be $N \div p$, where $N$ is the number of bars in the commutator and $p$ the number of poles. The material of the brushes may be too hard or too soft, as will usually be indicated by the surface of the commutator and the contact surface of the brushes. If the commutation is good, the commutator surface will assume a dark, glossy, chocolate color; the surface of the brushes will also be smooth and glossy, and the brushes will usually emit a characteristic squeaky noise when the commutator is turned slowly. If the commutator surface appears raw and copper-colored or has small furrows cut in it, and if the brush surfaces are roughened or covered with bits of copper, relief can often be obtained by removing the brushes, boiling them in vaseline, wiping dry, and resetting. In setting brushes, draw a strip of fine sandpaper a few times between the brushes and the commutator, with the rough side of the paper next to the brushes. If brushes are too thick, they will short-circuit too many coils, and heating and sparking will result; if too thin, the inductive voltage of commutation will be too high and sparking will result.

The *commutator* surface should be kept clean, smooth, and glossy; very little oil or grease should be applied to it. A rough commutator can sometimes be ground down with sandpaper (not emery paper) held on by means of a stock curve, after which the commutator and all surroundings should be thoroughly cleaned. A piece of sandstone cut with the same curvature as the commutator is sometimes used successfully to grind down a commutator surface. An eccentric commutator, or one that has high or low bars, must be trued in a lathe or with a special turning tool. Loose bars should be driven into place and the commutator tightened.

The *armature* may have short circuits, open circuits, grounds, etc. as described elsewhere. The shaft may be sprung, giving the same effect as an eccentric commutator.

A *poor foundation* will permit vibration and may cause sparking.

A *weak field* will permit field distortion and consequent shifting of the neutral region with each change of load.

## HEATING

**The Armature.**—*Overheated windings* are generally caused by overloads, but there may be other causes. The armature may have one or more short-circuited or grounded coils, which will cause local development of heat. Starting the machine when cool and stopping again after a run of a few moments may heat the locality of a short-circuited coil so that it can be felt. Two grounds may act the same as a short circuit. Excessive eddy currents in the armature core will make the core heat faster than the conductors, or the core will heat when the machine is running without load. In such a case, the only remedy is to rebuild the core of good sheet iron, with at least every alternate sheet dipped in japan and baked. The heat from hot bearings may sometimes affect the armature.

**The Field.**—Shunt-field coils will overheat if the voltage at the terminals is too high; that is, if the coils are overloaded. An overload on a shunt field may also occur if one or more coils are short-circuited. Eddy currents in the pole pieces may also cause overheating.

**The Bearings.**—Overheated bearings may be caused by lack of good, clean oil. The supply of oil to the bearings should be of good quality, abundant in quantity, though not sufficient to overflow and get into the commutator or windings, and perfectly clean and free from dirt. The bearings should also be cleaned out occasionally, especially if they heat. The heating may be due to rough journals or bearings, to the bearings being too tight, to the shaft being sprung, the bearings out of line, belt too tight, armature out of center, so that the magnetic pull one way is excessive, or anything else that may cause excessive friction.

## MISCELLANEOUS DISEASES

**Noise** cannot be entirely eliminated from rotating machinery, but well-designed and well-mounted dynamos and motors should run without excessive noise. Loose or vibrating parts, as an unbalanced pulley, the armature striking against the pole pieces, or the shaft collars against the bearings, etc., will cause unnecessary noise, usually enough to indicate the cause. *Humming* is usually caused by vibrations of the laminations of the pole pieces or the armature core.

**High** or **low speed** in a dynamo is the fault of the motive power; high speed is likely to cause sparking, and low speed will cause either low voltage or overheated shunt fields if the resistance is cut out to keep up the voltage. The speed of a motor is affected both by the voltage and by the field strength.

**Failure to Generate.**—A dynamo will *fail to generate* (1) if the residual magnetism has been lost; (2) if the field excitation and the residual magnetism are not in the same direction; (3) if the shunt-field circuit is open or has too much resistance; (4) if the resistance of the external circuit of the dynamo is too low, that is, the load on the dynamo too great.

**Stopping and Failure to Start.**—A motor will *stop* or *fail to start* if there is too little current in its armature conductors, or if its field magnetism is too weak, or if the load is too great. The power may be off the line, a fuse may be burned out, or a circuit-breaker open. The brushes may be short-circuited, so that current does not go through the armature conductors. The field circuit may be short-circuited or open, so that there is little or no field excitation, etc.

# ELECTRIC BATTERIES

## PRIMARY BATTERIES

A *primary battery* is a group of one or more *voltaic*, or *galvanic*, *cells*. Each cell consists of two dissimilar *elements*, called the *voltaic couple*, immersed in a saline or acidulated solution called the *electrolyte*. The elements are such that one, called the *positive element*, or *anode*, is acted on by the electrolyte more readily than the *negative element*, or *cathode*. The anode is the element at which the current enters the electrolyte, and the cathode is the element at which the current leaves the electrolyte. In the chemical action, the anode is consumed by oxygen from the electrolyte, and hydrogen gas liberated from the electrolyte gathers on the surface of the cathode. This gas is a non-conductor; a layer of it on the surface of the cathode prevents the passage of current, and the cell is said to be *polarized*. A *depolarizer* is a material that will combine readily with the liberated hydrogen; this is placed near the cathode so that the gas may be removed therefrom. Depolarizers may be solid or liquid. When solid, the material is placed around the cathode in a porous cup, through which the electrolyte can enter; when liquid, the material may be contained in a porous cup around the cathode, or it may be of different specific gravity than the electrolyte, so that one liquid will remain above the other, or it may be mixed with the electrolyte.

### PRIMARY BATTERY CELLS

The accompanying table gives the names of some of the most common battery cells, with the principal materials entering into each. The chemical symbol $NH_4Cl$ means ammonium chloride (sal ammoniac), and $H_2SO_4$ is the symbol for sulphuric acid. In the column headed Remarks is given the kind of work for which each cell is most suitable. Reference to bichromate-acid solution means potassium or sodium bichromate or chromic acid dissolved in water with the addition of a little sulphuric acid. Zinc anodes are used

# COMMON BATTERY CELLS

| Name of Cell | Electrolyte | Cathode | Depolarizer | E. M. F. Volts | Remarks |
|---|---|---|---|---|---|
| Law | Solution of $NH_4Cl$ | Carbon | None | 1.3 to 4 | For open-circuit work |
| Hercules | Solution of $NH_4Cl$ | Carbon | None | 1.3 to 4 | For open-circuit work |
| Bunsen | Dilute solution of $H_2SO_4$ | Carbon | Dilute nitric acid | 1.9 | For open or closed circuits |
| Bichromate, Grenet, Poggendorfs | Bichromate-acid solution | Carbon | The bichromate dissolved in the electrolyte | 1.9 to 2.1 | For open or closed circuits. Elements should be removed from electrolyte when not in use |
| Fuller | Bichromate-acid solution | Carbon | Bichromate | 2.14 | For open or closed circuits |
| Partz | Solution of common salt or magnesium sulphate | Carbon | Bichromate | 1.9 to 2 | For open or closed circuits. Electrolyte floats on top of depolarizer |
| Daniel (Crowfoot) | Zinc sulphate | Copper | Copper sulphate | 1.07 | For closed circuits. Electrolyte floats on top of depolarizer |
| Leclanche | Solution of $NH_4Cl$ | Carbon | Manganese dioxide | 1.4 to 1.7 | For open circuits |
| Edison-Lalande | Caustic potash | Cupric dioxide | Cupric oxide | .7 | For closed circuits and large current |
| Gordon | Caustic soda | Iron | Copper oxide | .7 | For open or closed circuits and large currents |
| Harrison | Dilute sulphuric acid or bisulphate of sodium or potassium. Must be pure | Lead | Lead peroxide | 2.5 | For closed circuits. Liable to local action |
| Latimer-Clark | Zinc sulphate | Mercury | Paste of mercurous sulphate and zinc sulphate | 1.4333 at 15° C. | Standard cell |

for all the cells given in the table. The elements assume various forms according to the type of cell.

**Applications.**—*Open-circuit primary batteries* are used where electric current is required in small quantities and very intermittently, as for ringing bells, lighting gas, operating telephone transmitters, etc. *Closed-circuit batteries* are used where a small but steady current is required for long periods of time, as in telegraphing, in operating fire-alarms, railway signals, etc. For open-circuit work, the Leclanché cell is much used. The electrolyte should have all the sal ammoniac that will readily dissolve and no more. For closed-circuit work, crowfoot, Gordon, Fuller, and Edison-Lalande cells are all popular.

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## STORAGE BATTERIES

A *storage battery, secondary battery*, or *accumulator*, as it is variously called, is an electrical device in which chemical action is first caused by the passage of electric current, after which the device is capable of giving off electric current by means of secondary reversed chemical action. Any voltaic couple that is reversible in its action is a storage battery. The process of storing electric energy by the passage of current from an external source, is called *charging* the battery; when the battery is giving off current, it is said to be *discharging*. A storage-battery cell has two elements, or plates, and an electrolyte. The two plates are usually made of the same material, though they may be of two different materials.

**Lead Accumulators.**—In all storage batteries of large capacity, lead is used for both plates, hence the name *lead accumulators*. There are two processes of preparing the plates, the *Planté process* and the *Faure process*. In the Planté process, pure lead plates are *formed* by electrolytic action caused by repeated charging and discharging until the surface of the plates for a considerable depth becomes porous, or spongy. During the process of charging, lead oxide forms on the positive plate and hydrogen collects on the negative plate; during discharge, the lead oxide is

reduced to spongy lead, and more lead oxide is formed on the negative plate. In the Faure process, the active substance, an oxide of lead, is pasted on the plates before they are put into the electrolyte. In both processes, it is an object to expose as much lead surface as possible to the action of the electrolyte; for this reason, the plates, or grids, are usually filled with perforations or grooves. All the positive plates of a cell are interconnected at one side of the cell, and all the negative plates at the other.

## INSTALLATION AND CARE OF STORAGE CELLS

**Location.**—Storage cells should be located in a well-ventilated room of moderate temperature, say from 50° to 75° F. The floor should be of cement, with drainage facilities, and the room should be light enough to allow easy inspection of the cells.

**Method of Supporting Cells.**—The cells are usually mounted on racks made of heavy wooden framework securely braced. If arranged in a single tier, which is the better way when there is space enough, all the framework that is necessary is a set of stringers properly fastened together. If the containing

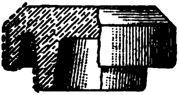


Fig. 1
Insulator

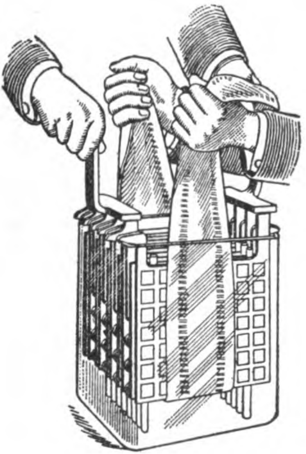


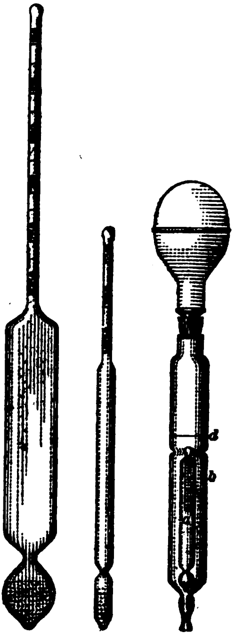
Fig. 2
Placing Plates in Glass Jars

vessels are glass, they should be set in shallow wooden trays filled with sand, in order to distribute the stresses; if lead-lined, wooden tanks are used, trays are not necessary. Each tray or tank is supported on glass petticoat insulators, Fig. 1. Fig. 2 shows a method of placing plates in glass jars.

**Mixing the Electrolyte.**—When the cells are fully charged, the specific gravity of the electrolyte, as indicated by the hydrometer, should be 1.2 to 1.24 at 60° F. The final density on discharge should not fall below 1.15; from 1.185 to 1.195 is the usual practice. The electrolyte can be obtained ready-mixed, but it is cheaper to buy concentrated acid and dilute it. None but sulphur or brimstone acid should be used. Acid made from pyrites is liable to contain impurities. When diluting, the acid must be poured into the water slowly and with great caution. Never pour the water into the acid. The specific gravity of commercial sulphuric acid is 1.835, and 1 part of such acid should be mixed with 5 parts (by volume) of pure water. Care should be taken that no impurities enter the mixture. The vessel used for the mixing must be a lead-lined tank or one of wood that has never contained any other acid; a wooden washtub or spirits barrel answers very well. The electrolyte when placed in the cell should come ½ in. above the top of the plates. Before putting the electrolyte in the cells, the positive pole of each cell should be connected to the negative pole of the next cell in the series, and the whole battery of cells should be connected, through a main switch, to the charging source—the positive pole of the battery to the positive side of the charging source, and the negative pole to the negative side. After adding the electrolyte the battery should be charged at once or at least inside of 2 hr. A little pure water should be added occasionally to the electrolyte to make up for evaporation, and a small quantity of acid should be added about once a year to make up for that thrown off in the form of spray or that absorbed by the sediment in the cells.

**Hydrometers**, to determine the density of the electrolyte, may be had in different forms. A bulb containing mercury

sinks into the electrolyte to a depth corresponding to the density, and the scale on the glass tube to which the bulb is attached is graduated to read the density. Of the three styles shown in Fig. 3, the largest (*a*) can be read with the greatest accuracy, and is preferred where it can be used. The syringe style (*c*) has the hydrometer *a* inside a glass tube *b*; by means of the rubber bulb the tube is filled to the mark *d* on the glass with the liquid to be tested, and the reading is taken at the point where the floating tube *a* emerges from the liquid. This style permits readings to be taken where styles (*a*) or (*b*) could not be placed, as in automobile batteries.



(a) (b) (c)

FIG. 3. HYDROMETER

## CHARGING

The normal charging rate is usually the same as the 8-hr. discharge rate specified by manufacturers. The charge should be continued uninterruptedly until complete; but if repeatedly carried beyond the full-charge point, unnecessary waste of energy, a waste of acid through spraying, a rapid accumulation of sediment, and a shortened life of the plates will result. At the end of the first charge, it is advisable to discharge the battery about one-half, and then immediately recharge it. Repeat this operation two or three times, and the battery will then be in condition for

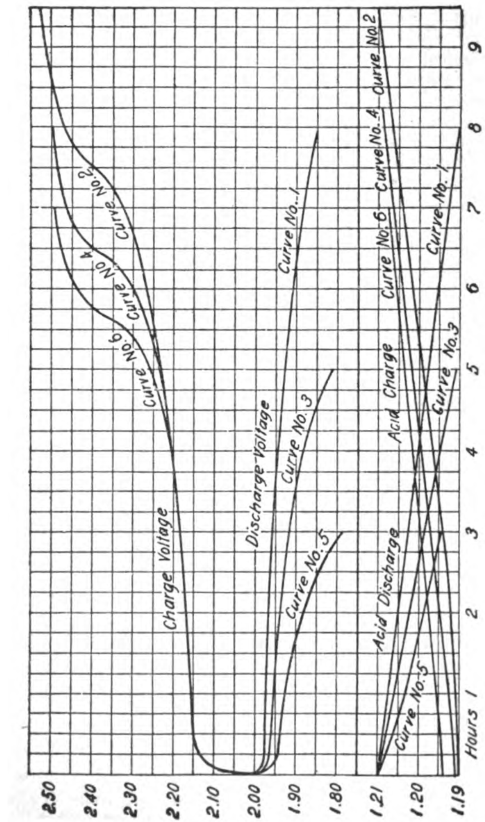



FIG. 4. CURVES OF TYPE W-5 "UNIT-ACCUMULATOR" STORAGE CELL

regular use. It is advisable to overcharge the batteries slightly about once a week, in order that the prolonged gassing may thoroughly stir up the electrolyte, and also to correct inequalities in the voltages of the cells. If the discharge rate is very low, or if the battery is seldom used, it should be given a freshening charge weekly.

**Indications of a Complete Charge.**—A complete charge should be from 12 to 15% greater in ampere-hours than the preceding discharge. The principal indications of a complete charge are: (1) The voltage reaches a maximum value of 2.4 to 2.7 per cell, and the specific gravity of the electrolyte a maximum value of 1.2 to 1.4 per cell. If all the cells are in good condition and the charging current is constant, maximum voltage and specific gravity are reached when there is no further increase for ¼ to ½ hr. (2) The amount of gas given off at the plates increases, and the electrolyte assumes a milky appearance, or is said to *boil*. (3) The positive plates become dark brown in color and the negative plates light gray.

The accompanying curves, Fig. 4, show graphically the action of a typical cell. The following data applies to the curves:

### Rate of Charge and Discharge

*(National Battery Company)*

No. 1. Discharge—8 hr. at 20 amperes, 310 watt-hr.; 84% ampere-hr. efficiency.

No. 2. Charge—9½ hr. at 20 amperes, 428 watt-hr.; 72.4% watt-hr. efficiency.

No. 3. Discharge—5 hr. at 28 amperes, 268 watt-hr.; 85% ampere-hr. efficiency.

No. 4. Charge—8½ hr. at 20 amperes, 373 watt-hr.; 71.8% watt-hr. efficiency.

No. 5. Discharge—3 hr. at 40 amperes, 228 watt-hr.; 85.7% ampere-hr. efficiency.

No. 6. Charge—7 hr. at 20 amperes, 311 watt-hr.; 73.2% watt-hr. efficiency.

ACID DENSITY

No. 1. Start, 1.210; finish, 1.190; temperature at finish 72° F.

No. 2. Start, 1.190; finish, 1.210; temperature at finish, 74° F.

No. 3. Start, 1.210; finish, 1.191; temperature at finish, 80° F.

No. 4. Start, 1.191; finish, 1.209; temperature at finish, 82° F.

No. 5. Start, 1.210; finish, 1.194; temperature at finish, 77° F.

No. 6. Start, 1.194; finish, 1.207; temperature at finish, 83° F.

**Voltage Required.**—The voltage at the end of a charge depends on the age of the plates, the temperature of the electrolyte, and the rate of charging; at normal rate of charge and at normal temperature, the voltage at the end of the charge of a newly installed battery will be 2.5 volts per cell or higher; as the age of the battery increases, the point at which it will be fully charged is gradually lowered and may drop as low as 2.4 volts. The voltage at the end of the charge will be approximately .05 volt less for each 25% decrease in the rate; for example, if the final voltage were 2.5 at the normal rate, say, of 1,000 amperes, it would be 2.45 at 750 amperes, and 2.4 at 500 amperes. The final charging voltage is noticeably lower at high temperatures, irrespective of the age of the plates. All voltage readings are taken with the current flowing; readings taken with the battery on open circuit are of little value and are frequently misleading. After the completion of a charge and when the current is off, the voltage per cell will drop rapidly to 2.05 volts and remain there for some time while the battery is on open circuit. When the discharge is started, there will be a further drop to 2 volts, or slightly less, after which the decrease will be slow. Cells should never be charged at the maximum rate except in cases of emergency, the final voltage per cell will then be about .05 volt higher than if charged at normal rate.

On beginning a charge, each cell requires about 2 volts. If $N$ is the number of cells to be charged, $E$ the supply voltage, and $I$ the charging current, the voltage effective in the circuit through the battery is $E-2N$; and the resistance ot the circuit is

$$R=\frac{E-2N}{I},$$

which is practically equal to the amount of resistance required in the rheostat, because the resistance of the cells is very low. For example, if twenty storage cells are to be charged from a 220-volt circuit with a charging current of 5 amperes, the regulating resistance must be

$$R=\frac{220-2\times 20}{5}=36 \text{ ohms}$$

This resistance should be adjustable, so that some of it can be cut out as the voltage of the cells increases, and it must be made of wire large enough to carry at least 5 amperes without overheating.

**Connections for Charging.**—Storage batteries cannot be charged with alternating current, and when this is the only available supply, it must be converted to direct current. This conversion is made by means of motor-generators, rotary converters, or mercury-vapor converters. Batteries can be charged from direct-current lighting mains by connecting as shown in Fig. 5. An ammeter in the circuit is advisable but not essential. The lamps or the rheostat serve to adjust the current. A 16-c. p., 110-volt, carbon-filament lamp has about 220 ohms resistance and will carry .5 ampere; a similar lamp of 32-c. p. rating has about 110 ohms resistance and will carry 1 ampere. Therefore, the charging current from 110-volt mains can be limited to, say, 5 amperes by connecting five 32-c. p. lamps in parallel, or from 500-volt mains by connecting in parallel five series of lamps, each series containing five 32-c. p. lamps. In both cases, two 16-c. p. lamps in parallel can be used in place of each 32-c. p. lamp.

**Direction of Current.**—The charging current must always flow through the battery from the positive pole to the negative pole. If it is necessary to test the polarity of the line

wires when no instruments are available, attach two wires to the mains, connect some resistance in series to limit the current, and dip the free ends of the wires into a glass of

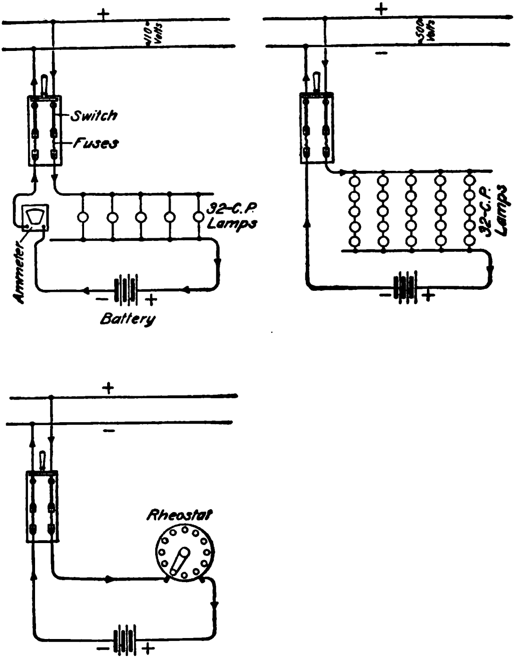



Fig. 5. Charging Storage Batteries From Direct-Current Lighting Circuits

acidulated water, keeping the ends about 1 in. apart. Bubbles are given off most freely from the negative end.

## DISCHARGING

Heavy overdischarge rates maintained for a considerable time, are almost sure to injure the cells. The normal discharge rate should not be exceeded except in case of emergency. The amount of charge remaining available at any time can be determined from voltage and specific-gravity readings. During the greater part of a complete discharge, the drop in voltage is slight and very gradual; but near the end the falling off becomes much more marked. Under no circumstances should a battery ever be discharged below 1.7 volts per cell, and in ordinary service it is advisable to stop the discharge at 1.75 or 1.8 volts. If a reserve is to be kept in the battery for use in case of emergency, the discharge must be stopped at a correspondingly higher voltage. The fall in density of the electrolyte is in direct proportion to the ampere-hours taken out, and is, therefore, a reliable guide as to the amount of discharge.

## MISCELLANEOUS POINTS

**Restoring Weakened Cells.**—There are several methods of restoring cells that have become low: (1) Overcharge the whole battery until the low cells are brought up to the proper point, being careful not to damage other cells in the battery. (2) Cut the low cells out of circuit during one or two discharges and in again during charge. (3) Give the defective cells an individual charge. Before putting a cell that has been defective into service again, care should be taken to see that all the signs of a full charge are present.

**Sediment in Cells.**—During service, small particles drop from the plates and accumulate on the bottom of the cells. This sediment should be carefully watched, especially under the middle plates where it accumulates most rapidly, and should never be allowed to touch the bottom of the plates and thus short-circuit them. If there is any free space at the end of the cells, the sediment can be raked from under the plates and then scooped up with a wooden ladle or other non-metallic device. If this method is impracticable, the electrolyte, after the battery has been fully charged, should be drawn off into clean containing vessels; the cells should

then be thoroughly washed with water until all the sediment is removed, and the electrolyte should be replaced at once before the plates have had a chance to become dry. In addition to the electrolyte withdrawn, new electrolyte must be added to fill the space left by the removal of the sediment; the new electrolyte should be of 1.3 or 1.4 sp. gr. in order to counteract the effect of the water absorbed by the plates while being washed. If at any time any impurities, especially any metal other than lead or any acid other than sulphuric acid, gets into a cell, the electrolyte should be emptied at once and the cells thoroughly washed and filled with pure electrolyte.

**Idle Batteries.**—If a battery is to be idle for, say, 6 mo. or more, it is usually best to withdraw the electrolyte, as follows: After giving a complete charge, siphon or pump the electrolyte into convenient receptacles, preferably carboys that have previously been cleaned and have never been used for any other kind of acid. As each cell is emptied, immediately refill it with water; when all the cells are filled, begin discharging and continue until the voltage falls to or below 1 volt per cell at normal load, and then draw off the water.

**Putting Battery Into Commission.**—To put an idle battery into commission, first make sure that the connections are right for charging; then remove the water, put in the electrolyte, and begin charging at once at the normal rate. From 25 to 30 hr. continuous charging will be required to give a complete charge.

**Sulphating.**—Lead sulphate is practically an insulator. Some of this material is formed in all lead-sulphuric-acid storage cells on discharge and is reconverted to lead oxide or lead peroxide on recharging the cell. If present in excessive quantities, the sulphate adheres to the plates, especially the positive, in white insoluble patches, preventing chemical action, increasing the resistance of the cell, and causing unequal mechanical stresses that may buckle the plates. The most frequent causes of sulphating are overdischarging, too high specific gravity of electrolyte, and allowing the battery to stand for a considerable length of time in a discharged condition. In case white insoluble sulphate appears

on the plates, the battery should be given a long-continued charge at somewhat below the normal 8-hr. rate, until the cells give all the signs of a full charge and the plates have resumed their normal color. In badly sulphated cells, the color of the positive becomes lighter than normal and the negatives considerably darker.

At the end of a complete charge, the gassing sometimes throws off small particles from the plates; these particles lodge on top of the cell as a white powder that may be easily brushed off and that is not injurious unless allowed to accumulate in too great quantities.

## GENERAL DATA ON STORAGE CELLS

*Storage-battery cells* can be made up of almost any number of plates desired. Both outside plates in each cell are negative; hence, the number of plates in a cell is always odd. From the data given in the accompanying tables, the capacity of cells with any number of plates can be calculated. For example, a nine-plate type F cell has four pair of plates and a 40-ampere, 8-hr. capacity, or 10 amperes per pair; hence, a fifteen-plate (seven-pair) cell must have a 70-ampere, 8-hr. capacity, and a twenty-seven-plate (thirteen-pair) cell a 130-ampere, 8-hr. capacity. In making estimates of the room occupied by a given battery, about $1\frac{7}{8}$ in. clearance should be allowed between glass jars, $2\frac{3}{8}$ in. between metal tanks, and 2 in. between wooden tanks.

## STORAGE BATTERY REGULATION

**End-Cell Switches.**—When a battery begins to discharge after receiving a full charge, the E. M. F. is about 2 volts per cell, but this drops slowly to about 1.9 volts per cell, and then more rapidly to the point where discharge should cease. In order to keep the voltage at the battery terminals constant, some of the cells near the end of a battery may be arranged to be cut into circuit, one by one, as the voltage falls, by means of *end-cell switches*. Some end-cell switches are operated by a worm-gear and screw run by a small motor controlled

## GENERAL DATA ON CHLORIDE ACCUMULATORS

| Type of Cell | Size of Plates Inches | Number of Plates | Normal Charge Rate Amperes | 8-Hr. Discharge Rate | Weight of Cell Complete With Acid, Glass Jar Pounds | Weight of Cell Complete With Acid, Metal Tank Pounds | Weight of Cell Complete, Lead-Lined Tank Pounds | Outside Dimensions of Glass Jars Inches | | | Outside Dimensions of Lead-Lined Tanks Inches | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | Width | Length | Height | Width | Length | Height |
| C | 4[illegible] × 4 | 3 | 1¼ | 1¼ | 11 | | | 3½ | 5[illegible] | 7[illegible] | | | |
| C | 4[illegible] × 4 | 5 | 2½ | 2½ | 15.1 | | | 4[illegible] | 5[illegible] | 7[illegible] | | | |
| C | 4[illegible] × 4 | 7 | 3¾ | 3¾ | 19.2 | | | 5[illegible] | 5[illegible] | 7[illegible] | | | |
| D | 6 × 6 | 3 | 2½ | 2½ | 20 | | | 3[illegible] | 7[illegible] | 9½ | | | |
| D | 6 × 6 | 5 | 5 | 5 | 28 | | | 4[illegible] | 7[illegible] | 9½ | | | |
| D | 6 × 6 | 7 | 7½ | 7½ | 38 | | | 6½ | 7[illegible] | 9[illegible] | | | |
| D | 6 × 6 | 13 | 15 | 15 | 63 | | | 11 | 8[illegible] | 9[illegible] | | | |
| E | 7¾ × 7¾ | 5 | 10 | 10 | 49 | 85 | | 5½ | 9[illegible] | 11[illegible] | | | |
| E | 7¾ × 7¾ | 9 | 20 | 20 | 74 | 124 | | 8 | 9[illegible] | 11[illegible] | | | |
| E | 7¾ × 7¾ | 15 | 35 | 35 | 112 | 180 | | 11 | 9[illegible] | 11[illegible] | | | |
| F | 11 × 10½ | 9 | 40 | 40 | 174.5 | 256 | 250 | 9 | 12[illegible] | 17 | 13[illegible] | 15 | 20[illegible] |
| F | 11 × 10½ | 15 | 70 | 70 | 260 | 377 | 372 | 12 | 12[illegible] | 17 | 18[illegible] | 15 | 20[illegible] |
| F | 11 × 10½ | 27 | 130 | 130 | | 618 | 615 | | | | 28[illegible] | 15 | 20[illegible] |
| G | 15 5/16 × 15 5/16 | 11 | 100 | 100 | | | 568 | | | | 15[illegible] | 19[illegible] | 26 |
| G | 15 5/16 × 15 5/16 | 25 | 240 | 240 | | | 1,165 | | | | 27[illegible] | 20[illegible] | 26[illegible] |
| G | 15 5/16 × 15 5/16 | 55 | 540 | 540 | | | 2,475 | | | | 53[illegible] | 21[illegible] | 27[illegible] |
| G | 15 5/16 × 15 5/16 | 75 | 740 | 740 | | | 3,300 | | | | 69[illegible] | 21[illegible] | 27[illegible] |
| H | 30[illegible] × 15 5/16 | 21 | 400 | 400 | | | 1,967 | | | | 25[illegible] | 21[illegible] | 48[illegible] |
| H | 30[illegible] × 15 5/16 | 41 | 800 | 800 | | | 3,538 | | | | 41[illegible] | 21[illegible] | 49[illegible] |
| H | 30[illegible] × 15 5/16 | 75 | 1,480 | 1,480 | | | 6,215 | | | | 69[illegible] | 21[illegible] | 49[illegible] |

Weights of type C cells complete with electrolyte in rubber jars are: C–3, 6½ lb.; C–5, 10 lb.; C–7, 13 lb.

## GENERAL DATA ON GOULD STORAGE CELLS

| Type of Cell | Size of Plates Inches | Number of Plates | Normal Charge Rate Amperes | 8-Hr. Discharge Rate Amperes | Weight of Cell Complete With Acid, Rubber Jar Pounds | Weight of Cell Complete With Acid. Lead-Lined Tank Pounds | Outside Dimensions of Rubber Jar Inches | | | Outside Dimensions of Glass Jar Inches | | | Outside Dimensions of Lead-Lined Tank Inches | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | Width | Length | Height | Width | Length | Height | Width | Length | Height |
| K | 3 × 3 | 3 | .75 | .63 | 4. | | 2¼ | 3¾ | 5½ | 3 | 4 | 6½ | | | |
| K | 3 × 3 | 5 | 1.5 | 1.25 | 7. | | 3⅞ | 3¾ | 5½ | 5¼ | 4¼ | 6½ | | | |
| L | 4 × 4 | 3 | 1.5 | 1.25 | 6.5 | | 2¼ | 4¾ | 6½ | 2½ | 5 | 7 | | | |
| L | 4 × 4 | 7 | 4.5 | 3.75 | 16.5 | | 5¼ | 4¾ | 6½ | 7½ | 5½ | 7 | | | |
| M | 6 × 6 | 3 | 3 | 2.5 | 14.0 | | 2⅝ | 6¾ | 8¾ | 3⅝ | 8 | 9½ | | | |
| M | 6 × 6 | 7 | 9 | 7.5 | 35.0 | | 5⅝ | 6¾ | 8¾ | 7¾ | 7¾ | 9½ | | | |
| M | 6 × 6 | 11 | 15 | 12.5 | 55.5 | | 8⅛ | 6¾ | 8¾ | 11⅛ | 8⅜ | 9½ | | | |
| N | 7¾ × 7¾ | 5 | 10 | 10 | 39. | | 4 | 8½ | 10½ | 5¾ | 9¼ | 12½ | | | |
| N | 7¾ × 7¾ | 9 | 20 | 20 | 73. | | 7½ | 8½ | 10½ | 9⅜ | 9⅝ | 12½ | | | |
| N | 7¾ × 7¾ | 13 | 30 | 30 | 107. | | 12¾ | 8½ | 10½ | 15½ | 10 | 12½ | | | |
| O | 10½ × 10½ | 5 | 20 | 20 | 112.5 | | | | | 6½ | 12¾ | 15½ | | | |
| O | 10½ × 10½ | 11 | 50 | 50 | 208. | 285 | | | | | | | 12⅞ | 14⅝ | 20¾ |
| O | 10½ × 10½ | 17 | 80 | 80 | | 420 | | | | | | | 13 11/16 | 14⅝ | 20¾ |
| S | 15½ × 15½ | 7 | 60 | 60 | | 370 | | | | | | | 9 | 20⅝ | 28¾ |
| S | 15½ × 15½ | 19 | 180 | 180 | | 934 | | | | | | | 21⅛ | 20⅞ | 29 |
| S | 15½ × 15½ | 35 | 340 | 340 | | 1,654 | | | | | | | 36⅝ | 20⅞ | 29 |
| S | 15½ × 15½ | 67 | 660 | 660 | | 3,178 | | | | | | | 68⅝ | 21⅝ | 29¾ |
| T | 15½ × 31 | 21 | 400 | 400 | | 1,950 | | | | | | | 23 13/16 | 21⅝ | 48½ |
| T | 15½ × 31 | 53 | 1,040 | 1,040 | | 4,850 | | | | | | | 55 11/16 | 22¾ | 49 |
| T | 15½ × 31 | 83 | 1,640 | 1,640 | | 7,475 | | | | | | | 84 | 22¾ | 49 |

GENERAL DATA ON ELECTRIC VEHICLE CELLS

| Type of Cell | Size of Plates Inches | Number of Plates | Discharge for 4-Hr. Amperes | Weight of Cell Complete With Acid Pounds | Outside Dimensions of Hard-Rubber Jar Inches | | |
|---|---|---|---|---|---|---|---|
| | | | | | Width | Length | Height |
| Exide M V | 5 3/4 × 8 5/8 | 7 | 21 | 22 | 2 3/4 | 6 1/8 | 12 3/8 |
| Exide M V | 5 3/4 × 8 5/8 | 9 | 28 | 28 3/4 | 3 1/2 | 6 1/8 | 12 3/8 |
| Exide M V | 5 3/4 × 8 5/8 | 11 | 35 | 35 1/4 | 4 1/4 | 6 1/8 | 12 3/8 |
| Exide M V | 5 3/4 × 8 5/8 | 15 | 49 | 46 3/4 | 5 3/4 | 6 1/8 | 12 3/8 |
| Exide M V | 5 3/4 × 8 5/8 | 19 | 63 | 60 1/4 | 7 1/4 | 6 1/8 | 12 3/8 |
| Exide P V | 4 13/16 × 8 5/8 | 5 | 12 | 14 1/4 | 2 | 5 5/16 | 11 7/8 |
| Exide P V | 4 13/16 × 8 5/8 | 7 | 18 | 19 1/2 | 2 3/4 | 5 5/16 | 11 7/8 |
| Exide P V | 4 13/16 × 8 5/8 | 11 | 30 | 29 3/4 | 4 1/4 | 5 5/16 | 11 7/8 |
| Gould T P | 5 3/4 × 8 3/4 | 7 | 21 | 24 1/2 | 6 1/8 | 2 3/4 | 12 |
| Gould T P | 5 3/4 × 8 3/4 | 9 | 28 | 31 1/2 | 6 1/8 | 3 1/2 | 12 |
| Gould T P | 5 3/4 × 8 3/4 | 11 | 35 | 38 1/2 | 6 1/8 | 4 1/4 | 12 |
| Gould T P | 5 3/4 × 8 3/4 | 13 | 42 | 45 1/2 | 6 1/8 | 5 | 12 |

from the front of the switchboard; incandescent lamps on the front of the board indicate to the attendant the exact position of the switch and the number of cells in circuit. The connection can be made so that the motor, after being started by the attendant, will stop automatically as soon as one additional cell has been cut into or out of circuit.

**Boosters and Booster Connections.**—Since the final voltage of a cell when charging is from 2.4 to 2.7 and the voltage on

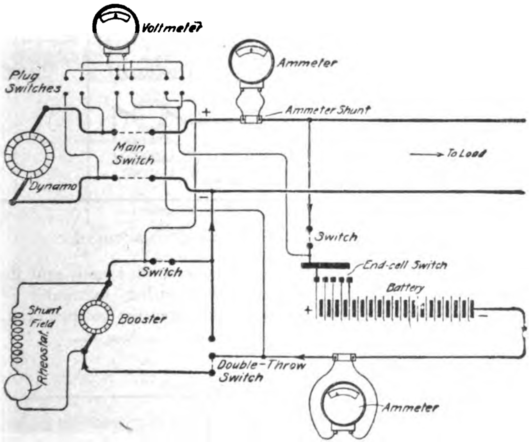



FIG. 6. SHUNT BOOSTER CONNECTIONS

beginning to discharge is only 2, the charging voltage must be about 35% higher than the line voltage across which the battery is used. If a circuit is supplied sometimes by a dynamo and sometimes by a storage battery that is charged by energy from the dynamo, it must be possible either to raise the voltage of the dynamo while charging the battery, or else to use an auxiliary dynamo in series with the main dynamo to raise, or *boost*, the voltage. The field of the

booster dynamo is easily made reversible, so that during discharge the battery voltage can be boosted if necessary.

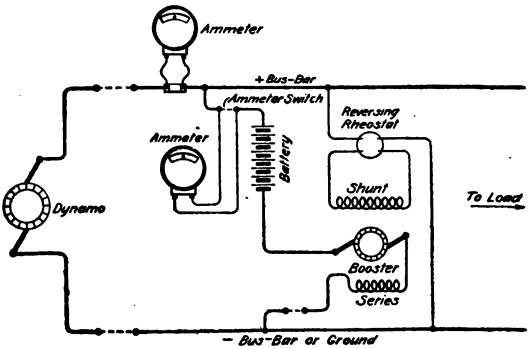



Fig. 7. Compound Booster Connections

The booster is usually driven at a constant speed, and its voltage is varied by field regulation, either manually or automatically, from maximum in one direction to zero or to

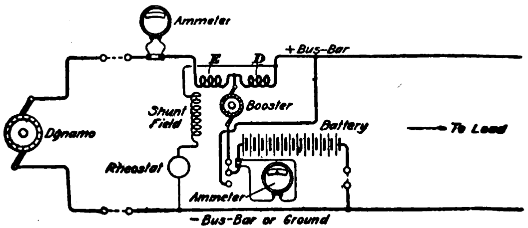



Fig. 8. Differential Booster Connections

maximum in the opposite direction, as may seem necessary. The shunt and compound booster-connection diagrams,

Figs. 6 and 7, need no explanation. The differential booster series field, in two sections $E$ and $D$, Fig. 8, is connected so as to oppose the shunt field; section $E$ carries the dynamo current, which is nearly constant, and section $D$ carries the line current, which may vary considerably. The adjustments are such that when the line current equals the full dynamo capacity, the line voltage balances the combined voltage of the battery and the booster and no current flows through the battery. When the line current exceeds the dynamo capacity, the effect of the series field predominates, and the booster helps the battery to discharge into the line; when the line current is less than the dynamo capacity, the

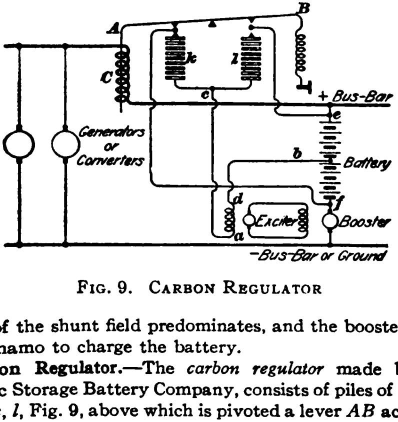



FIG. 9. CARBON REGULATOR

effect of the shunt field predominates, and the booster helps the dynamo to charge the battery.

**Carbon Regulator.**—The *carbon regulator* made by the Electric Storage Battery Company, consists of piles of carbon disks $k$, $l$, Fig. 9, above which is pivoted a lever $AB$ actuated by a solenoid $C$ connected in series with the main line. The pull of the solenoid $C$ is balanced by an adjustable spring. The conductivity of the piles of disks depends on the force with which they are pressed together, which, in turn depends on the main-line current through the solenoid $C$. The booster field is excited by a small dynamo, the field of which

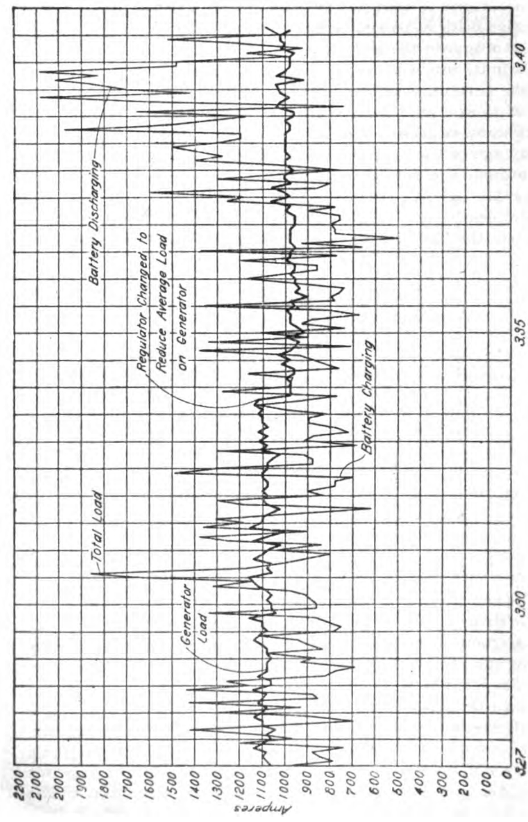



**Fig. 10. Control of Storage Battery by Carbon Regulator**

is connected between the middle point *b* of the battery and a point *c* between the piles of disks. During heavy load, disks *k* are compressed and disks *l* are allowed to separate; current then flows through the path *b–d–a–c–k–f*. During light load, the spring pulls the other end of the lever down, disks *l* are compressed, and disks *k* allowed to separate; current then flows through the path *e–l–c–a–d–b*, that is, in the reverse direction through the exciter field. The exciter field is thus automatically excited in either direction, according to the line load, and the booster is made to assist

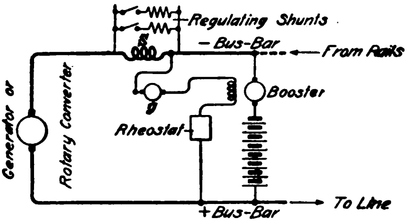



FIG. 11. COUNTER E. M. F. SYSTEM

or to oppose the battery. The regulator is adjusted by varying the tension on the spring. Fig. 10 shows the effect of a regulator.

**Counter E. M. F. System.**—In the *counter E. M. F. system* used by the Gould Storage Battery Company, the booster-field current passes through the armature of the *counter E. M. F. generator g*, Fig. 11, the field of which is excited by a coil *S* in series with the main line. Both machines are driven at a constant speed, and the voltage of *g* depends on the strength of the line current; it may be greater than the line voltage, thus causing current to flow through the booster field in one direction, or less than the line voltage, thus permitting the current to flow in the opposite direction. The voltage of *g* may be made to assist or to oppose the battery voltage. Adjustments are made by means of the regulating shunts and the booster-field rheostat.

# ALTERNATING-CURRENT APPARATUS

## ALTERNATING CURRENT

An *alternating current* is a current consisting of equal half waves in successively opposite directions; it flows back and forth in a circuit with as great regularity as a piston moves to and fro in the cylinder of a steam engine, but with much greater rapidity. Alternating currents are usually represented by curved lines, as in Fig. 1, that indicate the successive positive values of the current by loops, or half waves, *ab*, above a horizontal line, and negative values by loops *bc*, below the horizontal line. Distances along the horizontal line represent time. The points *a*, *b*, *c* represent instants when the current is zero; from *a* the current increases in a positive direction to a maximum value, falls to zero at *b*, increases to a maximum negative value, and again decreases to zero at *c*, thus completing a *cycle*. These cycles are continuously repeated, usually 25 to 60 or more times per sec.; the number of cycles per sec. is the *frequency* of the current, and the time required for the current to complete one cycle is a *period*. An *alternation* is a half cycle, or a half wave, and is represented by the curve between *a* and *b* or that between *b* and *c*. Frequency is sometimes expressed as the number of alternations per min., which equals $2 \times 60 \times$ the number of cycles per sec. Distances along the horizontal time line are usually expressed in degrees; thus, from *a* to *b* is 180° and from *a* to *c*, one complete cycle, is 360°.

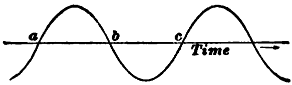



FIG. 1
ALTERNATING-CURRENT CURVE

The curve may also represent an E. M. F., as both the current and the E. M. F. pass through similar series of values. The shape of the wave depends on the construction of the alternator producing it, but is usually so nearly that of a sine curve that little error is introduced

if the laws governing the sine curve are used in making alternating-current calculations.

The *average value* of the current or E. M. F. is the average of all the ordinates of the curve of one half wave. The *effective value* of an alternating current is that which will produce the same heating effect as the same strength of continuous current. Since the heating effect depends on the square of the current, the effective value of an alternating current is the average of the squares of the instantaneous currents of one half wave. The effective E. M. F. is the value corresponding to the effective current. Effective values are the ones always measured and the ones referred to in speaking of alternating currents, unless otherwise

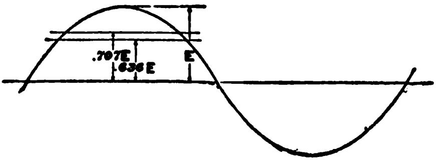



FIG. 2. MAXIMUM, AVERAGE, AND EFFECTIVE VALUES

mentioned. If $\mathbf{E}$, Fig. 2, is the maximum E. M. F. of a half cycle of a sine curve,

$$\text{average value} = .636\mathbf{E}$$

$$\text{effective value} = .707\mathbf{E} = 1.11 \times \text{the average value}$$

When an alternating E. M. F. and its resulting current pass through corresponding values simultaneously, they are said to be *in phase* with each other. When the E. M. F. reaches a definite value in the cycle sooner than the current reaches its corresponding value, the two are *out of phase* with each other; the E. M. F. is said to be *leading* and the current to be *lagging*.

Phase difference is always expressed in degrees; a complete cycle is 360°. If two currents or E. M. F.'s differ in phase by 180°, or ½ cycle, they will be in direct opposition to each other, one being maximum positive at the same instant that the other is maximum negative; a difference of

$\frac{1}{4}$ cycle, or 90°, would make one a maximum and the other zero simultaneously. Currents or E. M. F.'s differing from each other 90° are said to be at *right angles* to each other, or *in quadriture.*

*Self-induction* or *inductive resistance* in a circuit in which alternating current is flowing causes a lagging current; *capacity* causes a leading current. Both dampen the flow of current and prevent it from rising to the same value as would direct current with the same pressure; that is, they act, with alternating current, as additional resistance in the circuit. With ohmic resistance alone in the circuit, the current is in phase with the E. M. F.; with inductive resistance alone, the current lags 90° behind the E. M. F.; with capacity alone, the current is 90° ahead of the E. M. F.

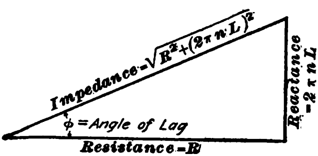



FIG. 3

The alternating E. M. F. $E_i$ impressed on a circuit may be considered as made up of components, one in phase with the current and one at right angles to it. The *resistance* $R$ in the circuit multiplied by the current gives the component of the E. M. F. in phase with the current. The *reactance* multiplied by the current gives the component at right angles to the current. The *impedance* of a circuit is that quantity which multiplied by the current gives the impressed E. M. F. If alternating current with frequency $n$ is flowing in a circuit containing resistance $R$ ohms and self-induction $L$ henrys, but no capacity, the reactance is $2\pi nL$ and the impedance

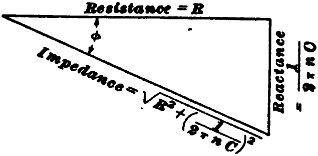



FIG. 4

$\sqrt{R^2+(2\pi nL)^2}$. (See Fig. 3.) In a circuit containing resistance $R$ ohms and capacity $C$ farads, but no self-induction, the reactance is $\frac{1}{2\pi nC}$ and the impedance. $\sqrt{R^2+\left(\frac{1}{2\pi nC}\right)^2}$. (See Fig. 4.) In a circuit containing resistance, inductance, and capacity, the reactance is $2\pi nL - \frac{1}{2\pi nC}$ and the impedance is

$$\sqrt{R^2+\left(2\pi nL-\frac{1}{2\pi nC}\right)^2}.$$

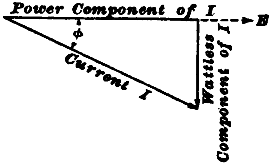



Fig. 5

In any case $\tan\phi = \frac{\text{reactance}}{\text{resistance}}$, or, for inductive circuits, $\tan\phi = \frac{2\pi nL}{R}$, and $L = \frac{R\tan\phi}{2\pi n}$, where $\phi$ is the angle of phase difference.

The *power factor* of a circuit is the ratio of the real watts to the apparent watts. The real power in watts equals the product of the volts $E$, the amperes $I$, and the cosine of the angle $\phi$, or $EI\cos\phi$. For example, if, in an inductive circuit, the ratio of the watts $EI\cos\phi$, measured by a wattmeter, to the volt-amperes $EI$, measured by a voltmeter and an ammeter, respectively, is .866, then the power factor $\cos\phi = .866$, $\phi = 30°$. $\tan\phi = .57735 = \frac{2\pi nL}{R}$ and $L = \frac{.57735R}{2\pi n}$, from which, if $R$ and $n$ are known, $L$ may be determined.

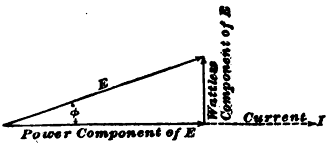



Fig. 6

**Wattless Current.**—If the current and the E.M.F. are out of phase, either may be considered as made up of two components. For example, if the current lags by an angle of

## RELATIONS OF RESISTANCE, INDUCTANCE, AND CAPACITY IN ALTERNATING-CURRENT CIRCUITS

| Circuit Contains | Reactance | Impedance | E. M. F. for a Current $I$ Amperes | Power Factor $=\cos\phi$ |
|---|---|---|---|---|
| $R$ ........ | $0$ | $R$ | $IR$ | $1$ |
| $L$ ........ | $2\pi nL$ | $2\pi nL$ | $2\pi nLI$ | $0$ |
| $C$ ........ | $\frac{1}{2\pi nC}$ | $\frac{1}{2\pi nC}$ | $\frac{I}{2\pi nC}$ | $0$ |
| $R$ and $L$ in series ... | $2\pi nL$ | $\sqrt{R^2+(2\pi nL)^2}$ | $I\sqrt{R^2+(2\pi nL)^2}$ | $\frac{R}{\sqrt{R^2+(2\pi nL)^2}}$ |
| $R$ and $C$ in series ... | $\frac{1}{2\pi nC}$ | $\sqrt{R^2+\left(\frac{1}{2\pi nC}\right)^2}$ | $I\sqrt{R^2+\left(\frac{1}{2\pi nC}\right)^2}$ | $\frac{R}{\sqrt{R^2+\left(\frac{1}{2\pi nC}\right)^2}}$ |
| $R$, $L$, and $C$ in series. | $2\pi nL-\frac{1}{2\pi nC}$ | $\sqrt{R^2+\left(2\pi nL-\frac{1}{2\pi nC}\right)^2}$ | $I\sqrt{R^2+\left(2\pi nL-\frac{1}{2\pi nC}\right)^2}$ | $\frac{R}{\sqrt{R^2+\left(2\pi nL-\frac{1}{2\pi nC}\right)^2}}$ |

$\phi$ degrees, the *power component* $I \cos\phi$ is in phase with the E. M. F. and the *wattless component* $I \sin\phi$ is at right angles to the E. M. F., as in Fig. 5. In like manner the components of the E. M. F. with reference to the current are the power component $E \cos\phi$ and the wattless component $E \sin\phi$, as in Fig. 6.

Wattless current adds very little to the load on a system, but its heating effect on electric circuits is as great as if it were in phase with the E. M. F.; hence, it is an object to keep the power factor ($\cos\phi$) as high as possible.

## PROPERTIES OF ALTERNATING-CURRENT CIRCUITS

In the accompanying table, $R$ is the resistance in ohms, $L$ the inductance in henrys, $C$ the capacity in farads, and $n$ the frequency.

For example, if 3 amperes of 60-cycle alternating current is flowing through 10 ohms, .5 henry, and 500 microfarads (.0005 farad) in series, the reactance is

$$2\times 3.1416\times 60\times .5-\frac{1}{2\times 3.1416\times 60\times .0005}=183.2$$

The impedance is $\sqrt{10^2+183.2^2}=183.5$; the E. M. F. for a current of 3 amperes is $3\times 183.5=550.5$ volts; the power factor, $\cos\phi=\frac{10}{183.5}=.0546$; and $\phi = 86° 52'$.

---

# ALTERNATORS

*Alternators* are *single-phase* if they generate a single alternating E. M. F., and *polyphase* if they generate two or more E. M. F.'s that differ in phase by a fixed amount. *Two-phase*, or *quarter-phase*, *alternators* generate two E. M. F.'s that differ in phase 90°, or ¼ cycle. *Three-phase alternators* generate three E. M. F.'s that differ in phase 120°, or ⅓ cycle.

Since it is not necessary for the brushes on the collector rings of an alternator to occupy any definite relation with regard to the poles, alternators can be built with a stationary armature and revolving field as readily as with a revolving armature and stationary field. The stationary armature offers better facilities for thorough insulation and is now

very widely used, particularly for machines of large output and high voltage.

If a coil $C$, Fig. 7, is moved horizontally so that its center falls successively under the centers $a$, $b$, $c$ of field poles, there will be generated in the coil an E. M. F. represented by the curve $a'\ b'\ c'$. During this movement, the coil will pass one pair of poles, or twice the *pole pitch* $a$, $b$, and the E. M. F. will pass through one complete cycle of 360°.

Since the frequency $n$ is the number of cycles per sec.,

$$n = \frac{p}{2}s = \frac{p}{2} \times \frac{\text{R. P. M.}}{60} = \frac{p \times \text{R. P. M.}}{120},$$

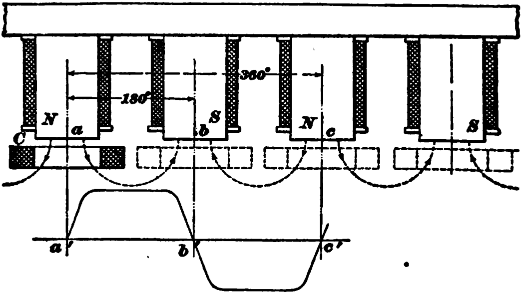



FIG. 7. SECTION OF ALTERNATOR DEVELOPED

in which $p$ is the number of poles, $s$ the revolutions per second, and R. P. M. the revolutions per minute.

## ELECTROMOTIVE FORCE OF ALTERNATORS

The form of the E. M. F. wave of an alternator is affected by the ratio of the pole arc to the pole pitch; if this ratio is small and the flux thereby concentrated into a comparatively small space, the curve will be more peaked than if the pole arc is greater, thus permitting the flux to be more widely distributed. In modern alternators, the ratio $\frac{\text{pole arc}}{\text{pole pitch}}$ is usually .5 to .7, and the pole pieces are usually so shaped

that the E. M. F. is very nearly in the form of a sine wave, in which case the E. M. F. is expressed by the formula

$$E = \frac{4.44 \Phi T n}{10^8} k_w,$$

where $\Phi$ is the flux per pole, $T$ the turns in series per phase

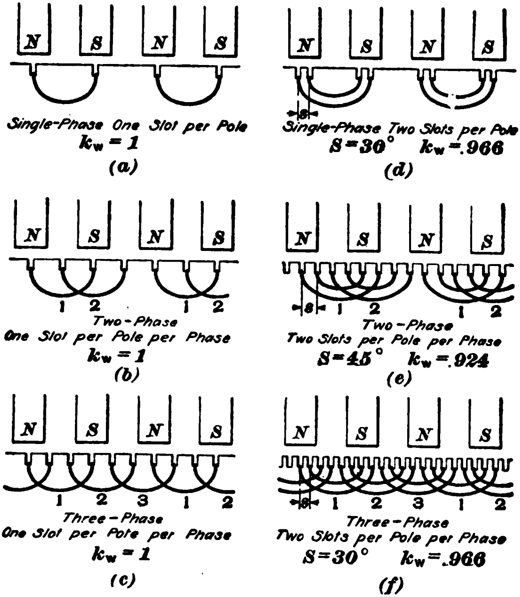



Fig. 8. Alternator Armature Windings

on the armature, $n$ the frequency, and $k_w$ a constant depending on the style of winding.

The value of the constant $k_w$ is given in the following table and in the diagrams shown in Fig. 8.

| Number of Phases | Number of Slots per Pole per Phase | Angular Distance Between Slots Degrees = S | $k_w$ | Diagram |
|---|---|---|---|---|
| Any | 1 | | 1 | (*a*), (*b*), (*c*) |
| 1 | 2 | 30 | .966 | (*d*) |
| 1 | 2 | 60 | .866 | |
| 1 | 3 | 60 | .667 | |
| 2 | 2 | 45 | .924 | (*e*) |
| 2 | 3 | 30 | .911 | |
| 2 | 4 | 22.5 | .906 | |
| 3 | 2 | 30 | .966 | (*f*) |
| 3 | 3 | 20 | .96 | |
| 3 | 4 | 15 | .958 | |

## SIZE OF EXCITERS FOR ALTERNATORS

The power required for the field excitation of alternators seldom exceeds 2% of the alternator capacity, and in large machines is less than 1%. Where two alternators and two exciters are installed, each exciter should be large enough to

| Alternator | | | Exciting Current | Size of Exciters |
|---|---|---|---|---|
| Poles | K. W. | Speed | Amperes | Kilowatts |
| 10 | 30 | 1,500 | 4.5 | 2½ |
| 10 | 60 | 1,500 | 6 | 2½ |
| 12 | 90 | 1,250 | 8 | 2½ |
| 14 | 120 | 1,070 | 10 | 2½ |
| 20 | 180 | 750 | 15 | 2½ |
| 32 | 300 | 470 | 50 | 7½ |
| 40 | 400 | 375 | 65 | 9 |
| 8 | 60 | 900 | 6.5 | 2½ |
| 8 | 90 | 900 | 7.5 | 2½ |
| 8 | 120 | 900 | 9.5 | 2½ |
| 12 | 180 | 600 | 14.5 | 2½ |
| 16 | 300 | 450 | 20 | 3½ |

excite both alternators. The accompanying table shows the approximate exciting current (at 110 volts) for a number of General Electric alternators, together with the size of exciter used.

## TWO-PHASE ALTERNATORS

A *two-phase armature winding* has two sets of coils *A*, *B*, Fig. 9; each set forms a series, of which the terminals are

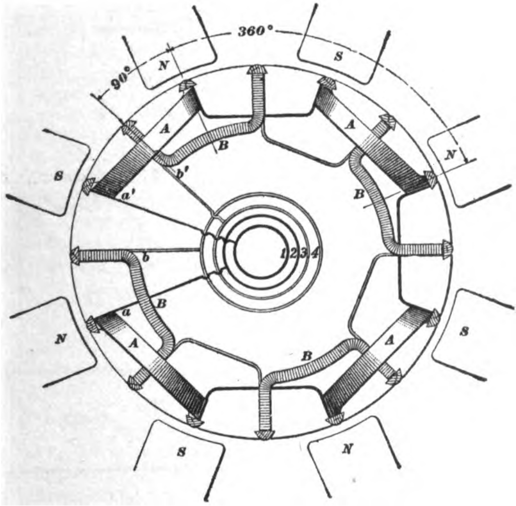



FIG. 9. TWO-PHASE WINDING

usually connected to separate insulated collector rings, thus forming a two-phase, four-wire system, shown diagrammatically in Fig. 10. Each set of coils generates a single-phase E. M. F.; and, since the centers of the coils are always one-half a pole pitch, or 90°, from each other, the two

E. M. F.'s are 90°, or ¼ cycle, apart at all times, and may be represented by curves, as in Fig. 11. The windings of each

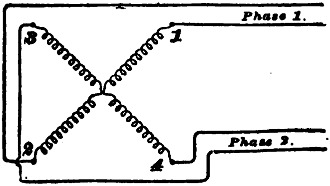



FIG. 10. TWO-PHASE, FOUR-WIRE SYSTEM

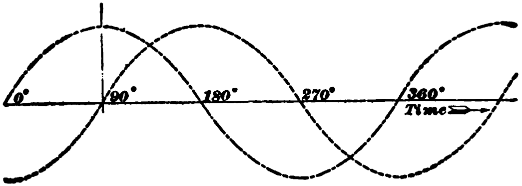



FIG. 11. TWO-PHASE CURRENT CURVES

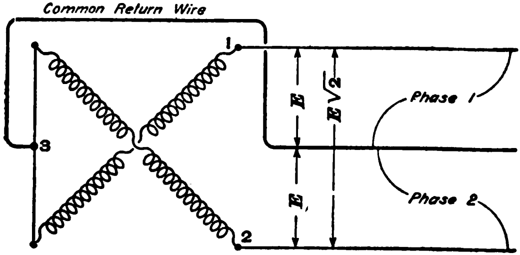



FIG. 12. TWO-PHASE, THREE-WIRE SYSTEM

phase may lie in one slot per pole per phase, as shown in Fig. 9, or in more.

If one end of each phase is brought to the same collector ring, as in Fig. 12, a two-phase, three wire system is formed; this system is easily unbalanced and is seldom used.

The *monocyclic system*, Fig. 13, is essentially a single-phase system with a so-called *teaser winding* displaced 90° from the main winding on the alternator; that is, the two windings occupy the same positions relative to each other, as do the two windings of a two-phase machine. The teaser winding, however, has only one-fourth as many turns as the main winding. One end of the teaser winding is connected to the middle point of the main winding, and the other to a collector ring. The object of the teaser winding is to produce an E. M. F. slightly out of phase with the main E. M. F., so as to produce a rotating field for motors. The system is not much used.

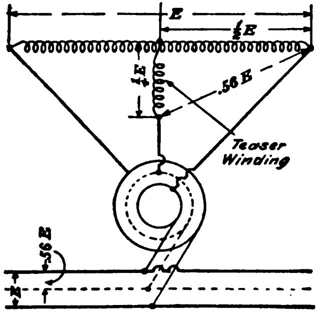



FIG. 13. MONOCYCLIC SYSTEM

## THREE-PHASE ALTERNATORS

*Three-phase armatures* have three sets of coils, $A, B, C$, Fig. 14, the coils of each set, or phase, being displaced from adjacent coils of each of the other sets 120°, or ⅓ cycle. The coils may be in one slot per pole per phase, as in Fig. 14, or in more, but seldom over four. One terminal of each phase may be joined to a common connection inside the machine and the other brought out to a collector ring, thus forming a three-wire Y, or *star*, *connection*, as in Fig. 15, where *1*, *2*, *3* represent collector rings; or, to each of three collector rings may be joined terminals of two phases, thus forming Δ (delta), or *mesh*, *connection*, as in Fig. 16.

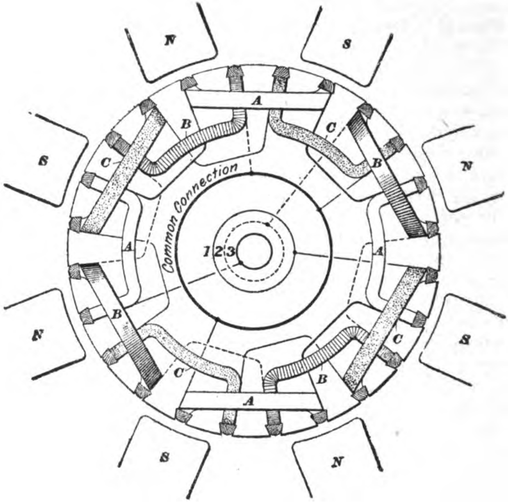



FIG. 14. THREE-PHASE WINDING

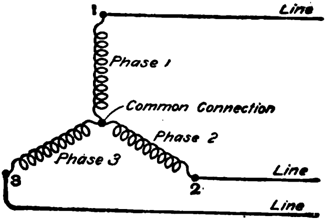



FIG. 15. Y CONNECTION

Sometimes, when an alternator is to supply current for both lamps and motors, the Y connections are used with a fourth collector ring, to which the common connection is joined and from which a fourth line wire leads to the external circuits. The three main wires lead to the motors, and the lamps are connected between the main wires and the fourth, or neutral, wire.

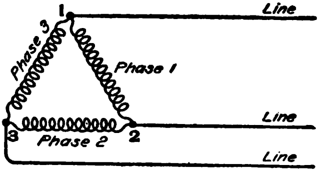



FIG. 16. Δ CONNECTION

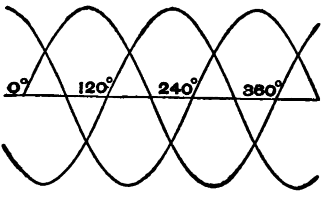



FIG. 17
THREE-PHASE CURRENT CURVES

As three-phase windings are fixed 120° apart, the three E. M. F.'s are always 120° apart. If the system is *balanced*, that is, the three currents exactly equal, the algebraic sum of the ordinates of the curves (Fig. 17) representing the currents is zero, and under such conditions no current would flow in the neutral of a Y-connected, four-wire system.

The E. M. F. between any two lines of a three-wire, Y-connected armature is the resultant of that generated in two phases, and may be calculated as shown in Fig. 18. The

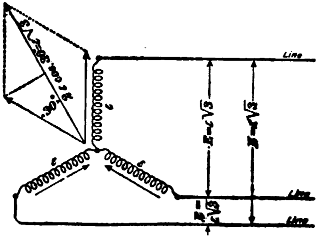



FIG. 18. THREE-WIRE Y CONNECTIONS

current in each line is the same as that through the phase to which it is connected. In a $\Delta$ system, the voltage between any two lines is that generated in a phase, but the current in a line is the resultant of the currents in the two phases supplying that line.

Let $E$ = volts between lines;
$\varepsilon$ = volts generated per phase;
$I$ = current in each line;
$i$ = current in each phase.

Then, for Y connections,

$$E=\varepsilon\sqrt{3};\ \varepsilon=\frac{E}{\sqrt{3}};\ \text{and } I=i$$

For $\Delta$ connections,

$$E=\varepsilon;\ I=i\sqrt{3};\ \text{and } i=\frac{I}{\sqrt{3}}$$

In either case, with a non-inductive load, the watts output $W$ is $3\varepsilon i=\sqrt{3}EI$. If the load is inductive, the formula for the output becomes

$$W=\sqrt{3}EI\cos\phi,$$

where $\cos\phi$ is the power factor. This formula is general, since with a non-inductive load $\phi=0$, $\cos\phi=1$, and $W=\sqrt{3}EI$, as before.

## ALTERNATORS WITH CLOSED-CIRCUIT ARMATURE WINDINGS

**Closed Circuit Windings.**—If a *closed-circuit armature winding* for a two-pole machine is tapped at four equidistant points, as shown in Fig. 19, and the points are connected to collector rings, a quarter-phase, or two-phase, current can be generated. If it is desired to operate a two-phase, three-wire system from such machine, three of the collector rings, or taps, are used. If a three-wire, two-phase system is operated from connections *1*, *2*, *3*, one phase is between *1* and *2* and the other between *2* and *3*. If the E. M. F. per phase with the four-wire arrangement is $E$, then the E. M. F. between adjacent connections one-half pole pitch apart is $\frac{E}{\sqrt{2}}$.

If a three-phase, two-pole machine is required, the closed-circuit winding is tapped at three equidistant points, or

points two-thirds pole pitch apart. Multipolar alternators with an ordinary multiple winding require a tap to each ring for each pair of poles; a series-wound, two-circuit, multipolar armature requires only one tap for each ring, the angular displacement of these taps depending on the number of poles and the number of phases. Alternators with closed-circuit armature windings are now seldom built.

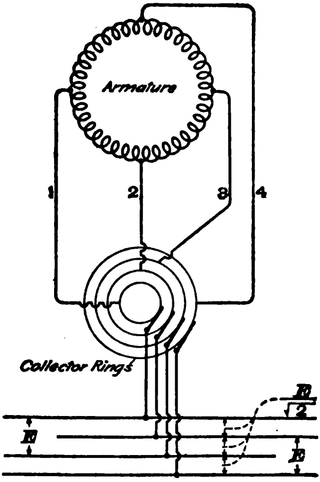


FIG. 19. CLOSED-COIL WINDING

## COMBINED OPERATION OF ALTERNATORS

Alternators are not operated in series for two reasons: (1) unless the armatures were rigidly connected together so that their phase relation would remain fixed, the machines would fall into exact opposition and would supply no energy to the circuit; and (2) series operation is necessary only for the purpose of increasing voltage, and with alternating current, the voltage is more easily raised by means of transformers.

### ALTERNATORS IN PARALLEL

In order to operate *alternators in parallel*, they must each generate the same voltage and they must be *in synchronism*, or *in step*. The state of synchronism of two alternators may be ascertained by means of incandescent lamps. If

the E. M. F.'s of the alternators are low enough so that their combined voltages can be safely impressed on a series of lamps, the series may be connected directly between corresponding terminals of the two machines, as shown by dotted lines, Fig. 20; if the voltages are high, a small reducing transformer is used with each alternator. The connections may be such that the lamps will be dark at synchronism, in which case the machines can be connected in parallel at the middle of a dark period that is several seconds long. By interchanging the connections of either

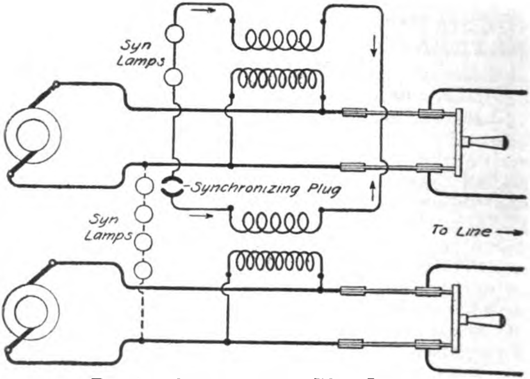



FIG. 20. SYNCHRONIZING WITH LAMPS

the primary or the secondary of either transformer, the lamps may be made to show light at synchronism.

When used to indicate synchronism of polyphase machines, the lamps must be connected across the same phase of the several machines, as in Fig. 21. The correct connections for two alternators can be found by using the regular pair of synchronizing transformers across one phase of each machine and an auxiliary pair across another phase; the phase connections should then be changed until the lamps of both

pairs of transformers act in unison; that is, become light or dark simultaneously, after which the auxiliary transformers may be removed, as the others are then known to be properly connected.

**Synchronizing With Voltmeters.**—Synchronizing lamps do not indicate the phase relation close enough to parallel large alternators, without a considerable momentary exchange of current. An arrangement of connections may be made so that at perfect synchronism the station voltmeter of the

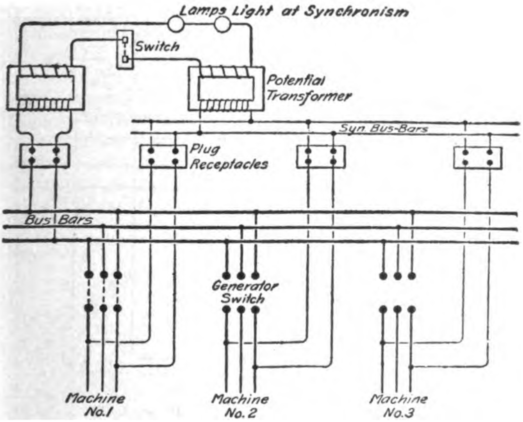



FIG. 21. SYNCHRONIZING THREE-PHASE ALTERNATORS

incoming machine will indicate normal E. M. F. of the system; that is, the indicator will be on that part of the scale where it will be sensitive to very slight phase differences. With the synchronizing connections shown in Fig. 22, voltmeter *d* is in series, through ground, with coils *e* and *h*. If the plug for reading voltage were substituted in receptacle *b*, voltmeter *d* would be in series with coils *g* and *h*. Hence, in

synchronizing machine No. 2, the effect of coil *e* on voltmeter *d* is substituted for that of coil *g* when reading voltage; and since coils *e* and *g* are symmetrically connected to the two alternators, voltmeter *d*, with connections as shown, will read full voltage when the machines are in synchronism. Synchronizing lamps are also shown connected so that they will be dark at synchronism.

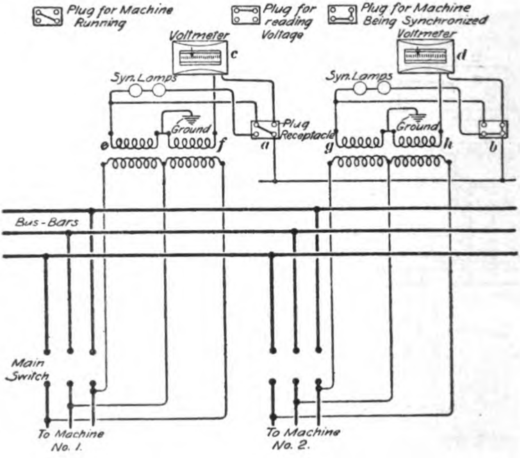
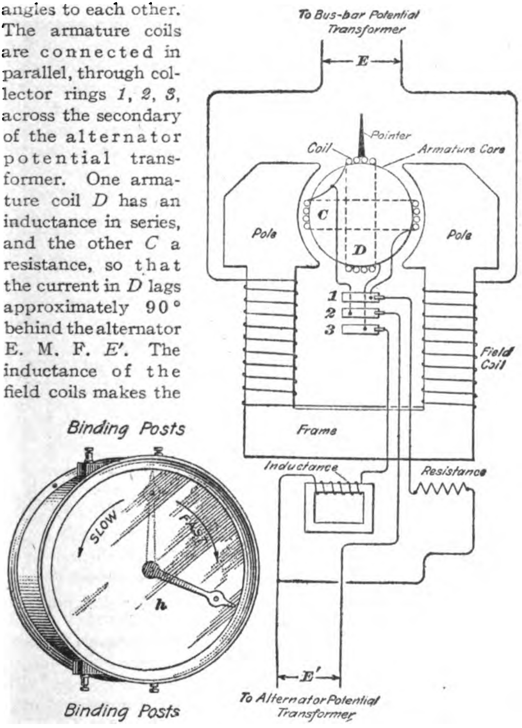



FIG. 22. SYNCHRONIZING WITH VOLTMETERS

**Lincoln Synchronizer.**—The *Lincoln synchronizer*, Fig. 23, by the vertical position of its pointer *h*, indicates exact synchronism, and also by slow movement of the pointer clockwise or counterclockwise, indicates whether the incoming machine is running too fast or too slow. The instrument contains a laminated-iron field frame and poles, the exciting coils of which are connected in series across the secondary of the bus-bar potential transformer, and a

laminated-iron armature core having two coils $C$, $D$ at right angles to each other. The armature coils are connected in parallel, through collector rings *1*, *2*, *3*, across the secondary of the alternator potential transformer. One armature coil $D$ has an inductance in series, and the other $C$ a resistance, so that the current in $D$ lags approximately 90° behind the alternator E. M. F. $E'$. The inductance of the field coils makes the



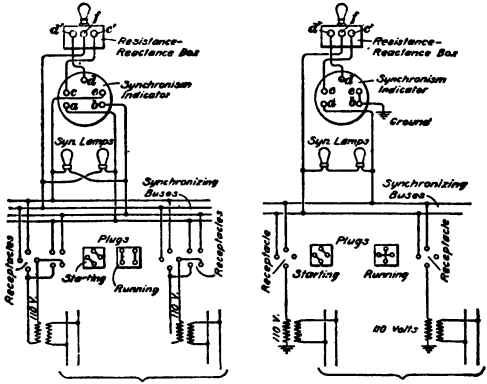

(a) *Face* (b) *Connections*

FIG. 23. LINCOLN SYNCHRONIZER

field current lag nearly 90° behind the bus-bar E. M. F. $E$

so that when the incoming machine is in exact synchronism with the machines already running, the magnetism set up by the field current will cause armature coil *D* to assume a vertical position.

**Synchronizer Connections.**—The diagrams in Fig. 24 show synchronizer connections as viewed from the back of the switchboard. Binding posts *a*, *b* are for the field coils; post *e* is connected inside the instrument through a collector ring with the junction of the two armature coils; the two




*Ungrounded Secondaries* *Grounded Secondaries*

FIG. 24. LINCOLN SYNCHRONIZER CONNECTIONS

other armature terminals are connected through collector rings inside with binding posts *c* and *d*, which are connected with posts *c'* and *d'* on the resistance-reactance box. The lamp on top of the box is connected between posts *c'* and *f*, and serves as a non-inductive resistance; the box also contains an inductance connected between posts *d'* and *f*. With connections as shown, the synchronizing lamps will be dark at synchronism.

**Automatic Synchronizer.**—The *automatic synchronizer*, Fig. 25, made by the Westinghouse Electric and Manufacturing Company, consists essentially of two solenoids acting on cores hung on opposite ends of a rocker-arm. The right-hand solenoid is connected so that it receives full current at synchronism, like a lamp that shows light, and the other so that it receives no current, like a lamp that is dark at synchronism; therefore, the right-hand end of the rocker-arm is at its lowest position at synchronism. One element of a dashpot is hinged to the rocker-arm near the left-hand end, and the other element is connected through a lever and link to a disk *a* of insulating material mounted on a short shaft in line with the pivot of the rocker-arm. On the disk is a contact piece *b* long enough to bridge the gap between a stationary finger *c* and a finger *d*, which oscillates with the rocker-arm, when the distance between the two fingers is a minimum.

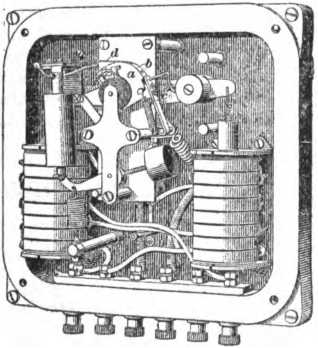


FIG. 25. AUTOMATIC SYNCHRONIZER

When the two E. M. F.'s are approaching synchronism, the oscillations of the rocker-arm are so rapid that both elements of the dashpot move almost in unison, thus drawing the contact piece *b* away from the finger *c* every time finger *d* approaches finger *c*. The nearer synchronism is approached, the slower become the oscillations, thus permitting the lower element of the dashpot to remain nearer its lowest position, until, at synchronism, piece *b* and fingers *c* and *d* are in positions to close a circuit, which, by means of a relay switch, causes the electrically operated main switch to close.

This condition cannot occur unless the two voltages are also equal, for otherwise the left-hand solenoid will receive some current even at synchronism, and the rocker-arm will not assume the correct position.

**Conditions for Parallel Operation of Alternators.**—Connections for parallel operation of three-phase alternators are shown in Fig. 26. When alternators are run in parallel, the output of each machine is proportional to the power supplied it by its prime mover. In addition to a voltmeter and an ammeter, each machine should therefore have an indicating wattmeter to show its output. If either machine fails to carry its share of the load, its prime mover should be adjusted to supply more power. The field current of each alternator should be adjusted until, for a given total output from the station, the sum of the readings of the machine ammeters will be a minimum; that is, until the exchange of wattless currents between the two machines is a minimum.

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# ALTERNATING-CURRENT MOTORS

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## SYNCHRONOUS MOTORS

*Synchronous motors* are so called because they run in synchronism with the alternator supplying the energy to drive them. Their construction is almost identical with that of alternators, and the fields must be excited by direct current the same as alternators.

*Single-phase synchronous motors* are not self-starting, and hence are little used; *polyphase synchronous motors* start with about 25% of full torque, but require a very large starting current. Because of their low starting torque and excessive starting current, synchronous motors are used only in the larger sizes and where frequent starting with a load is unnecessary. Their most frequent use is as rotary converters, which are never required to start with load and are usually provided with means for limiting the starting current.

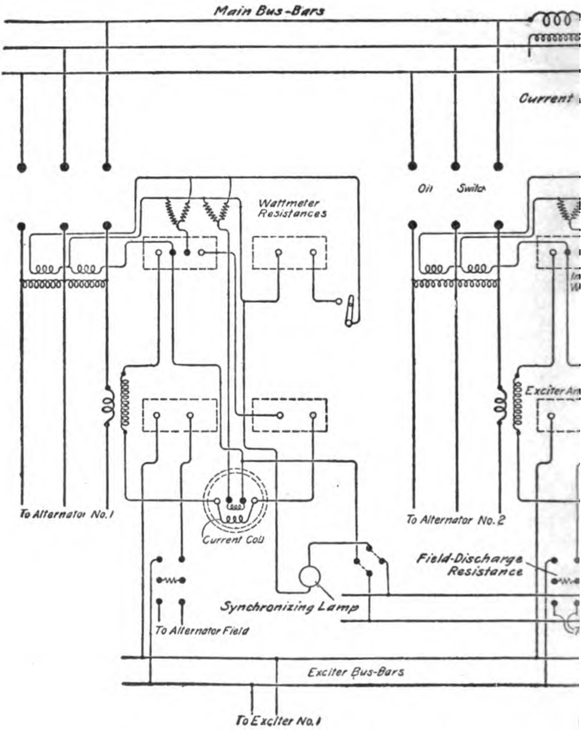


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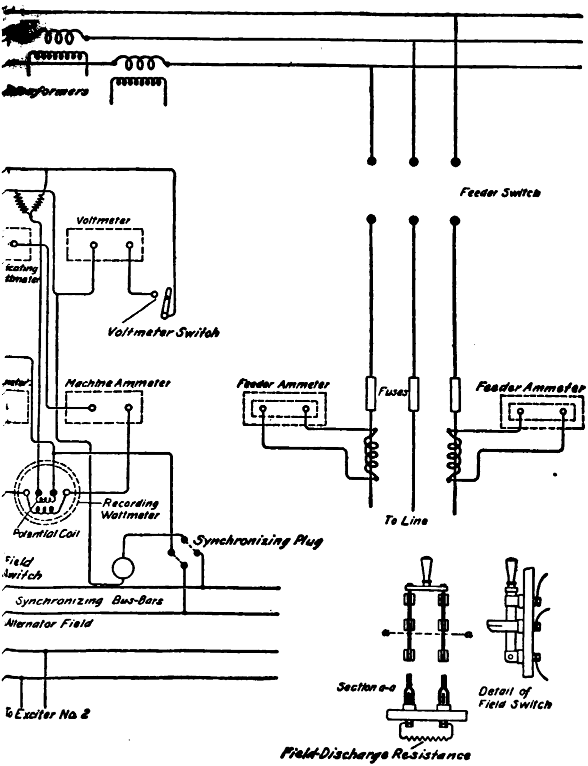
FIG. 26. CONNECTIONS FOR PARALLEL O




ERATION OF THREE-PHASE ALTERNATORS

When starting a polyphase synchronous motor with alternating current, an alternating magnetism is set up in the field poles. This magnetism lags behind the current causing it, so that the magnetism set up by one phase reacts enough on the current in the following phase to produce a small starting torque. With solid pole pieces, eddy currents in the pole faces will increase the starting torque.

The alternating magnetism in the field poles induces E. M. F.'s in the field coils, and if all the coils are in series, these E. M. F.'s are added, making a pressure between field terminals so high that the insulation may be punctured. For this reason, a *field break-up switch* is usually provided, so that the field circuit can be broken up into several parts when starting with alternating current.

The speed of a synchronous motor is fixed by the frequency of the supply current, but the amount of current taken from the line depends on the motor excitation. For a given load, the excitation can be so adjusted that the current is in phase with the line E. M. F.—that is, so that the power factor will be unity—in which case the current will be a minimum. If the fields are overexcited, the motor current will lead the line E. M. F.; if underexcited, the motor current will lag. A synchronous motor can therefore be made to act like a condenser of large capacity or like an inductance.

If $s$ is the speed of a synchronous motor in revolutions per sec., $p$ the number of poles, and $n$ the frequency of the supply current, $s = 2n \div p$, and the revolutions per min. $= 120n \div p$.

## INDUCTION MOTORS

*Polyphase induction motors* are self-starting and have considerable torque. They always consist of two essential parts; namely, the *primary*, or *field*, to which the line is connected, and the *secondary*, or *armature*, in which currents are induced by the action of the primary. Either of these parts may be the revolving member, but, generally, the field is stationary and the armature revolves, giving rise to the names *stator* and *rotor*, respectively.

Two or more currents differing in phase are led into the stator, thus producing a revolving magnetic field that induces currents in the rotor conductors. In order to have any induced currents in the rotor, there must be a difference between the speed of the rotor and that of the revolving field. When the load is very light, the motor runs very nearly in synchronism, but the speed drops off as the load is increased. This difference between the speed of the rotor and that of the field for any given load is called the *slip*. In well-designed motors, the slip varies from 2 to 7% of the synchronous speed, depending on the size of the motor. If $s'$ represents the speed of the rotor and $s$ the synchronous speed, or speed of the revolving field in revolutions per sec., then

$$\text{slip (rev. per sec.)} = s - s',$$

or, expressed as a percentage of synchronous speed,

$$\text{slip } (\%) = \frac{(s-s')100}{s}$$

Synchronous speed is determined the same as for synchronous motors by the frequency $n$ and the number of poles $p$; thus,

$$s = \frac{2n}{p}$$

**Reversing Direction of Rotation.**—To reverse the direction of rotation of an induction motor, it is necessary to reverse the rotation of the revolving magnetism set up by the field windings. In a two-phase four-wire motor this can be done by reversing the current in either of the phases; that is, by interchanging the connections of the two leads of one phase. To reverse a two-phase, three-wire motor, interchange the two outside leads. A three-phase motor can be reversed by interchanging the connections of any two motor leads.

**Speed Regulation of Induction Motors.**—Generally speaking, the induction motor is not so well adapted for variable speed as the direct-current motor, although its speed can be varied through a considerable range. This is usually accomplished by providing the rotor with a three-phase Y-connected winding and connecting a variable resistance

in series with each phase, as shown in Fig. 27. The three terminals of the winding are connected to collector rings mounted on the shaft, and the rotor current flows by way of the rings through the resistances. By means of a three-pronged arm, the resistance in series with each phase of the rotor winding can be varied from the full amount to zero, thus varying the speed from minimum to full speed.

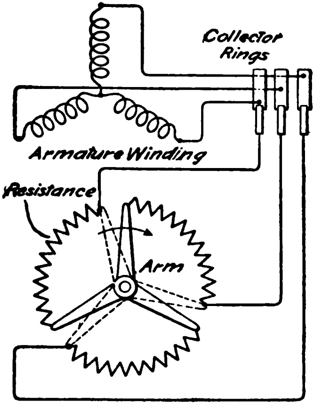



FIG. 27
SPEED-REGULATOR CONNECTIONS

### METHODS OF STARTING INDUCTION MOTORS

A polyphase induction motor can be started by connecting its stator directly to the line, but the sudden start and the large rush of current makes this method objectionable for any but small motors. To obtain a smooth start, the voltage applied to the primary may be reduced either by inserting a resistance in the primary circuit or by an autotransformer; or, a resistance may be inserted in the secondary circuit at starting and cut out when the motor comes up to speed.

**Starting Compensator.**—The rotor windings of most induction motors consist of copper bars lying in slots on the peripheries of the cores; all the bars of an armature are connected to short-circuiting rings—one ring at each end of the core. The winding resembles the wheel of a squirrel cage, hence the name *squirrel-cage winding*. It would be difficult to use a variable resistance in series with a squirrel-cage winding, and such a motor is therefore started by cutting

down the voltage applied to the primary, usually by means of an autotransformer, called, in this connection, a *starting compensator*, or *autostarter*, connections for which are shown in Fig. 28.

When the double-throw starting switch is closed in the starting position, the current for each phase of the motor flows through a portion of an autotransformer coil, which portion can be changed to suit the starting conditions of the motor with which the compensator is to be used. The compensator thereby reduces the starting voltage, and when the

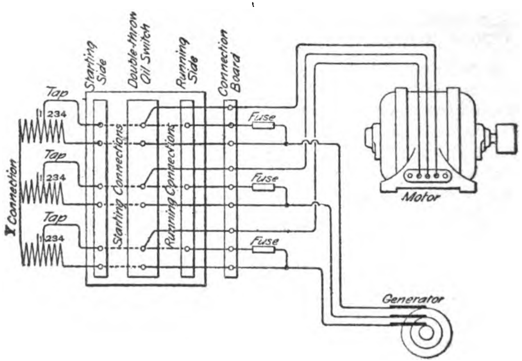


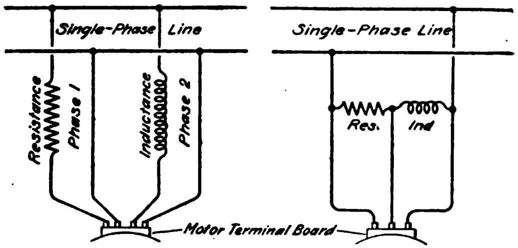

FIG. 28. STARTING COMPENSATOR

motor has attained full speed on the reduced voltage, the switch is thrown over to the running position, thus applying the full line voltage to the motor. The fuses are usually connected so that they will be in circuit in the running position, but not in the starting position, where they might be blown by the large starting current.

Since the torque of an induction motor decreases as the square of the applied voltage, starting compensators cannot be used where very strong starting torque is desired. In such a case, one manufacturing company mounts a resistance

of sufficient capacity on the rotor spider to carry the rotor current while starting, the arrangement being such that the resistance can be cut out as soon as full speed is attained, leaving the rotor winding short-circuited.

**Single-Phase Induction Motors.**—Single-phase current will not produce a revolving magnetic field in an induction motor if the armature is stationary. If, however, the armature is started in rotation by repulsion effect or otherwise, a form of revolving magnetic field is set up and the armature will continue to rotate. Motors are also started by displaced E. M. F.'s that are obtained from single-phase circuits by *splitting the phase.* This is done by providing the motor




*Two Phase—Motor—Three Phase*

FIG. 29. METHODS OF PHASE SPLITTING

stator with a polyphase winding; all the phases are connected to the single-phase line, but the several currents are made to differ from each other in phase by the use of resistances and inductances as shown in Fig. 29. The current in the phase having the inductance will lag enough behind that in the phase having only resistance in series to cause the magnetic field to revolve. Phase splitting is needed only at starting; hence, the resistances and inductances are usually switched out of circuit as soon as full speed is attained, the connections being such that the motor will then run as a single-phase machine.

## FULL-LOAD POWER FACTORS OF INDUCTION MOTORS

The *power factors* of induction motors are always less than unity, being highest at full load and decreasing as the load decreases. The following are average full-load values:

| Brake H. P. . . . . | .02 | 5 | 10 | 25 | 50 | 75 | 100 |
|---|---|---|---|---|---|---|---|
| | Power Factors | | | | | | |
| Polyphase . . . . . . | .78 | .80 | .85 | .87 | .88 | .89 | .90 |
| Single-phase. . . . | .72 | .75 | .80 | .83 | .85 | | |

## ROTARY CONVERTERS

If two collector rings are mounted on the shaft of a closed-circuit bipolar armature, and connected to opposite points on the armature winding (or to opposite commutator bars), as in Fig. 30, single-phase alternating current can be taken

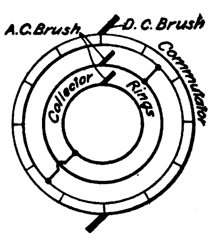



FIG. 30

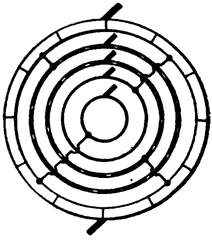


FIG. 31

from the collector rings at the same time that direct current is taken from the commutator. The voltage between the collector rings is a maximum when the bars to which the

rings are connected are under the brushes, and a minimum when these bars are midway between the brushes. The maximum alternating E. M. F. therefore equals the continuous E. M. F. $E_c$, and the effective alternating E. M. F. is

$$E_a = .707 E_c$$

If the machine is run as a direct-current motor on 100 volts, single-phase alternating current at 70.7 volts may be taken from the collector rings; or, if run as a single-phase synchronous motor on 70.7 volts, continuous current at 100 volts may be taken from the commutator.

If two pair of collector rings are used, the connections of each pair being midway between those of the other pair, as in Fig. 31, two-phase alternating current can be taken from the rings when direct current is taken from the commutator or when the machine is run as a direct-current motor. Or, the machine can be run as a two-phase synchronous motor, and direct current taken from the commutator. In this case, as with a single-phase converter,

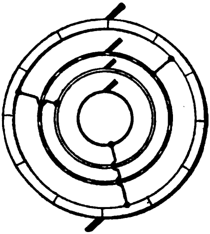


Fig. 32

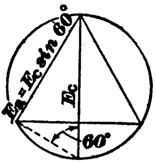



Fig. 33

$$E_a = .707 E_c$$

By connecting three rings to three equidistant points on the winding, as in Fig. 32, both direct current and three-phase alternating current can be taken from the armature simultaneously; or the machine may be run as a motor with either kind of current and made to supply the other kind. If a circle is drawn so that its diameter represents the direct-current voltage $E_c$ (see Fig. 33), then the maximum alternating E. M. F. is

$$E_a = E_c \sin 60° = .866E_c$$

and the effective alternating E. M. F. is

$$E = .707 \times .866E_c = .612E_c$$

Rotary converters are always built multipolar, because bipolar machines would have to run at excessive speeds to obtain commercial frequencies. The armatures are either series- or parallel-drum wound. If series-wound, only one connection is needed to each collector ring for any number of poles; if parallel-wound, each ring must have one connection for each pair of poles. Fig. 34 shows connections for a six-pole, single-phase converter, and Fig. 35, those for a six-pole, three-phase converter. Rotary converters are nearly always run as synchronous motors; hence, the

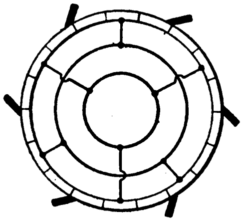


FIG. 34

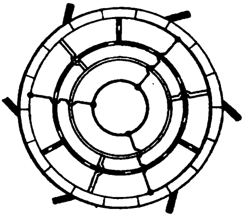


FIG. 35

speed depends on the frequency of the current supply and not on the load taken from the direct-current end.

Since the ratio of direct to alternating E. M. F. in a converter remains constant under all conditions, in order to change the E. M. F. of the current delivered by the converter, the applied E. M. F. must be changed. This may be done by supplying the converter from transformers having a number of taps, so that the number of effective turns can be changed, or by the use of potential regulators or compensating autotransformers. If converters are used for supplying direct current where the load is subject to such sudden fluctuations that hand regulation is impracticable, as in electric-railway work, the alternating current supplied to

the collector rings is made to flow through reactance coils. The field coils of the converter are compound-wound, so that at increased loads, the field excitation is increased, thus raising the power factor, bringing the current and E. M. F. more nearly into phase, and increasing the effective voltage at the collector rings. When the load decreases the current lags and lowers the applied E. M. F.

**Inverted Rotary Converter.**—The name *inverted rotary* is sometimes applied to a converter that is run as a direct-current motor and used to supply alternating current. When run in this way, the speed increases when the field is weakened and decreases when the field is strengthened, but the ratio of the two E. M. F.'s—direct and alternating—to each other remains fixed. In order to vary the alternating E. M. F. delivered by the converter, it is necessary to vary the direct E. M. F. applied to the commutator.

**Heating Effect in Converter Armature.**—With the exception of single-phase converters (which are not used), the *heating effect* in a rotary-converter armature is less than if the same machine were driven as a dynamo at the same output. The ratios of the output as a converter to that as a dynamo with the same heating effect are approximately as follows: Single-phase, .85; three-phase, 1.34; two-phase, four-wire, 1.64; six-phase, 1.96. Where large units are required, this has led to the very general use of six-phase converters.

**Double-Current Generator.**—A rotary converter may be run by an external motive power and made to deliver both direct and alternating current, in which case it is called a *double-current generator.* The full output of the machine may be delivered as direct current, as alternating current, or partly each, provided the combined output of both does not exceed the capacity of the machine.

## MERCURY-VAPOR CONVERTERS

**Cooper Hewitt Converter.**—The *Cooper Hewitt mercury-vapor converter*, built for charging storage batteries, etc., consists of a converter bulb, a panel, a frame, and an auto-transformer. The bulb, Fig. 36, is an exhausted glass vessel

containing two iron anodes in bottle-shaped glass shields, a small liquid mercury starting anode, and a mercury cathode.

The connections are shown in Fig. 37. The panel and supporting frame carry the bulb with its supporting ring resting on knife edges; the necessary switches and instruments; a resistance for use in series with the auxiliary anode at starting; a cut-out switch that opens the circuit through the starting switch after the converter is properly started; a tilting magnet that automatically tilts the bulb until the mercury in the starting anode comes in contact with that in the cathode, thus starting the flow of current; and a regulating coil for adjusting the alternating E. M. F. applied to the auto-transformer.

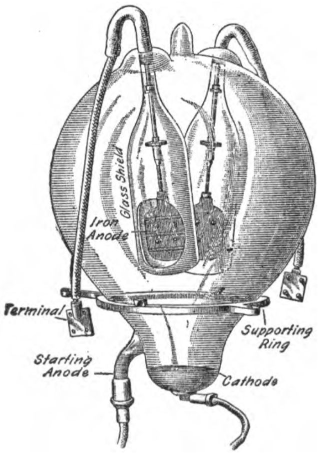



FIG. 36
MERCURY-VAPOR CONVERTER BULB

Near the frame, but not supported by it, are the auto-transformers, the terminal board of which is supplied with a number of plug receptacles into which corresponding plugs connected with the alternating-current circuit may be placed. With a given alternating E. M. F., the direct E. M. F. may thus be varied between wide limits. When the transformer plugs are in place and the switches closed, the starting switch being so placed that the resistance is connected directly with the negative battery terminal, direct current flows from the battery through the ammeter–cut-out coil–sustaining coil-

tilting magnet–resistance–to battery. This current causes the bulb to tilt until the mercury forms a conducting path between the starting anode and the cathode. Most of the current follows this path, shunting the tilting magnet and permitting the bulb to right itself. This action breaks the mercury path and starts the arc, thus causing the formation

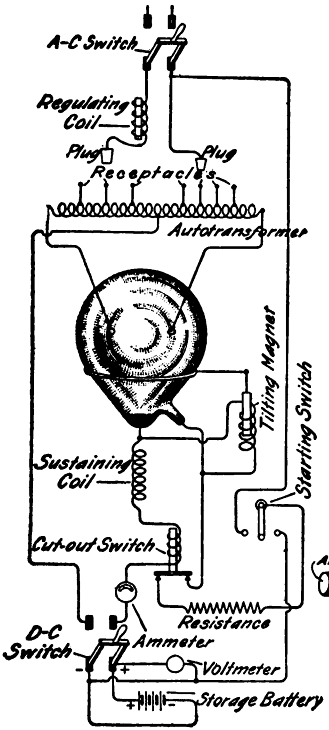



FIG. 37
CONVERTER CONNECTIONS

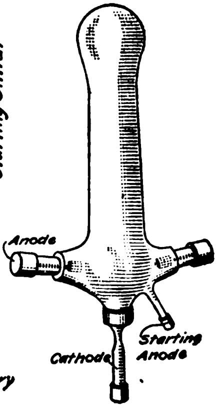



FIG. 38
RECTIFIER BULB

of vapor, through which current passes alternately from the

two main anodes to the cathode and so back through coils, ammeter, and battery to the autotransformer, causing the cut-out switch to open. The alternating current can pass in only one direction through the bulb—always from the anode that is momentarily positive to the cathode.

To start with a *dead load* in place of the storage battery, the starting switch is so placed that the resistance is connected with the alternating-current line, and the alternating current tilts the bulb and starts the arc.

**General Electric Rectifier.**—The *General Electric mercury-arc rectifier* operates on the same general principles as those just described, though the bulb or tube, Fig. 38, is different in shape and the connections differ somewhat. Mercury-arc rectifiers are used not only for charging storage batteries and operating small direct-current appliances, but also for supplying direct current to series arc-lamp circuits; for this purpose bulbs are made to supply a few amperes (about 4) at several thousand volts.

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## TRANSFORMERS

A *transformer* consists essentially of the parts illustrated in Fig. 39; namely, a laminated-iron core upon which are wound two separate insulated coils, the primary $P$ and the secondary $S$. The primary coil is connected to the alternator or supply mains and has impressed upon it the primary E. M. F. $E_p$; the alternating current thereby set up causes an alternating magnetic flux, which not only sets up a counter E. M. F. equal and opposite $E_p$ in the primary coil, but also sets up an E. M. F. $E_s$ in the secondary coil. If the magnetic flux is $\Phi$, the frequency $n$, and the number of primary turns $T_p$, the counter E. M. F. when no current is flowing in the secondary is

$$E_p = \frac{4.44 \Phi T_p n}{10^8}$$

The only current then flowing in the primary is the magnetizing current $I_m$ necessary to set up the flux $\Phi$ in the closed magnetic circuit; the magnetizing current is usually a

very small percentage of the full-load primary current of the transformer.

Assuming no *magnetic leakage*, the secondary E. M. F. is

$$E_s = \frac{4.44 \Phi T_s n}{10^8}$$

and $\frac{E_p}{E_s}$, the ratio of transformation, is $\frac{T_p}{T_s}$, or

$$E_s = \frac{T_s}{T_p} E_p$$

In a well-designed transformer, there is very little mag-

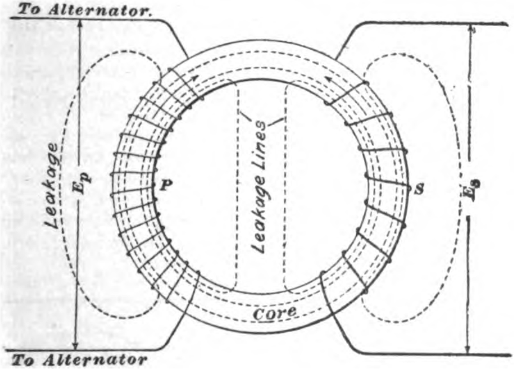



FIG. 39. TRANSFORMER

netic leakage. The effect of the leakage is to cause a decrease of secondary E. M. F. when the transformer is loaded. When a current $I_s$ flows through the secondary in phase with the secondary E. M. F., a corresponding current $I_p$ flows through the primary in addition to the magnetizing current $I_m$. The magnetizing effects of the current $I_s$ and $I_p$ are equal and opposite; that is,

$$I_p T_p = I_s T_s$$

In a perfect transformer—that is, one having no hysteresis or eddy-current losses, no resistance in its windings,

and no magnetic leakage—the magnetizing effects of the primary load current and the secondary current neutralize each other, leaving only the constant primary magnetizing current $I_m$ effective in setting up the constant flux $\Phi$. Such a transformer, if supplied with a constant primary pressure, would maintain constant secondary pressure at all loads. The best modern transformers approach this condition closely, the drop in secondary pressure from no load to full load, or the *regulation*, being not more than 1 to 3%, depending on the size.

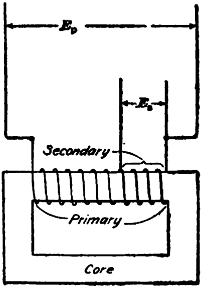



FIG. 40
AUTOTRANSFORMER

In an *autotransformer*, Fig. 40, there is only one coil, any portion of which may be used as primary and any portion as secondary. The ratio of transformation depends on the portions used; if the whole winding is used as primary and one-third of it as secondary, and the losses, which are small, are neglected,

$E_p = 3E_s$ and $I_s = 3I_p$

As the primary and secondary coils are in direct electrical contact, either coil is exposed to the pressure that may exist in the other. Therefore, it would not be safe to use autotransformers to reduce high pressures for any purpose where the high pressure itself would be dangerous, as in ordinary lighting and power work. Autotransformers for a given capacity are considerable smaller and cheaper to build than ordinary transformers.

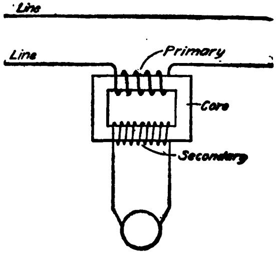



FIG. 41

and are very useful where there is little difference between primary and secondary pressures or where there is no danger incurred by the use of either.

In a *series*, or *current, transformer*, Fig. 41, the primary coil is connected in series with the circuit, and the secondary in series with the device to which the current is supplied. As the current through the primary increases, the magnetization in the core increases, thus increasing the voltage in the secondary. If the resistance of the secondary circuit is fixed, the secondary current will increase in proportion to the secondary voltage, and hence in proportion to the current in the main circuit. Series transformers are much used on alternating-current switchboards for operating ammeters and wattmeters

## OPERATION OF TRANSFORMERS ON DIFFERENT FREQUENCIES

Generally speaking, the lower the frequency the larger must a transformer be in order to deliver a given output without overheating. A transformer designed for a given output and frequency can therefore be operated at the same output on a higher frequency with somewhat reduced heating; but if operated on a lower frequency, it will run somewhat hotter and the efficiency will be slightly lower. Transformers designed for 120 or 133 cycles, unless of modern design, may not therefore give satisfactory service on a 60-cycle system. Modern transformers, however, usually have sufficient capacity to be operated at full rating on half frequency without injury.

At a given primary voltage, the product of the flux and the frequency must be constant (see formula for counter E. M. F.); hence, if the frequency is reduced from 120 to 60 cycles, the flux must be doubled. To set up the increased flux will require 2½ to 3 times as much magnetizing current, which will cause some increased heating. The eddy-current loss varies with the square of the frequency and the square of the density; since the frequency is reduced and the density increased in the same proportion, the eddy-current loss remains constant at all loads. The hysteresis loss varies

directly as the frequency and as the 1.6 power of the density; hence, halving the frequency and doubling the flux will increase the hysteresis loss from 30 to 35%. The combined increased losses in magnetizing current and hysteresis loss when using a 120-cycle transformer on 60 cycles is not great enough to cause injury if the transformer is well designed.

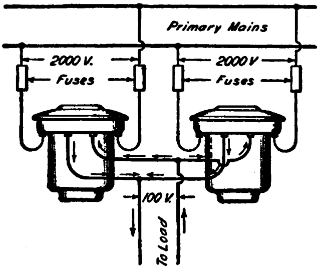



FIG. 42. TWO-WIRE SECONDARY

## TRANSFORMER CONNECTIONS

**Single-Phase Circuits.** — In connecting transformers in parallel, corresponding primary leads of each transformer must be connected to the same primary main wire, and the secondary leads must be so connected that the secondary voltages of the transformers shall at every instant oppose each other. If this is done, although the secondary coils are in series, no current can flow through them until the secondary load is applied; but if the leads are improperly connected, the secondaries will be short-circuited on each other. For this reason, after connecting the primaries to the line and two of the secondary leads together, it is well first to join the other two leads through

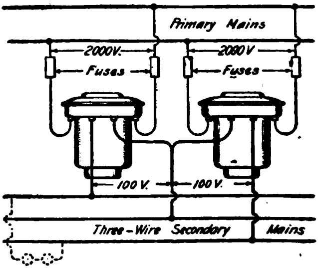



FIG. 43. THREE-WIRE SECONDARY

a fuse, which will blow if the connections are wrong. Connections for two-wire and for three-wire secondaries are plainly shown in Figs. 42 and 43. One secondary lead from each of a pair of transformers is connected to each outside wire of a three-wire system, and the other two leads are joined and connected to the middle wire. The connections must be so made that the secondaries are in series across the outside wires; if correct, two lamps in series across the outside wires, as shown by dotted lines, each lamp intended for the voltage of one side of the three-wire system, will burn at full candle-power.

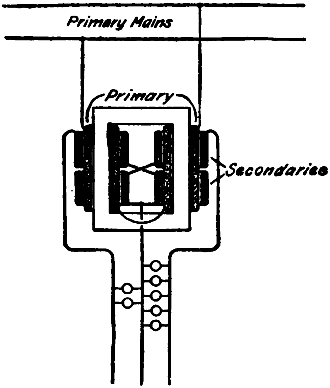


FIG. 44

If an ordinary core-type transformer is used on a three-wire system and the load becomes unbalanced, the current in the heavily loaded side will set up a large flux through the lightly loaded side, thus inducing therein a high E. M. F. To prevent this unbalancing, the secondary coils of core-type transformers for three-wire secondary systems are constructed in sections, as shown in Fig. 44, each core carrying the same number of sections of each secondary coil, so that the voltages remain balanced however unbalanced the load may be.

**Two-Phase Circuits.**—Two-phase circuits, Fig. 45, nearly always have four wires and are equivalent to two single-phase circuits in which the currents have the same frequency and always preserve a definite phase relation to each other. Both phases are used for motors, half the current

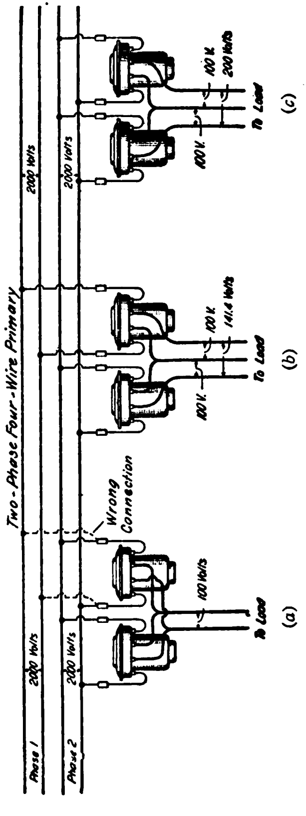
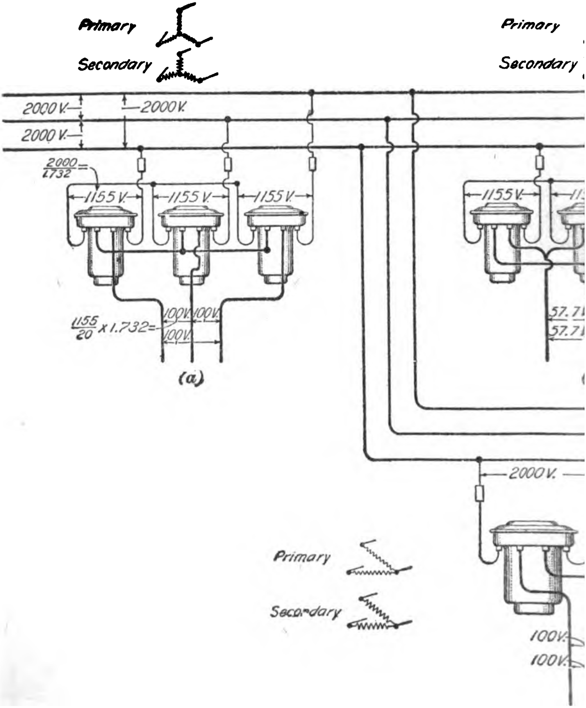
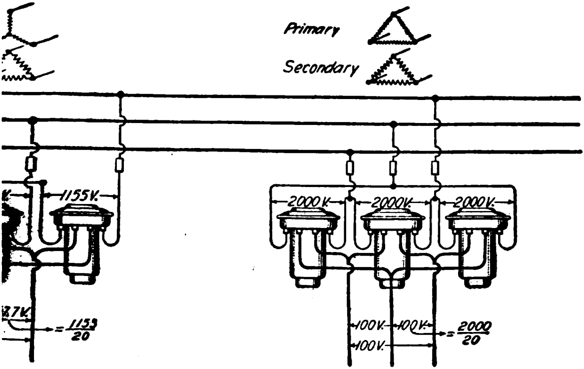



FIG. 45. TRANSFORMER CONNECTIONS TO TWO-PHASE CIRCUIT

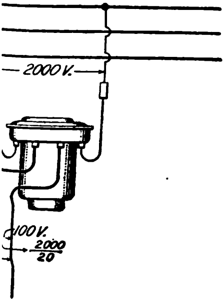



p. 287

FIG. 46. TRANSFORMERS




(c)




N Three-Phase Circuits

14

being drawn from each phase so that the same transformer capacity must be connected to each phase. For lamps, the transformers are connected the same as to single-phase circuits, care being taken to divide the load between the two phases as nearly equal as possible. If two transformers are connected in parallel, both primaries must be connected to the same phase, as in Fig. 45 (*a*); if connected wrong, as shown by dotted lines, the secondary currents will be out of phase and local currents will circulate through the secondary coils, resulting in waste of energy and unnecessary heating.

The secondaries of a pair of transformers may be joined in series, as in Fig. 45 (*b*), with one primary connected to each phase of the line circuit, thus forming a two-phase, three-wire secondary system; or the secondaries may be in series and both primaries connected to the same phase, as in Fig. 45 (*c*), forming the regular three-wire secondary system. In (*b*), the voltage between the outside secondary wires is $\sqrt{2}$, or 1.414, times that on either side, and in (*c*) the voltage between the outside secondary wires is the sum of the voltages on the two sides. The method shown in (*b*) is seldom used, since the voltages on the two sides of such a system are easily unbalanced.

**Three-Phase Circuits.**—When three transformers are connected **Y**, two coils are in series across each phase, and the voltage on each coil is the voltage per phase divided by $\sqrt{3}$, or 1.732. When the primaries are connected **Y** or Δ, Fig. 46 (*a*) or (*c*), the secondaries are usually connected in the same way. In special cases, the primaries may be connected **Y** and the secondaries Δ, as in (*b*), but this connection is not usual as it causes special secondary voltage. When the three transformers are connected Δ, each coil must be wound for the full voltage. It is possible to use only two transformers on a three-wire system in open Δ, or **V**, connection, as in (*d*), but if one fails, the service is crippled.

## PHASE-CHANGING TRANSFORMERS

Alternating current may be transformed from two to three phases by the C. F. Scott system. Two special transformers *A* and *B*, Fig. 47, have their primaries connected to

the two-phase circuit. The secondary of transformer $A$ contains turns enough to give, between its outside terminals, the voltage $E$ desired between lines on the three-phase circuit. The secondary of transformer $B$ contains turns enough to give .87 $E$; one end feeds directly into one side of the three-phase circuit, and the other to the middle point of the secondary of transformer $A$. One of the three phases

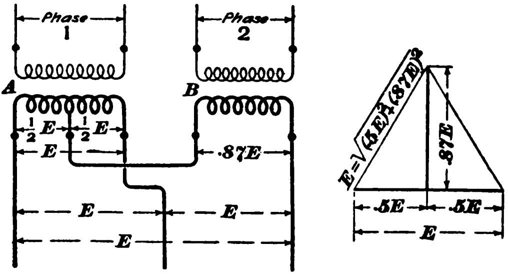



FIG. 47. SCOTT PHASE-CHANGING TRANSFORMER

then, has the voltage $E$ generated by the secondary of transformer $A$, and each of the other two has $\sqrt{(.5E)^2+(.87E)^2} = E$. The same system can be used to transform from three-phase to two-phase currents.

## CAPACITY OF TRANSFORMERS FOR THREE-PHASE INDUCTION MOTORS

A safe general rule to follow in installing transformers for running induction motors is to allow 1 kilowatt of transformer capacity for each horsepower of motor capacity; thus, a 20-H. P., two-phase motor would require two 10-kilowatt, single-phase transformers, while a 20-H. P., three-phase motor would require three 7.5-kilowatt, single-phase transformers—this being a standard size. The following table will serve as a guide:

| Horsepower of Motor | Capacity of Transformers Kilowatts | |
|---|---|---|
| | Two Transformers | Three Transformers |
| 1 | .6 | .6 |
| 2 | 1.5 | 1.0 |
| 3 | 2.0 | 1.5 |
| 5 | 3.0 | 2.0 |
| 7½ | 4.0 | 3.0 |
| 10 | 5.0 | 4.0 |
| 15 | 7.5 | 5.0 |
| 20 | 10.0 | 7.5 |
| 30 | 15.0 | 10.0 |
| 50 | 25.0 | 15.0 |
| 75 | | 25.0 |

# ALTERNATING-CURRENT WATT-METER MEASUREMENTS

## WATTMETERS

**Indicating Wattmeters.**—Since the product of volts and amperes does not give the true watts expended in an alternating-current circuit, unless the power factor is unity, voltmeters and ammeters are not well adapted for power measurements on such circuits, and wattmeters are almost universally used. The *Siemens dynamometer* type of wattmeter, Fig. 1, has a fixed coil that carries all the current flowing in the circuit, or a fixed portion of it, and a movable coil that carries a very small current proportional to the voltage of the circuit. These two coils are called respectively the *current*, or *series*, *coil* and the *potential*, or *pressure*, *coil;* either or both may be subdivided into several parts. The movable coil tends to rotate until the fields of the two coils coincide in direction. The average torque existing between the two coils determines the deflection, and the torque at

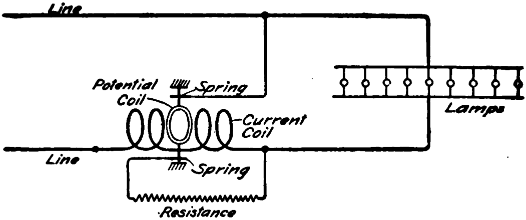



FIG. 1. SINGLE-PHASE WATTMETER CONNECTIONS

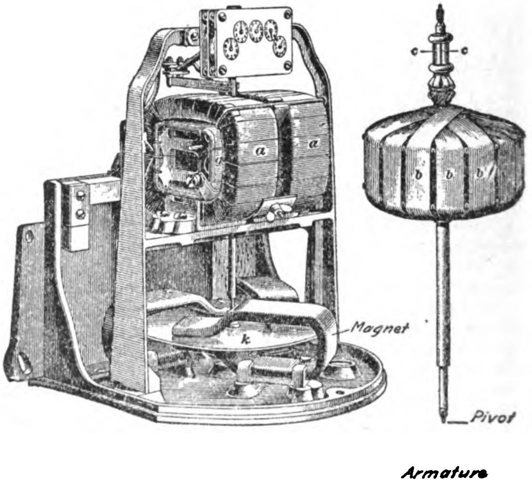



FIG. 2. THOMSON RECORDING WATTMETER

each instant is proportional to the product of the currents in the two coils at that instant. The product of the instantaneous currents in the two coils is proportional to the watts supplied to the lamps or other consuming devices, and the average torque is proportional to the average of the watts at each instant. If the power factor of the load is zero, the average of the instantaneous watts is zero, and there should be no deflection of the movable coil. If the phase difference between the currents in the two coils is greater than 90°, the movable coil reverses its deflection and the wattmeter gives a negative reading. In three-phase measurements, it is quite common to have a phase difference of more than 90° between the two currents, causing a reversal of the wattmeter deflection.

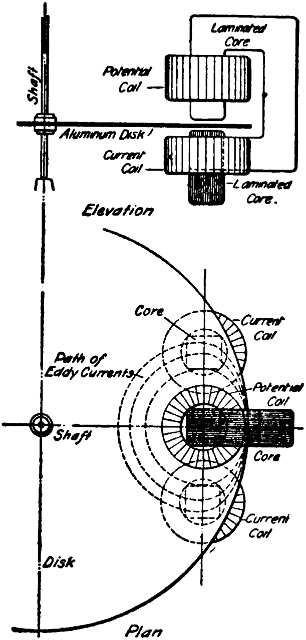



FIG. 3. INDUCTION WATTMETER

**Recording Wattmeters.**—By replacing the swinging coil of the indicating wattmeter by an armature that can rotate, and by providing a suitable braking device whereby the speed will always be proportional to the driving torque, the indicating instrument

can be transformed into a *recording meter* that will register the watt-hours expended during an interval of time.

A *Thomson recording wattmeter*, Fig. 2, consists of fixed coils *a, a* between and within which rotates the armature carrying the movable coils *b, b, b*. The ends of the movable coils are connected to the silver segments *c, c* of a commutator mounted on the upper end of the armature shaft. The braking device consists of an aluminum disk *k*, which is rotated by the shaft between the poles of permanent magnets. Adjustable shunt, or starting, coil *g*, in series with the armature, provides a field proportional to the friction, and thus makes the instrument more accurate on light loads. If the commutator, the pivot, and the jewel on which the pivot rotates are kept in good condition, the Thomson wattmeter will give accurate results on either direct or alternating current.

**Induction wattmeters,** as well as voltmeters and ammeters, can be used only with alternating current. In each instrument, alternating-current electromagnets are arranged near a disk, or vane, in which the alternating magnetism set up by the magnets causes eddy currents to flow. These eddy currents react on the magnetic field, and the indication, or record, made by the instrument is proportional to the force of the reaction.

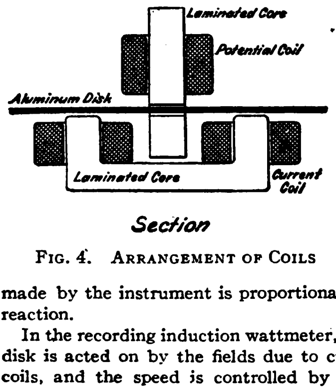



FIG. 4. ARRANGEMENT OF COILS

In the recording induction wattmeter, Fig. 3, an aluminum disk is acted on by the fields due to current and potential coils, and the speed is controlled by permanent magnets acting on the same disk. The instrument is practically a small induction motor, of which the disk is the armature.

The potential coil is on one laminated core, between the poles of which the disk rotates, and the two-current coils are on a separate core arranged so that the disk rotates past its poles. The arrangement is shown in Fig. 4.

The inductance of the potential coil, together with additional inductance usually connected in series with the coil, causes its current to lag approximately 90° behind the current in the current coils. Therefore, the eddy currents set up by the potential coil are reacted on by the field of the current coil, causing the disk to rotate.

## MEASUREMENT OF POWER ON TWO-PHASE CIRCUITS

If a polyphase system is perfectly balanced, that is, each phase carrying the same load at the same power factor—the

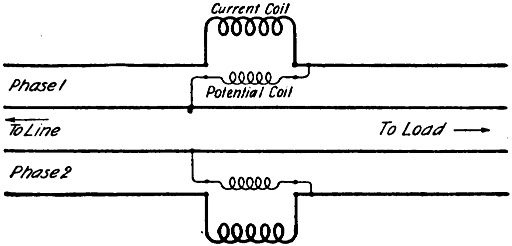



FIG. 5. TWO-PHASE, FOUR-WIRE SYSTEM

total power can be accurately determined by measuring that delivered to one phase and multiplying by the number of phases. Exact balance, however, is seldom maintained on any polyphase system.

**Two-Phase, Four-Wire System.**—To measure power on a two-phase, four-wire system, each phase is usually provided with a wattmeter connected as shown in Fig. 5. Since each wattmeter measures the power delivered by one phase entirely independent of the other phase, the sum of the readings of the two wattmeters is the power delivered by the system. On two-phase circuits, there are also in some

use recording wattmeters in which two sets of series coils and two potential coils act on one armature; this practically combines two single-phase instruments into one that will accurately measure the power delivered by a two-phase system whether balanced or not.

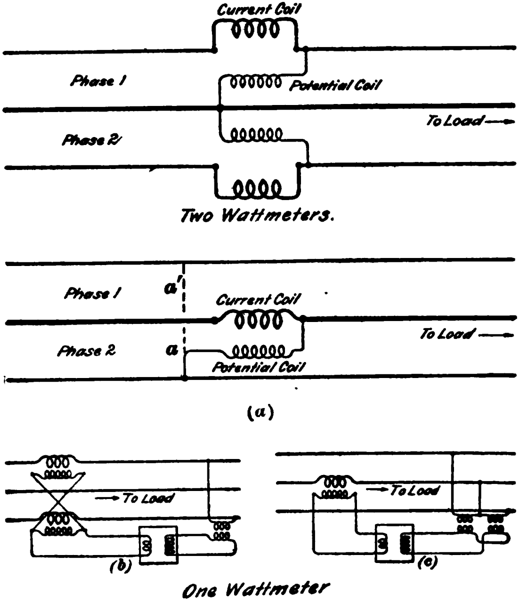



FIG. 6. TWO-PHASE, THREE-WIRE SYSTEM

**Two-Phase, Three-Wire System.**—In using two wattmeters on a two-phase, three-wire system, Fig. 6, the current coil of each meter is connected in circuit with one of the

outside wires, and the pressure coil, between the same wire and the common-return wire. For the sake of good regulation, it is quite essential that a two-phase, three-wire system be kept almost perfectly balanced, in which case the power can be measured with a single wattmeter by any one of the following methods: In (*a*), one end of the potential coil is connected to the return wire, and the other, first to one outside wire and then to the other, as at *a*, *a′*, thus measuring successively the voltage across each of the two phases. With the connections of the series transformers and shunt transformers as shown in (*b*) and (*c*), the resultant current and resultant voltage are in phase; the wattmeter will thus indicate the power.

## MEASUREMENT OF POWER ON THREE-PHASE CIRCUITS

**Use of Three Wattmeters.**—In a three-phase Y system where the neutral is accessible, three wattmeters can be used

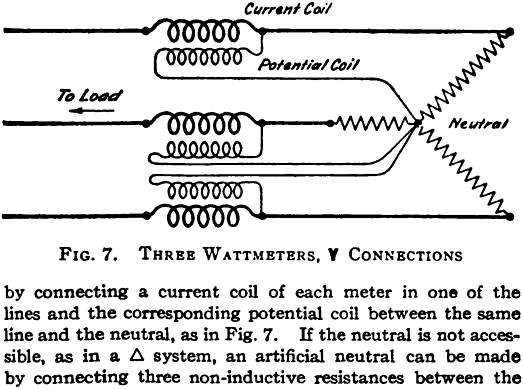



FIG. 7. THREE WATTMETERS, Y CONNECTIONS

by connecting a current coil of each meter in one of the lines and the corresponding potential coil between the same line and the neutral, as in Fig. 7. If the neutral is not accessible, as in a Δ system, an artificial neutral can be made by connecting three non-inductive resistances between the lines in the form of a Y, as shown in Fig. 8. In either case, the total power is the sum of the three wattmeter readings.

**Use of One Wattmeter.**—If a three-phase system is balanced, one wattmeter can be used by connecting the current coil in one line and the potential coil between the same line

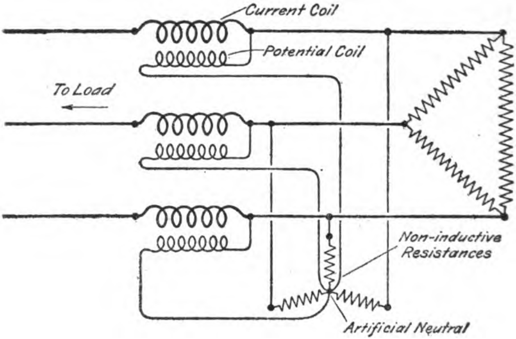



FIG. 8. THREE WATTMETERS, $\Delta$ CONNECTIONS

and the other two through a Y resistance, as in Fig. 9; the total power is the reading of the meter multiplied by 3. Or, if the load can be maintained constant long enough to trans-

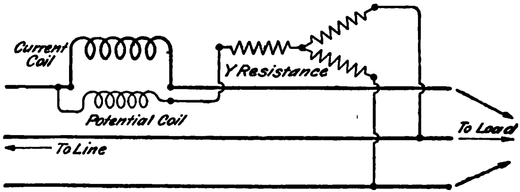



FIG. 9. ONE WATTMETER, VARIABLE LOAD

fer the connection of one end of the potential coil from one line to another, as from *c* to *c'*, Fig. 10, the other end remaining connected to the same line as the current coil, the total

power will be the sum of the two readings when the power factor is greater than .5, and the difference of the two readings when the power factor is less than .5.

**Use of Two Wattmeters.**—The best and most common method of measuring three-phase power is by means of two wattmeters. The current coils are connected in two of the

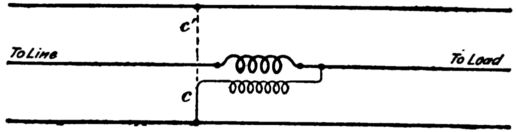



Fig. 10. One Wattmeter, Constant Load

lines, and the corresponding potential coils, between these two lines and the third line, as in Fig. 11. If the power factor is greater than .5 (angle of lag less than 60°), both meters give positive readings, and their sum is the total power; if the power factor is less than .5, one meter will read positive and the other negative, and the difference of the two readings is the total power. For example, if the angle of lag is 90°, both meters will register the same amount, but one

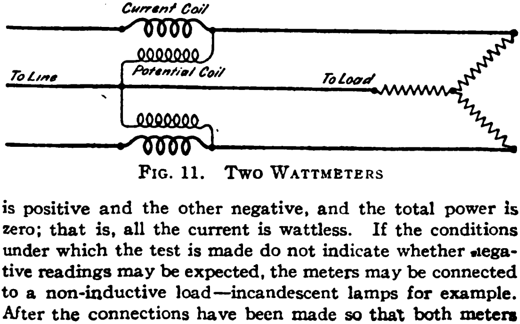



Fig. 11. Two Wattmeters

is positive and the other negative, and the total power is zero; that is, all the current is wattless. If the conditions under which the test is made do not indicate whether negative readings may be expected, the meters may be connected to a non-inductive load—incandescent lamps for example. After the connections have been made so that both meters

read positively, disconnect the lamps and apply the load under test. If one meter gives a reverse reading, the difference of the two readings should be taken. Another test is to interchange the positions of the two instruments, keeping the same relative connections of their current and potential coils. If the deflections of both needles are reversed, the difference of the readings gives the power; but if the deflections are in the same direction as before, the sum of the readings should be taken.

# PRACTICAL MANAGEMENT OF RECORDING WATTMETERS

## LOCATION

The *location of recording wattmeters* should be in easily accessible, dry, well-ventilated places, where there is as little vibration as possible. A location near a door that is being continually opened and closed should be avoided. Exposure to steam or to acid fumes is also to be avoided.

## WATTMETER CONNECTIONS

Both sides of the two-wire circuit pass through the small Thomson recording wattmeter, Fig. 12, while only one side passes through the larger size, Fig. 13. In either case, the armature is connected across the circuit in series with a resistance and the starting coil. On a three-wire circuit, Fig. 14, the two outside wires enter the meter, and the neutral wire is tapped so as to place the armature across one side of the circuit.

When wattmeters are used on high-tension circuits, the current, or field, coils are usually supplied from the secondaries of current transformers, and the potential, or armature, coils, from the secondaries of potential transformers. In such cases, the meters are calibrated to record the number of watt-hours or kilowatt-hours delivered by the high-tension circuit. Care must be taken not to confuse the connections of the field and armature coils; if the field coils are

connected where the armature should be, they will form a short circuit across the line and the meter will be ruined.

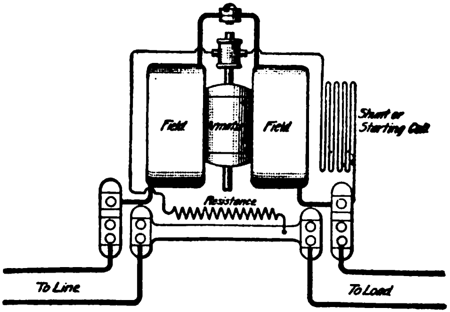



FIG. 12

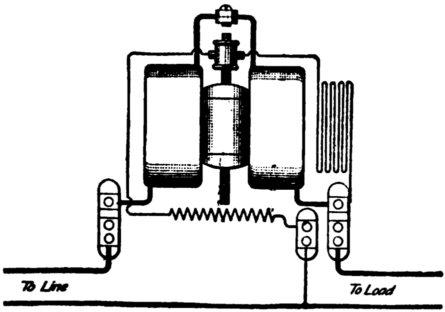



FIG. 13

The Stanley induction wattmeter field terminals are marked, as in Fig. 15; the black terminal *B* must always be

connected to the transformer, or line, and the white terminal *W* to the load. The potential wire is connected with the

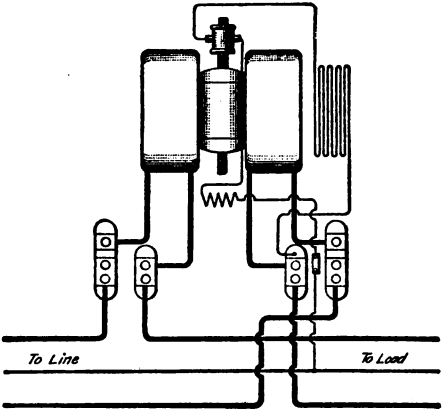



Fig. 14

line that does not enter the meter. The connections of all types of induction wattmeters are much the same. Fig. 16

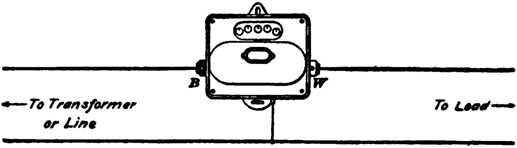



Fig. 15. Stanley Induction Meter

shows connections for measuring power on a three-phase circuit or a two-phase, three-wire circuit.

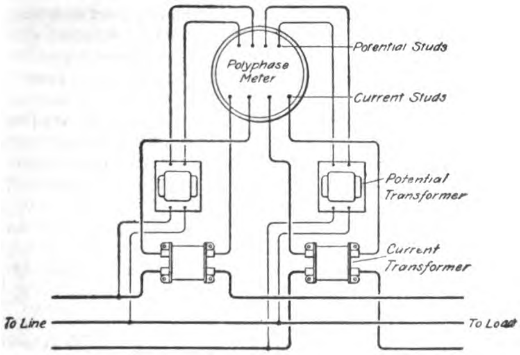



FIG. 16. THREE-PHASE INDUCTION WATTMETER

## TESTING AND ADJUSTING RECORDING WATTMETERS

**Testing.**—Recording wattmeters should be tested occasionally. A rough test may be made by loading the meter with a number of lamps of known power consumption. A more accurate test is made by comparing the recording

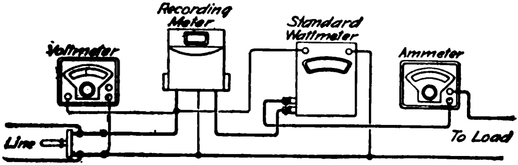



FIG. 17. TESTING A THOMSON WATTMETER

meter with a standard indicating wattmeter or with the watts obtained from the indications of a standard ammeter and voltmeter. The connections for both methods are shown in Fig. 17.

In checking a two-wire Thomson meter, a load of incandescent lamps or other convenient resistance is applied, a standard wattmeter is connected in the same circuit, and the revolutions $R$ of the recording wattmeter disk are counted for $T$ (usually from 40 to 60) sec. The load is kept as nearly constant as possible during the test, and the reading of the standard wattmeter is observed and recorded for comparison. The measurement $W$ of the recording instrument, which should agree with the indication of the standard wattmeter, is calculated by the formula

$$W = \frac{3{,}600RK}{T},$$

in which $K$ is the constant of the meter. This constant will

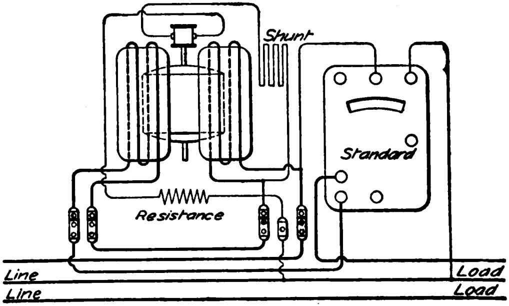



FIG. 18. TESTING A THREE-WIRE THOMSON METER

be found marked on the dial of the older types of meters, in which it was necessary always to multiply the dial reading by the constant; in the more modern meters, the constant is marked on the disk. Both the recording meter and the standard test meter must be so connected that their potential coils are subjected to the full-line voltage. In direct-current work, a voltmeter and an ammeter may be used instead of the standard wattmeter; it is well to have a voltmeter in either case.

In a Thomson recording wattmeter for a three-wire circuit (110–220 volts), the potential coil is wound for 110 volts. If the field coils of the meter under test and those of a standard meter are all connected in the same line of the circuit, and both potential circuits are subjected to the same voltage, as shown in Fig. 18, both meters measure the same power; but since the field coils of the recording meter are in series, one-half the meter constant is used in calculating the power measured.

The connections for testing any two-wire induction wattmeter are very similar to those for the Stanley meter, Fig. 19; a standard meter is connected in series with the meter to be

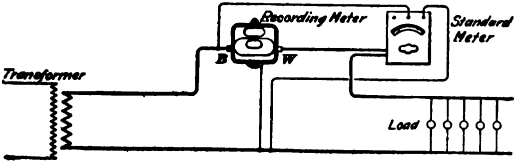



FIG. 19. TESTING A STANLEY INDUCTION METER

tested, and both potential circuits are subjected to the same voltage. In the Stanley meter, the meter watts is

$$W = \frac{100RK}{T},$$

in which $R$ is the number of revolutions in $T$ sec., and $K$ the constant marked on the meter case. The same formula applies to Fort Wayne induction meters.

**Adjustments.**—A wattmeter should not be tested until its potential circuit has been subjected to full voltage for at least 20 min.; if tested before this, it will run fast. Tests should be made for accuracy, both on light load and on full load; hence, two standard instruments are usually necessary. There should be no *creeping* of a wattmeter at no load, that is, with only the potential circuit on; if this occurs in a Thomson meter, it can usually be stopped by bending a short piece of approximately .035-in. iron wire into a **U** shape and slipping it over the edge of the disk just far enough to prevent

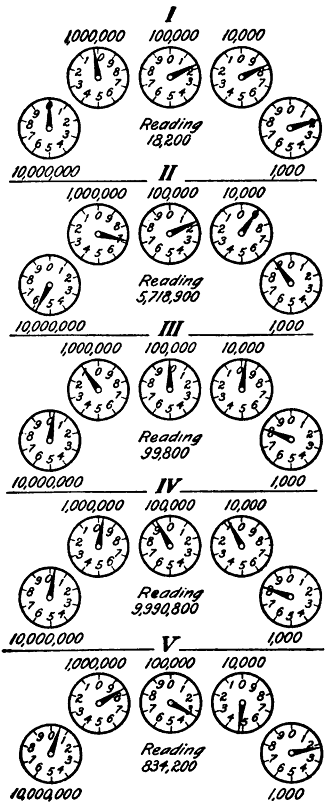



Fig. 20. Wattmeter Dials

rotation on potential alone. In an induction wattmeter, an iron wire is fastened to the disk adjacent to the hub. To counteract creeping, this wire is slightly inclined radially away from the shaft. If the wire is moved too far, the meter will not start on light load.

If a Thomson wattmeter runs slow on light loads, the starting coil should be moved a little nearer the armature, unless the slow reading is evidently due to a roughened commutator. The adjustments should be made with the commutator tarnished (seasoned) but not roughened. A roughened commutator should be cleaned and smoothed by means of tape, and the cause of the roughening should be found and removed if possible. If a meter runs slow on

full load, the damping magnets should be moved a trifle nearer the center of the disk.

## READING RECORDING WATTMETERS

Unless a constant is marked on the dial, a wattmeter may be assumed to be direct reading. The Thomson meter has five dials, which, beginning on the left, Fig. 20, may be referred to as Nos. 1, 2, 3, 4, and 5. Each dial has ten divisions, and each pointer on dials Nos. 1, 2, 3, and 4 moves one division while the pointer on its right is making one revolution. Each dial is marked with the number of watt-hours indicated by one complete revolution. Each division of dial No. 5 indicates 100 watt-hours, and each revolution of its pointer indicates 1,000 watt-hours, etc.

In reading a meter, read for each dial, beginning with dial No. 5, the figure from which its pointer is receding, being careful to allow for misplaced pointers, which are sometimes found on meters that have been subjected to rough handling. For example, in reading *I*, pointer on dial No. 5 indicates 200; pointer on No. 4 is receding from figure 8, thereby indicating 8,000+ the reading on No. 5; pointer on No. 3 is receding from figure 1, thereby indicating 10,000+ the readings on Nos. 4 and 5; and pointers on Nos. 2 and 1 are each receding from zero. Therefore, the reading is 10,000+8,000+200=18,200.

The statement of *II* is 5,718,900 (not 5,719,900, as it frequently would be read). Pointer on dial No. 4 should not be read 9 until pointer on No. 5 has completed its revolution and is again at 0. The statement of *III* is 99,800 (not 109,800); the 100,000 mark will not be reached until pointer on dial No. 5 has passed from 8 to 0, when pointers on Nos. 4 and 3 will be at 0, pointer on No. 2 at 1, and pointer on No. 1 just past the zero mark. In *IV*, pointer on dial No. 1 should be slightly to the left of 0 instead of to the right, making the reading 9,990,800. In *V*, pointer on dial No. 4 is too near figure 5; it should be two-fifths of a space past figure 4. The reading is 834,200.

To ascertain the number of watt-hours that has been used between two readings of a wattmeter, subtract the earlier

reading from the later and multiply the result by the constant of the meter, if one is marked on the dial. If no constant is marked, the meter is direct reading, and the result obtained by the subtraction is the number required. For example, if the reading of a meter on Jan. 30 is 8,619,900 and on Feb. 28, 9,990,800, and the meter is direct reading, 9,990,800 − 8,619,900 = 1,370,900 watt-hours have been supplied.

# ELECTRIC TRANSMISSION

## ELECTRIC CIRCUITS AND SYSTEMS OF DISTRIBUTION

An *electric circuit* is the complete conducting path of an electric current. The circuit is *closed* when the current can flow, *open* when it has one or more breaks that prevent the current from flowing, and *grounded* when some part is in contact with the earth or with some conductor leading to earth. A source of E. M. F. is *short-circuited* when its terminals are connected by a conducting path of very low resistance.

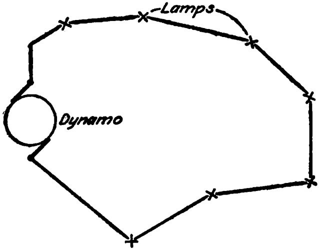


FIG. 1. SERIES SYSTEM

Electric energy may be distributed in any of several different ways: (1) By connecting all the receiving devices of the circuit in series, as shown in Fig. 1, and keeping the current through the circuit constant; for example, in street lighting, by means of a series of arc or incandescent lamps. (2) By connecting the receiving devices in parallel, as in Fig. 2, and subjecting all to a constant E. M. F., as in most systems of

incandescent lighting. (3) By connecting two or more devices in series, and a number of these series in parallel across constant-potential circuits, as shown in Fig. 3; for example, some special systems of incandescent lighting. For any of these three methods, either direct or alternating current may be used.

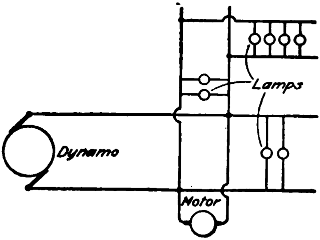



FIG. 2. PARALLEL SYSTEM

## ELECTRIC CONDUCTORS

*Electric conductors* are usually made of copper, though for some long-distance aerial transmission lines aluminum is used. Solid cylindrical wire, Fig. 4, is generally used, unless the diameter required is greater than $\frac{3}{8}$ to $\frac{1}{2}$ in. or unless great flexibility is desired, in which cases cable, Figs. 5 and 6, consisting of a number of smaller wires twisted together is better. For aerial work in sparsely settled territory, bare wire is considered good enough; but in most other cases, the wire or cable is insulated by a covering of two or three layers of cotton soaked in a weather-proof compound. For underground work, the conductor is first covered with layers of rubber or paper soaked in compound, and then all is covered with a lead

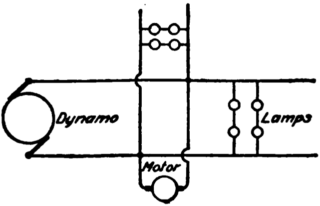



FIG. 3. PARALLEL-SERIES SYSTEM

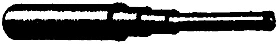


FIG. 4. INSULATED WIRE

sheathing, as shown in Fig. 6. Two or more conductors may be individually insulated and then all wrapped and insulated as one cable, as in Fig. 7; the several conductors may lie side by side in a cable or may be arranged concentrically. For telephone and telegraph work, a large number of insulated conductors are sometimes included in one cable.

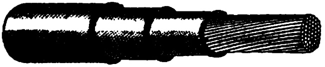


FIG. 5. INSULATED CABLE

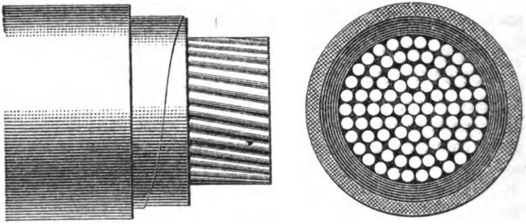


FIG. 6. LEAD-COVERED CABLE

## CONDITIONS GOVERNING SIZE OF CONDUCTORS

In order to send a current $I$ through a conductor of resistance $R$, there is required a pressure $e = IR$; that is, $e$ is the *drop of pressure* in the conductor. Since the product of

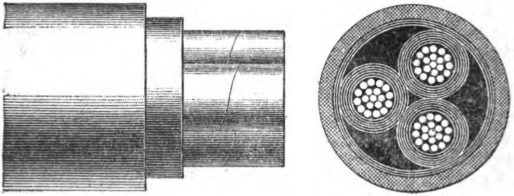


FIG. 7. THREE-CONDUCTOR CABLE

current and the pressure producing it is the work done, the product $Ie$ is the work done in sending the current through the conductor, and this work heats the conductor. The resistance of a cylindrical conductor is expressed by the formula

$$R=\frac{ml}{d^2},$$

in which $m$ is a constant, $l$ the length of the conductor in feet, and $d$ its diameter in mils (1 mil = .001 in.). For the grade of copper used for electric conductors, $m=10.65$ at 25° C. and 12.3 at 65° C.; for ordinary wiring calculations, $m$ may be taken at 10.8. The value of $m$ for aluminum wire may be taken at 17.2.

For a given length of conductor, the resistances can be made small by using a conductor of large diameter, or cross-section; but the cost of the installation will thereby be increased.

**Economical Conditions.**—In laying out a transmission system, the following points should be considered:

1. The conductors must be so proportioned that the energy transmitted through them will not cause an undue rise of temperature.

2. The conductors must have such mechanical properties as to enable them to be successfully erected, and so durable as to require a minimum of annual maintenance.

3. The conductors may be so chosen (*a*) that the first cost of line construction shall be a minimum; (*b*) that the station construction shall be a minimum; (*c*) that the cost of the plant shall be reduced and the cost of operation and maintenance be a minimum; (*d*) that the total first cost of installation shall be a minimum; (*e*) that the conditions of good service shall be a maximum; (*f*) that the total first cost of the plant shall be a minimum, consistent with maximum income.

The success of any system requires that conditions Nos. 1 and 2 be fulfilled. It is not possible to fulfil all of the conditions in No. 3; the success of a particular installation depends on the securing of a proper balance of these conditions and the selection of the proper one to be governing

or most prominent. The economical conditions may be considered in detail as follows:

1. Except in interior wiring, the question of heating of conductors used for electric transmission is seldom of importance, since the proper proportioning of the conductors to meet the other conditions will make them so large that they will not overheat. A table of safe carrying capacities of various sizes of wire will be found under Interior Wiring.

2. In every installation, thought should be given to the maximum stresses to which any of the parts are ever likely under ordinary conditions, to be subjected; that is, ordinary windstorms, snow and sleet, etc. bring to bear on aerial conductors and their supports stresses much greater than those to which they are subjected while being installed, and the various parts must be strong enough to withstand the more severe conditions. Ordinarily, conductors selected with a view of obtaining economical transmission will be strong enough mechanically; but this is not always the case, and special investigation should be made in each case of doubt.

3. (*a*) If a transmission line is installed for temporary purposes only, as for lighting or for running motors for construction work, it should be made as cheaply as possible, consistent with safety, regardless of the effect on the cost of the generating station; for the materials used in the line construction will be of little value second hand, while the engines and dynamos used in the station will not be much injured by the temporary use.

3. (*b*) In some cases it may be desirable to make the station cost a minimum. The line losses must then be a minimum, for the station must not be compelled to generate more energy than is absolutely necessary. This will require large line conductors with correspondingly high cost. For example, in city territory with underground lines, it should very seldom be necessary to take up a line for repair or renewal; hence, the transmission line should be installed large enough to care for future growth, while the station cost may be comparatively low with provision for installing increased capacity when needed.

3. (*c*) If by using small conductors the *cost of the transmission system* is made low, the transmission losses will be correspondingly high, thus increasing the *cost of the station* and the *cost of operation*. On the other hand, the cost of the station and of operation may be made low by using large conductors, so as to occasion small losses. In every plant, such a relation of these three costs to one another should obtain so that the sum of the three shall be a minimum. Sir William Thomson (Lord Kelvin) in 1881 deduced a law, an exact statement of which is made by Gisbert Kapp, as follows:

*The most economical area of conductor is that for which the annual cost of energy wasted is equal to the interest on that portion of the capital outlay which can be considered to be proportional to the weight of metal used.*

The specific weight of hard-drawn copper wire, such as used for electrical conductors, is .32 lb. per cu. in. The weight of a wire $d$ mils in diameter and $L$ thousand ft. long is, therefore,

$$\frac{d^2 \times .7854}{10^6} \times 1{,}000L \times 12 \times .32 = .003Ld^2$$

If $K$ is the cost per pound of copper wire in place on the line, and with interest at 1%, the annual interest on the cost of the conductors is

$$A = .01K \times .003Ld^2$$

Since the resistance of the line is $\frac{10.8 \times 1{,}000L}{d^2}$, the kilowatts lost in transmitting a current of $I$ amperes is $\frac{10.8I^2L}{d^2}$, and if $K'$ is the cost in dollars per kilowatt-year to generate the power, then the annual cost of the lost power is

$$B = \frac{10.8K'I^2L}{d^2}$$

According to Kelvin's law, for the most economical area of conductor, $A = B$, or

$$.01K \times .003Ld^2 = \frac{10.8K'I^2L}{d^2}$$

or,

$$d^2 = 600I\sqrt{\frac{K'}{K}} \qquad (1)$$

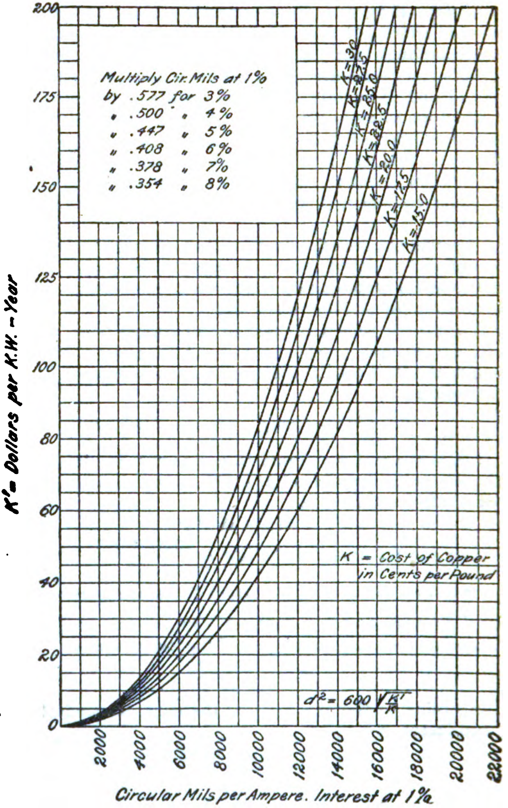



**Fig. 8. Sizes of Copper Conductors**

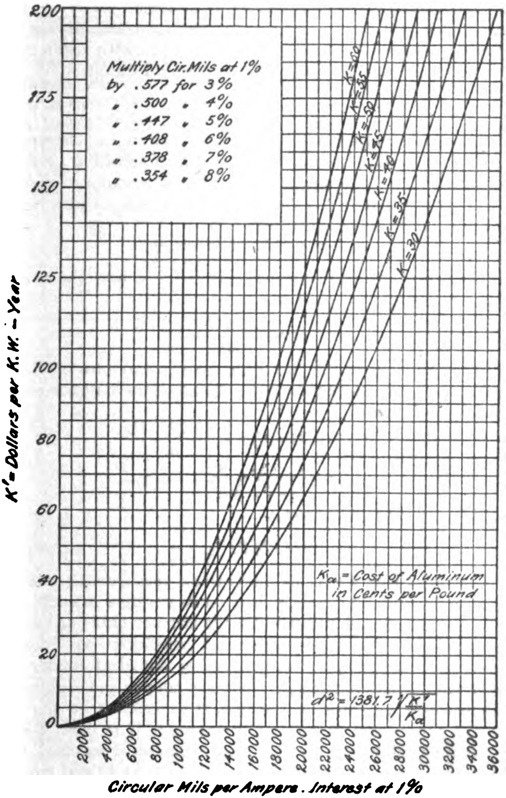



FIG. 9. SIZES OF ALUMINUM CONDUCTORS

This formula gives the most economical sectional area in circular mils $d^2$ of copper conductor for transmitting any current $I$ at any voltage for any distance, with interest at 1%; for any rate of interest $i$, the formula becomes

$$d^2 = 60I\sqrt{\frac{K'}{iK}}$$

Since aluminum wire weighs .096 lb. per cu. in., the weight of a wire $d$ mils in diameter and $L$ thousand ft. long is $.0009Ld^2$. If $K_a$ is the cost per lb. of aluminum wire in place on the line and interest is 1%, the annual interest on the cost of aluminum conductors is

$$A = .01K_a \times .0009Ld^2$$

The power lost in transmission is $\frac{17.2I^2L}{d^2}$, and the annual cost of the transmission losses is

$$B = \frac{17.2K'I^2L}{d^2}$$

With interest at 1%, the most economical area of aluminum conductor for any current, voltage, and distance occurs when $A = B$; that is,

$$.01K_a \times .0009Ld^2 = \frac{17.2K'I^2L}{d^2}$$

or, $$d^2 = 1,382I\sqrt{\frac{K'}{K_a}} \qquad (2)$$

or, for any rate of interest $i$,

$$d^2 = 138.2I\sqrt{\frac{K'}{iK_a}}$$

Formulas 1 or 2 may be made to express circular mils per ampere by dividing their second members by $I$ in each case. The curves, Figs. 8 and 9, show the circular mils per ampere required for the most economical conditions at different costs per kilowatt-year for generating electrical energy and at several different costs per pound of copper and aluminum, respectively, in place on the line, with interest on the cost of the conductors at 1%. For example, if it costs $60 to generate 1 kilowatt-year, the most economical sectional area of copper conductor at 20 ct. per lb. is about 10,400 circular mils per ampere, and of aluminum conductor at 45 ct. per lb. is about 15,950 circular mils per ampere.

The higher the rate of interest, the less should be the cost of the conductors, the sizes varying as $\sqrt{\frac{.01}{i}}$, where $i$ is expressed decimally. For example, at $40 per kilowatt-year and with copper at 20 ct. per lb., 8,480 circular mils per ampere should be used if the rate of interest is 1%; if the rate is 5%, the circular mils per ampere should be 8,480 $\times \sqrt{\frac{.01}{.05}} = 8{,}450 \times .447 = 3{,}780$. The curves can therefore be used for any rate of interest by multiplying the value found for 1% by a factor corresponding to the required rate.

**MEAN ANNUAL CURRENT**

| ¼ *I* | ½ *I* | ¾ *I* | *I* | Ratio |
|---|---|---|---|---|
| 0 | 0 | 0 | 1 | 1.000 |
| 0 | 0 | ¼ | ¾ | .944 |
| 0 | ¼ | 0 | ¾ | .901 |
| 0 | 0 | ½ | ½ | .884 |
| ¼ | 0 | 0 | ¾ | .875 |
| 0 | ¼ | ¼ | ½ | .838 |
| 0 | 0 | ¾ | ¼ | .820 |
| ¼ | 0 | ¼ | ½ | .810 |
| 0 | ½ | 0 | ½ | .790 |
| 0 | ¼ | ½ | ¼ | .771 |
| ¼ | ¼ | 0 | ½ | .760 |
| ¼ | 0 | ½ | ¼ | .744 |
| ½ | 0 | 0 | ½ | .729 |
| 0 | ½ | ¼ | ¼ | .718 |
| ¼ | ¼ | ¼ | ¼ | .685 |
| 0 | ¾ | 0 | ¼ | .661 |
| ½ | 0 | ¼ | ¼ | .650 |
| ¼ | ½ | 0 | ¼ | .611 |
| ½ | ¼ | 0 | ¼ | .586 |
| ¾ | 0 | 0 | ¼ | .545 |

Before determining the size of conductors by this method, it is necessary to know the cost of generating energy per kilowatt per year. This cost should include labor, material interest and depreciation, insurance and taxes, etc. on the station. This information, together with the cost per pound of the line copper in place—that is, cost of copper per pound

delivered plus cost per pound to put in place and insulate—is sufficient to enable the most economical cross-section in circular mils per ampere to be selected by means of the curves. The number found by the curves multiplied by the number of amperes to be transmitted gives the total cross-section required. It is possible to use copper 20% smaller than given by the curves without increasing the transmission losses more than 1 or 2%.

If less than full current $I$ is to be carried a portion of the time, the *mean annual current*, determined by multiplying the full-load current by the ratios in the accompanying table, should be used in making calculations:

For example, a line to carry 100 amperes ($I$) three-fourths of the time and 75 amperes ($\frac{3}{4}I$) one-fourth of the time should be calculated for $100 \times .944 = 94.4$ amperes. A line carrying $\frac{1}{2}I$ one-fourth of the time, $\frac{3}{4}I$ one-half of the time, and $I$ (full load) one-fourth of the time should be calculated for .771 $I$ amperes, etc.

3. (*d*) If the plant is installed for a temporary purpose only and the whole installation is chargeable to the temporary service, the cost of both station and line construction should be made low, regardless of increased operating and maintenance expenses.

3. (*e*) For supplying current to constant-potential devices in multiple, the size of the conductors must be calculated so that the pressure drop in the lines to any device shall not be excessive. The drop $e$ is

$$IR = \frac{mlI}{d^2}$$

Or, for copper,

$$e = \frac{10.8lI}{d^2} \text{ and } d^2 = \frac{10.8lI}{e},$$

in which $d$ is the diameter of the wire, in mils, and $d^2$ the circular mils area; $l$ is the length in feet, and $I$ the current, in amperes. If $D$ is the distance in feet (one way) for a two-wire circuit, the formula may be written

$$d^2 = \frac{21.6DI}{e}$$

Since $D = l \div 2$, the constant 10.8 is multiplied by 2.

For example, suppose it is desired to transmit 200 amperes 5,000 ft. with a drop of 50 volts; then,

$$d^2 = \frac{21.6 \times 5{,}000 \times 200}{50} = 432{,}000 \text{ cir. mils}$$

Reference to the table of B. & S. gauge copper wire shows that two No. 0000 wires would have about the correct cross-section. The size of wire required in a given case is sometimes more quickly determined by reference to wiring tables, such as found under Interior Wiring.

3. (*f*) If power is cheap and the cost of materials for the transmission line is high, the line installation is usually made as cheaply as is consistent with the service required. However, sufficient allowance should be made for growth, otherwise, taking on additional customers may cause excessive line losses, thus calling for expensive additions to the station.

## THREE-WIRE SYSTEMS

In the *Edison three-wire system*, Fig. 10, two dynamos alike in voltage and capacity are connected in series between the outside wires, and the neutral wire is connected with a point in the circuit between the two machines. Being in series, the E. M. F.'s of the two machines are added, making the voltage between the outside wires double that between either outside wire and the neutral wire. The lamps are connected between either outside wire and the neutral wire, and if an equal number is connected on each side—that is, if the system is *balanced*—no current flows in the neutral wire. In any case, the current in the neutral wire is the difference between the currents in the two sides, and this difference should be made as small as possible by connecting an equal number of lamps on each side. The direction of the current in the neutral wire depends on which side has the greater number of lamps burning. Motors for

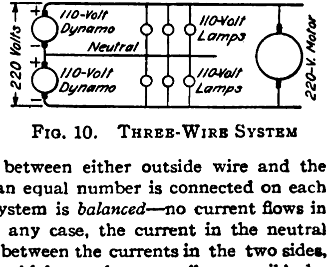



FIG. 10. THREE-WIRE SYSTEM

such a system are usually wound for the full voltage between the outside wires.

**Size of Wires.**—The outside wires are calculated as for a two-wire system having the higher voltage. The more perfect the balancing, the smaller may be the neutral wire, but as some unbalancing is always likely to occur, owing to switching on or off lamps from either side, it is customary to make the neutral wire at least one-half as large as either outside wire, and in many cases all three wires are the same size.

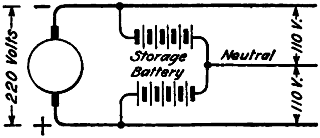



Fig. 11

**Special Three-Wire Systems.**—A three-wire system may be supplied from a single dynamo or from a circuit of double the desired lamp voltage, in any of several different ways. The neutral wire may be connected to the middle point of a *storage battery*, as in Fig. 11. Each half of the battery has cells enough to maintain the voltage desired at the lamps;

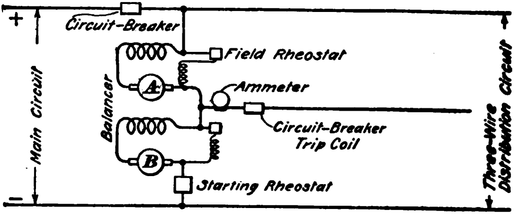



Fig. 12. Balancing Set

each half will then discharge when the number of lamps burning on its side greatly exceeds that on the other side, and will be charged under the reverse condition. If used continuously in this way, the cells should be inspected occasionally to see that they do not become unevenly discharged.

**Balancing Set.**—Two small dynamos, $A$, $B$, Fig. 12, with shafts continuous or rigidly coupled together, called a *balancing set*, or *balancer*, are often used to change from a two-wire to a three-wire system. Each armature is wound for the desired lamp voltage—one-half the voltage of the circuit across which the set is to be used—and the two are connected in series across the outside wires. The neutral wire is connected between the two armatures. Each armature acts at a generator when the lamps on its side require an excess of current, the other armature at the same time acting as a motor to drive the generator.

If the system is perfectly balanced, no current flows in the neutral wire and the set runs idle at a speed sufficient for each armature to generate a counter E. M. F. very nearly equal to one-half the E. M. F. across the outside wires. If the current required on one side $A$ exceeds that on the other $B$, the voltage on side $A$, without the effect of the balancer, would drop, while that on side $B$ would rise, their sum remaining at all times equal to the voltage across the outside wires. If the outside voltage is 220, as soon as the E. M. F. on side $A$ drops below, say, 109 volts, armature $A$ acts as a generator to maintain the voltage, while the rise of pressure on side $B$ makes armature $B$ continue to act as a motor, driving $A$ at nearly a constant speed. If each field of the balancer set has a few series turns so connected that when either machine is running as a generator its field is cumulatively compound-wound, and when running as a motor is differentially compound-wound, the voltage of the generator will be slightly increased, owing both to its increased field strength and to the increased speed of the set due to weakening the field of the motor. In this way, almost perfect balance of E. M. F. on the two sides can be maintained at any degree of unbalancing of the load. A trip coil in the neutral wire opens the main circuit-breaker if the current in the neutral wire becomes excessive.

Storage batteries and balancer sets can be placed near the lamps to be supplied. Only two wires, therefore, are necessary to lead from the station to a distant building or center from which a three-wire distribution may be fed.

**Dobrowolsky Three-Wire Dynamo.**—By mounting a pair of collector rings on the shaft of an ordinary direct-current dynamo, and connecting each ring to equipotential points (two for each pair of poles) on the commutator, or armature winding, an alternating E. M. F. will be impressed on the collector rings while the machine is operating; and if a choke coil is connected between the two rings, and the neutral

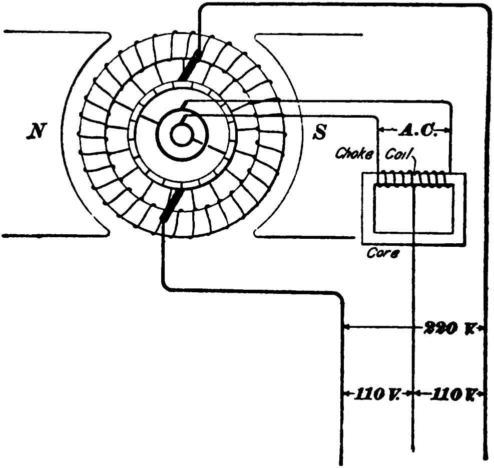



FIG. 13. THREE-WIRE DYNAMOS

wire of a three-wire system is connected to the middle point of the choke coil, the E. M. F.'s between the neutral wire and the two outside wires will always be approximately the same. Fig. 13 shows the connections. Owing to its self-induction, the choke coil will permit only a very small alternating current to flow between the two rings, but direct current readily passes to or from the neutral wire.

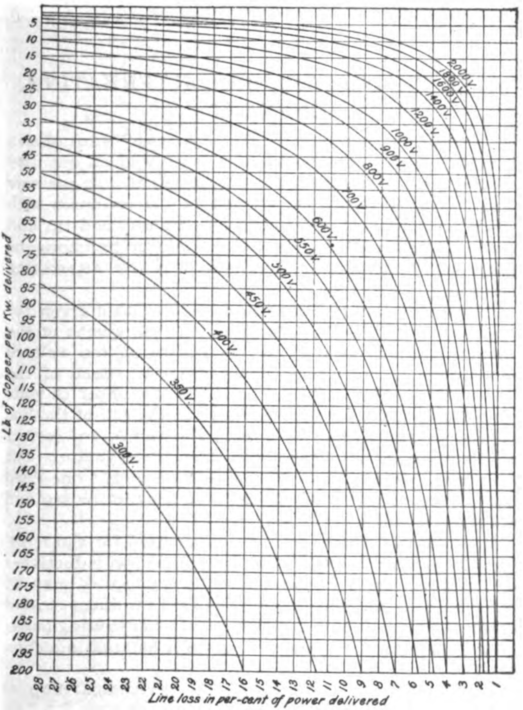



FIG. 14. WEIGHT OF COPPER FOR THREE-PHASE TRANSMISSION

Single dynamos with balancing sets or dynamos built on the Dobrowolsky plan are now generally installed for three-wire systems in preference to two dynamos.

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## LINE CONDUCTORS FOR ALTERNATING CURRENT

The size of the conductor for economical transmission depends on the strength of current; hence, a given quantity of energy can be transmitted much cheaper at high pressure than at low pressure. On account of the readiness with which alternating E. M. F.'s may be transformed up and down by means of step-up and step-down transformers, nearly all long-distance transmission is by alternating current.

To transmit a given quantity of energy with a given per cent. drop of potential and a given loss, the weight of the conductor varies inversely as the square of the voltage and directly as the square of the distance. Each of the curves in Fig. 14 is plotted for a different number of volts per mile; that is, total volts delivered at the end of the line divided by the number of miles. The curves are correct only for three-phase current with 100% power factor. For two-phase, single-phase, or continuous-current, one-third more copper is required; 5% has been allowed for sag and waste in weights.

If alternating current is to be transmitted for short distances, say 1 or 2 mi., and if the load is non-inductive—incandescent lamps for example—the conductors may be determined by the same methods as used for direct-current conductors; but for long distances or for loads causing power factors less than 1, the effect of capacity and self-induction must be considered.

In direct-current transmission, the percentage drop of pressure is the same as the percentage loss of power but in alternating-current transmission the percentage drop, when the power factor is less than 1, is greater than the percentage loss. This is because the current is greater than

## VALUES OF CONSTANTS $T$ AND $t$

| Kind of System | Power Factor (Approximate) | $T$ | $t$ |
|---|---|---|---|
| Single-phase, operating incandescent lamps only | .95 | 1.052 | 2,400 |
| Single-phase, operating incandescent lamps and motors | .85 | 1.176 | 3,000 |
| Single-phase, operating motors only | .80 | 1.250 | 3,380 |
| Two-phase, four-wire, operating incandescent lamps only | .95 | .526 | 1,200 |
| Two-phase, four-wire, operating incandescent lamps and motors | .85 | .588 | 1,500 |
| Two-phase, four-wire, operating motors only | .80 | .625 | 1,690 |
| Three-phase, three-wire, operating incandescent lamps only | .95 | .607 | 1,200 |
| Three-phase, three-wire, operating incandescent lamps and motors | .85 | .679 | 1,500 |
| Three-phase, three-wire, operating motors only | .80 | .725 | 1,690 |

proportional to the real watts by the ratio cos $\phi$ (the power factor); that is, the real watts are $EI$ cos $\phi$, while the apparent watts are $EI$.

If the power in watts delivered by the line is represented by $W$, the volts delivered by $E$, the distance in feet one way by $D$, and the per cent. of the delivered power lost in the line by $P$, then the following formulas give very nearly correct results:

$$\text{Current in amperes} = \frac{W}{E}T$$

$$\text{Area of conductor section in circular mils} = \frac{DW}{PE^2}t,$$

in which $P$ is always expressed as a whole number and $T$ and $t$ are constants, as given in the accompanying table.

For example, if it is desired to deliver 300 H. P. by means of 4,000-volt, three-phase current to motors 5 mi. away with a loss of 10% of the delivered power,

$$\text{current} = \frac{300 \times 746}{4{,}000} \times .725 = 40.6 \text{ amperes in each line wire,}$$

and

$$\text{circular mils} = \frac{5 \times 5{,}280 \times 300 \times 746}{10 \times 4{,}000 \times 4{,}000} \times 1{,}690 = 62{,}407,$$

or about No. 2 B. & S. wire.

**Estimation of Line Drop.**—The drop in a line with alternating current is greater than with the same strength of direct current by a factor $M$, the value of which depends on the frequency, the power factor, and the size of the line wire. If $P$ is the per cent. drop with direct current and $E$ the volts delivered, the approximate drop with alternating current is

$$e'_a = \frac{PE}{100}M$$

EXAMPLE.—600 K. W. of 3-phase, 60-cycle current is to be delivered at 6,000 volts to a mixed load of motors and lights 6 mi. from the power station with a loss of 10% of the delivered power. Calculate (*a*) the current in each line wire; (*b*) the size of line wires; and (*c*) the volts drop in the line.

SOLUTION.—$W = 600{,}000$ watts; $D = 6 \times 5{,}280 = 31{,}680$ ft.; $T = .679$; $t = 1{,}500$; $P = 10$; and $E = 6{,}000$.

**RATIO (M) OF ALTERNATING-CURRENT DROP TO DIRECT-CURRENT DROP**

| No. of Wire B. & S. | Area Circular Mils | 30 Cycles | | | 60 Cycles | | | 125 Cycles | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | Lamps Only | Motors and Lamps | Motors Only | Lamps Only | Motors and Lamps | Motors Only | Lamps Only | Motors and Lamps | Motors Only |
| 0000 | 211,600 | 1.26 | 1.27 | 1.24 | 1.64 | 1.85 | 1.85 | 2.44 | 3.06 | 3.14 |
| 000 | 167,805 | 1.20 | 1.17 | 1.14 | 1.49 | 1.63 | 1.62 | 2.15 | 2.62 | 2.67 |
| 00 | 133,079 | 1.15 | 1.08 | 1.05 | 1.39 | 1.46 | 1.42 | 1.92 | 2.25 | 2.29 |
| 0 | 105,534 | 1.10 | 1.00 | 1.00 | 1.30 | 1.32 | 1.28 | 1.73 | 1.96 | 1.99 |
| 1 | 83,694 | 1.06 | 1.00 | 1.00 | 1.23 | 1.21 | 1.16 | 1.57 | 1.74 | 1.73 |
| 2 | 66,373 | 1.03 | 1.00 | 1.00 | 1.16 | 1.11 | 1.06 | 1.44 | 1.54 | 1.53 |
| 3 | 52,634 | 1.02 | 1.00 | 1.00 | 1.11 | 1.04 | 1.00 | 1.35 | 1.38 | 1.38 |
| 4 | 41,742 | 1.00 | 1.00 | 1.00 | 1.07 | 1.00 | 1.00 | 1.26 | 1.26 | 1.22 |
| 5 | 33,102 | 1.00 | 1.00 | 1.00 | 1.04 | 1.00 | 1.00 | 1.19 | 1.16 | 1.11 |
| 6 | 26,250 | 1.00 | 1.00 | 1.00 | 1.02 | 1.00 | 1.00 | 1.14 | 1.08 | 1.03 |
| 7 | 20,816 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.09 | 1.01 | 1.00 |
| 8 | 16,509 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.06 | 1.00 | 1.00 |

(a) Current $=\frac{600,000}{6,000}\times .679=67.9$ amperes. Ans.

(b) Circular mils $=\frac{31,680\times 600,000}{10\times 6,000^2}\times 1,500=79,200$, or about No. 1 B. & S. wire. Ans.

(c) For No. 1 wire, 60 cycles, and a combined lamp and motor load, $M=1.21$; hence,

$$e_a=\frac{10\times 6,000}{100}\times 1.21=726 \text{ volts. Ans.}$$

The foregoing methods are not exact, but will usually give practical results.

**Polyphase Lines.**—A two-phase, four-wire transmission line, if so arranged that there is no inductive interaction between circuits, is equivalent, so far as loss and regulation are concerned, to two single-phase lines, each carrying one-half the total energy. The same rule applies to a three-phase, three-wire circuit with conductors symmetrically arranged, and is practically correct for a two-phase, three-wire circuit. Therefore, in laying out any of these systems, calculate first a single-phase circuit to carry one-half the load at the given voltage, and then use four such wires instead of two for a four-wire, two-phase circuit, and three wires instead of two for a three-wire, three-phase circuit, in all cases keeping the distance between wires the same as obtained for the single-phase circuit.

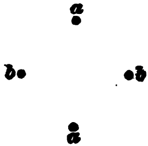


Fig. 15

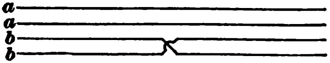


Fig. 16

To make the circuit of two-phase, four-wire lines inductively independent, the wires of the two circuits must be arranged either as shown in Fig. 15—that is, wires *a*, *a* of one phase on opposite ends of one diagonal of a square, and wires *b*, *b* of the other phase on the ends of the other diagonal—or the wires of one phase must be transposed at their middle point, as shown in Fig. 16, so that the mutual inductive effect of one half is neutralized by the opposite effect of

the other half. The three wires of a three-phase line are usually arranged in the form of an equilateral triangle, as in Fig. 17; they may, however, be arranged in one plane and transposed at one-third and at two-thirds of the transmission distance, as shown in Fig. 18. On a long line, the transpositions shown in Figs. 16 and 18 should be repeated between each two points where power is taken off.

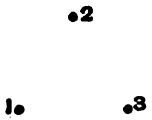


FIG. 17

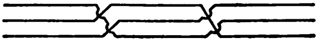

FIG. 18

If several triangularly arranged three-phase lines are adjacent to one another, one line can run straight through, provided those near it are transposed often enough to make the mutually inductive effects neutralize each other. If side by side, as in Fig. 19, where line *a* runs straight through, the next line *b* should be transposed twice, and line *c* eight times; the fourth line would be transposed twenty-six times, etc. The two circuits of a two-phase, three-wire transmission cannot be made inductively independent; the ar-

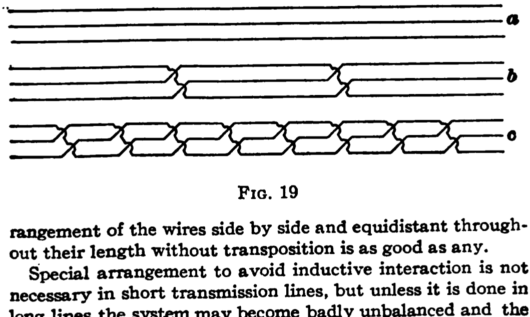


FIG. 19

rangement of the wires side by side and equidistant throughout their length without transposition is as good as any.

Special arrangement to avoid inductive interaction is not necessary in short transmission lines, but unless it is done in long lines the system may become badly unbalanced and the drop and the losses may differ widely from calculations.

# ELECTRIC LAMPS

## INCANDESCENT LAMPS

*Electric lamps* may be considered under two general heads: *incandescent lamps* and *arc lamps*. The light from each type is caused by heating a conductor to incandescence by the passage through it of an electric current. In an incandescent lamp, the conductor is solid and continuous whether the current is flowing or not; in an arc lamp, the conductor is gaseous and exists only while the current is flowing. There is no sharp dividing line between the two, and the classification is made only for convenience.

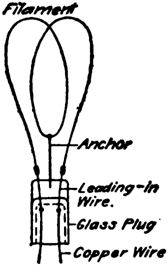


FIG. 1

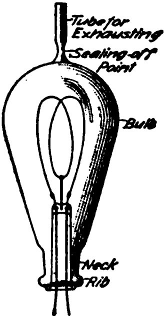


FIG. 2

The term *incandescent lamp* is ordinarily understood to apply to an exhausted glass vessel in the form of a bulb, globe, or tube containing the conductor, which glows when sufficient current is passed through it. This conductor is usually called the *lamp filament* (Fig. 1). Lamp filaments are made of some form of carbon or of some of the rare metals having very high melting points, such as tantalum, tungsten, and osmium.

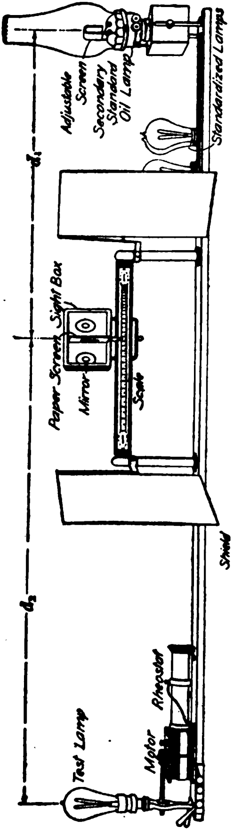


FIG. 3. DESHLER-MCALLISTER PHOTOMETER

The diameter and length of the filament depend on the voltage for which the lamp is made; the ends are joined to platinum leading-in wires that pass through the glass plug in the neck of the bulb. To the lower end of the leading-in wires are attached the copper wires for connection with the lamp base. The filament is then pushed in through the neck of the bulb, Fig. 2, the plug sealed into the neck, and the air pumped from the bulb through a glass tube, which is then sealed off, leaving the tip. Tipless bulbs are exhausted through a tube sealed into the plug in the neck.

**Rating.**—In English-speaking countries, it is the prevailing custom to rate all light sources in *candlepower* (abbreviated c. p.), this expression coming from the spermaceti candle formerly used in England as a standard light source. In the United States and Germany, the accepted standard is the *hefner*, the light produced by an amyl-acetate flame adjusted until its tip is 40 mm. above the top of the wick tube. One c. p. = [illegible] hefners. Another standard

in some use is the Methven screen, an argand burner of specified dimensions, before which is a screen having a slit that permits light equal to 2 British candles to pass. The Carcel lamp, a French standard, burns colza oil and gives off light equal to 9½ British candles.

**Light Measurement.**—The output from a light source is usually measured by comparing it to some other source that has been standardized by comparison with an original unit, such as the hefner amyl-acetate lamp. A *photometer* is a device for comparing the outputs from two light sources. The Deshler-McAllister photometer, Fig. 3, largely used in lighting stations, illustrates the principles of all. In this photometer, an oil lamp with an adjustable screen for the flame is used as a secondary standard in preference to a standardized incandescent lamp, so as to avoid the necessity of keeping the voltage accurately adjusted. A standardized lamp is first put in the place to be occupied by the test lamp, the sight box is moved along the scale until the pointer stands over the number on the scale indicating the candlepower marked on the standardized lamp, and the screen over the lamp flame is adjusted until the grease spot on the paper screen in the sight box throws exactly the same intensity of shadow on the mirrors located each side of the paper screen, showing that the light from the oil lamp is exactly the same as that from the standardized lamp. The standardized lamp is then removed and the test lamp put in place and rotated about 180 times a minute by a small motor, the voltage being adjusted to a constant value by means of a rheostat. Suitable shields keep the light from the operator's eyes while he adjusts the sight box along the scale. When the grease spot is balanced, as shown by the shadows on the two mirrors, the candlepower of the test lamp can be read directly from the scale. The standardized lamp is in use only for adjusting the oil lamp.

Other photometers use a standardized lamp in place of the oil lamp, and in some the scale is graduated to read distances from each of the two lamps instead of candlepower. *The intensity of illumination falling on any object from a concentrated source of light varies inversely as the square of the*

*distance of the object from the source.* If $C_1$ is the candlepower emitted by the standardized oil lamp and $d_1$ its distance from the paper screen, and $C_2$ the candlepower of the test lamp and $d_2$ its distance, then, when the same intensity of illumination falls on each side of the paper screen,

$$\frac{C_2}{C_1}=\frac{d_2^2}{d_1^2}$$

or,

$$C_2=C_1\frac{d_2^2}{d_1^2}$$

From this formula, the candlepower can be calculated or read from curves when $C_1$ $d_1$, and $d_2$ are known.

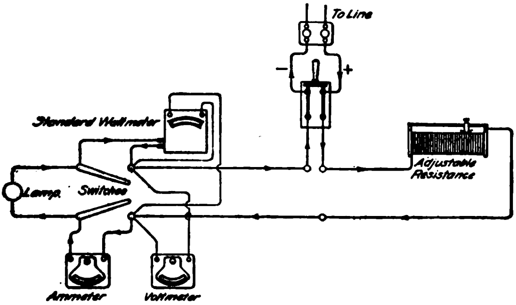


FIG. 4. CONNECTIONS FOR TESTING

The rapid rotation of the test lamp makes the measurement of its candlepower the mean of that thrown off in all directions in a horizontal plane; that is, the *mean horizontal candlepower* is measured. Unless otherwise stated, this is the measurement usually meant when referring to the candlepower of incandescent lamps. Other measurements sometimes referred to are *mean spherical candlepower*, the average of the intensities of light given off in all directions, and *mean hemispherical candlepower*, the average of the intensities in all directions in a hemisphere.

Fig. 4 shows connections for testing lamps. Accurate instruments should be used; and accurate records should be kept of the time the lamps burn and of their consumption of energy. The ammeter, or milliammeter, should read to $\frac{1}{1000}$ ampere and the voltmeter to $\frac{1}{10}$ volt. A wattmeter may also be used, but it is not necessary if the other instruments are accurate. Switches should be arranged to short-circuit the ammeter and the current coil of the wattmeter.

**Efficiency.**—According to the standardization rules adopted in 1907 by the American Institute of Electrical Engineers, *the efficiency of an incandescent lamp is the ratio of its mean spherical candlepower to the watts consumed.* Thus, an ordinary 16-c. p. lamp consuming 50 watts and giving off, say, 12 mean spherical c. p., has an efficiency of $\frac{12}{50}$ or .24 mean spherical c. p. per watt.

The efficiencies of electric lamps vary with the temperatures of the light-giving bodies and, to some extent, with the materials of which these bodies are made; that is, some materials radiate a larger proportion of the energy consumed by them as light than others. Carbon-lamp filaments are soon destroyed if operated at temperatures higher than 1,800° to 1,950° C.; metallized filaments will stand somewhat higher temperatures, tantalum filaments about 2,000° C., and tungsten filaments about 2,300° C.

Under ordinary conditions, 110-volt, carbon-filament lamps consume 3.1 to 3.5 watts per mean horizontal c. p., metallized-filament lamps about 2.5 watts, and metallic-filament lamps 1 to 2 watts. Lamps for very high or very low voltages are less efficient than indicated by the figures just given.

**Useful Life of Lamps.**—The *useful life* of an incandescent lamp is usually understood to mean the length of time it will burn before its light output decreases more than 20%. The length of useful life varies considerably, according to the degree of perfection of the lamp construction and the temperature at which it is operated. Increased voltage causes greatly increased candlepower, decreased watts per candlepower, and rapid deterioration, as shown by the curves in Fig. 5. For example, increasing the voltage of an

ordinary carbon-filament lamp to 101% of normal causes the candlepower to increase to 106% and the life to decrease to 80% of normal; if the voltage is decreased 1%, the candlepower decreases 6% and the life increases 23%. It is therefore very important that the voltage be kept constant.

Metallic-filament lamps are not so sensitive to changes in the voltage, because the resistance of the filaments increases as the temperature rises, creating a tendency to oppose excessive increase of current with increased voltage.

The useful life of carbon-filament lamps is from 500 to 700 hr. Metallized-filament lamps are good for 500 to 1,000

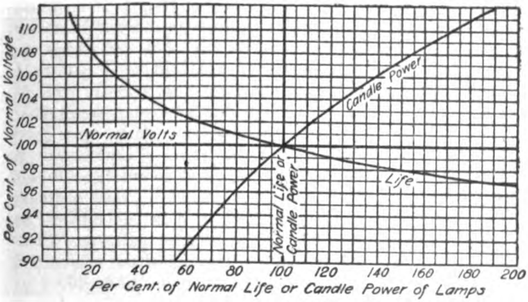


FIG. 5. LIFE, CANDLEPOWER, AND VOLTAGE

hr., and some of the metallic-filament lamps are also sold with a guaranteed useful life of 1,000 hr.

**Current Required.**—If the watts per candlepower $w$, the candlepower rating $L$, and the voltage $E$ of electric lamps are known, the current required by each lamp can be determined by the formula

$$\text{current} = \frac{wL}{E}$$

For example, a 16-c. p., 3.1-watt, 110-volt lamp requires $\frac{3.1 \times 16}{110} = .45$ ampere, approximately.

**Units of Illumination.**—*Illumination* is utilized light. Several units of illumination are in use, among them being the *lux*, which is the illumination produced by a source of 1 hefner of light on a normal surface at a distance of 1 meter from the source, and the *candle-foot* (abbreviated c.-ft.), the unit most used in English-speaking countries, which is the illumination produced by a standard candle on a normal surface 1 ft. away.

**Intensity of Illumination.**—The *intensity of illumination* in candle-feet at any point is the quotient of the candle-power given off by the source of light in the direction of the point divided by the square of the distance in feet from the source to the point. If a lamp gives off 16 c. p., the illumination 1 ft. away is 16 c.-ft.; 2 ft. away, 4 c.-ft.; 4 ft. away, 1 c.-ft., etc. For mild general illumination, by which the objects in a room can be readily distinguished, .5 c.-ft. i sufficient. For reading, 1 c.-ft. will do, but 2 c.-ft. is much better; more than 3 c.-ft. is too brilliant, unless very thoroughly diffused. The illumination in candle-feet from a few common sources of light is about as follows: From ordinary moonlight, .025; in a street lighted by gas, .03; on the stage of a theater, 2.9 to 3.8; brilliantly lighted drafting rooms, 8 to 12; and diffused daylight, 10 to 40.

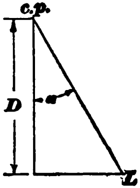

FIG. 6

**Calculating Quantity of Light.**—In calculating the quantity of light to install in a room, an estimate must first be made of the average number of candle-feet required in the plane where the light is to be used; then, the candlepower required in a given direction can be calculated from the formula

$$\text{c. p.} = \frac{LD^2}{K \cos^3 \alpha},$$

in which c. p. is the candlepower given off by the source in the desired direction, $L$ the estimated number of candle-feet, $D$ the perpendicular distance of the source from the plane, $\alpha$ the angle between the perpendicular and the direction of the light rays to the point (see Fig. 6), and $K$ the coefficient

of reflection from the walls, ceiling, and surrounding objects. Values of $K$ are approximately as follows:

| | |
|---|---|
| All reflecting surfaces black........ | $K=1$ |
| Dark-brown paper and dull ceiling.. | $K=1.1$ to 1.25 |
| Blue-tinted paper and ceiling...... | $K=1.25$ to 1.4 |
| Plain white plastered walls and ceiling or pink- or yellow-tinted walls and ceiling.................... | $K=1.5$ to 1.8 |
| Very light-colored wallpaper....... | $K=1.75$ to 1.9 |
| Walls covered with white reflecting paint.......................... | $K=2.5$ to 5. |
| Mirrors and polished metal......... | $K=5$ to 20. |

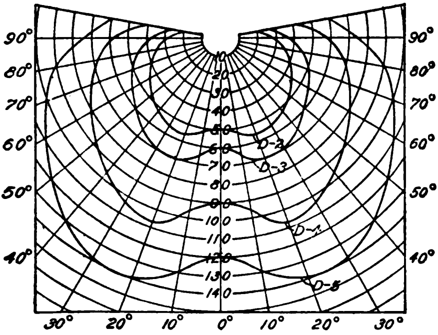


Fig. 7. Light-Distribution Curves

For example, suppose a room with white plastered walls and ceiling is to be lighted so that the illumination on tables shall not be less than 2 c.-ft. at any point, and suppose that Gem high-efficiency lamps are to be used. Since an even distribution is desired, the form D reflectors will be preferable to the form C, which concentrate the light more in a downward direction. The curves in Fig. 7 show the distribution with form D reflectors. If the lamps are hung

10 ft. above the tables and 16 ft. apart each way, the point in the plane of the table tops most distant from any lamp is the point directly under the center of a square (Fig. 8) outlined by four lamps. The triangle to be considered, Fig. 9, has an altitude of 10 ft. and a base of 11.3 ft.; $\cos a = \frac{10}{\sqrt{10^2+11.3^2}} = .663$, $\cos^3 a = .291$, and $a =$ about 48° 30′. Since the point under consideration receives light from four lamps, it should receive at least .5 c.-ft. from each lamp. Assume that $K = 1.65$, then

$$\text{c. p.} = \frac{.5 \times 10^2}{1.65 \times .291} = 104$$

The D-5 Gem lamp gives off about 120 c. p. at $a = 48° 30$ (see distribution curve, Fig. 7), and the next smaller size D-4

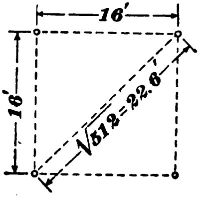


Fig. 8

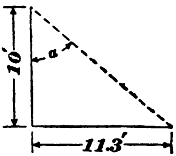


Fig. 9

only 90 c. p.; hence, the D-5 size would be used, and the point considered would receive $\frac{120}{104} \times .5 \times 4 = 2.31$ c.-ft.

A point under the middle of a side of the square receives light principally from two lamps. The triangle to be considered has an altitude of 10 ft. and a base of 8 ft.; $\cos a = \frac{10}{\sqrt{10^2+8^2}} = .78$, $\cos^3 a = .475$, and $a = 39°$. At 39° from the vertical, the D-5 lamp gives off about 131 c. p. and from each lamp the illumination is

$$L = \frac{1.65 \times .475 \times 131}{10^2} = 1.03 \text{ c.-ft.},$$

or 2.06 c. ft. from the two lamps.

Directly under each lamp, the illumination is $\frac{120 \times 1.65}{10^2} = 1.98$ c.-ft.; but enough light will reach this point from neighboring lamps to make the illumination slightly exceed the requirements.

## THE NERNST LAMP

In the *Nernst lamp*, one or more pencils of highly refractory oxide of a rare metal, such as thorium, are heated to incandescence in air by the passage of an electric current. The essential parts of the Nernst lamp are the *glowers*, or light-giving portion; the *heaters*, for raising the temperature of the glowers until they become conductors; the *ballast*, a resistance that increases rapidly as its temperature increases; and the *cut-out device*, for opening the circuit through the heaters after the glowers have reached the proper temperature. Fig. 10 shows some of the parts of a one-glower Nernst lamp.

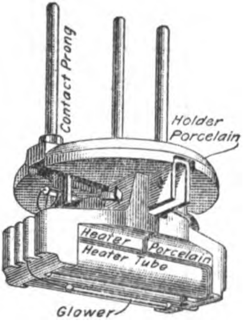


FIG. 10

The glowers are made by forcing through suitable dies a dough made of the oxide employed. The dough issues from the dies in pencils about $\frac{1}{32}$ in. in diameter. These pencils are dried, cut into suitable lengths (about 1 in.), and baked, after which terminal wires are soldered into platinum beads embedded in the ends of the pencils, or glowers.

The heaters consist of two or more porcelain tubes, each about $\frac{1}{4}$ in. in diameter, wound with fine platinum wire and coated with a white refractory material that assists in reflecting the light downwards.

The ballast consists of iron wire in glass tubes, from which the air is exhausted and nitrogen or other inert gas

substituted. Each lamp contains a ballast tube in series with each glower.

The resistance of the glowers at ordinary temperatures is very high, but falls rapidly as their temperature increases. At 600° to 700° C. they become good conductors, and if subjected to constant E. M. F., the increasing current through the glowers will rapidly bring them to a very high temperature. The resistance of the ballast is low at ordinary

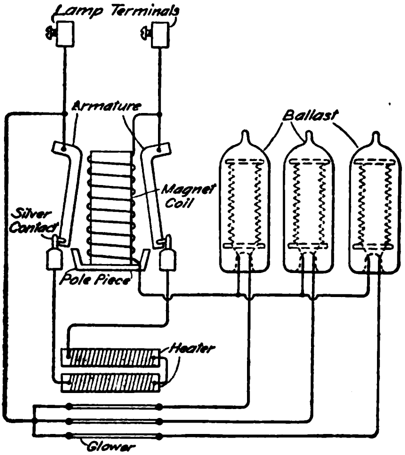


FIG. 11. CONNECTIONS OF A THREE-GLOWER LAMP

temperatures, but increases rapidly as the wire becomes heated. A ballast tube and a glower being in series, a condition is soon reached where further reduction of the resistance of the glowers is balanced by an increase in the resistance of the ballast, and the current becomes constant. Fig. 11 shows the connections and relative location of the parts of a three-glower Nernst lamp.

The cut-out device consists of an electromagnet in series with the ballasts and glowers, with hinged armatures so adjusted that when the glower current has reached a predetermined strength, the armatures are drawn in toward the magnet pole pieces, breaking the circuit through the heaters, at silver contact points.

Nernst lamps are made in one-, two-, three-, four-, and six-glower sizes, giving mean hemispherical candlepowers of about 35, 75, 125, 190, and 300, respectively, at specific consumptions of about 2.4, 2.2, 2.1, 1.85, and 1.75 watts per c. p. On account of the reflecting surfaces just above the glowers, nearly all the light is distributed in the lower hemisphere.

## ARC LAMPS

An *arc lamp* consists of one or more pairs of electrodes and a system of magnets and springs by which, when electric current is applied, an arc is drawn and maintained between the electrodes, which usually consist of carbon. Constant-potential arc lamps are used in multiple, each lamp independent of any other; constant-current lamps are used in series, the same current flowing through all.

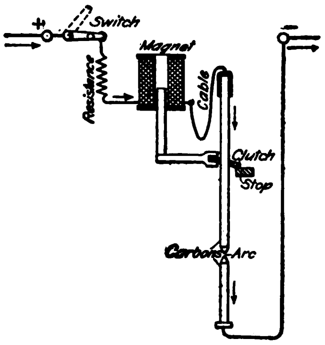

FIG. 12. CONSTANT-POTENTIAL LAMP

**Constant-Potential Lamps.** — In a *constant-potential arc lamp*, Fig. 12, one electromagnet, connected in series with the arc, suffices. A resistance also in series with the arc steadies the current

and consumes excess voltage not needed at the arc. In an alternating-current, constant-potential lamp, an inductive resistance is used.

**Differential Lamps.**—In a *constant-current, differential lamp*, Fig. 13, one magnet winding is used in series with the arc and one in shunt with it. The two windings may be combined in one electromagnet. Since the current is constant, the series winding alone would cause a constant pull, tending to hold the carbons apart; but the shunt winding is so connected as to cause an opposing effect, and since this effect gradually increases as the length of the arc increases, the common magnet core is finally drawn down until the carbons move closer together, thus decreasing the arc voltage. In both styles of these lamps, the carbons remain in contact while the lamp is idle, and are drawn apart by the series coil when the lamp is started.

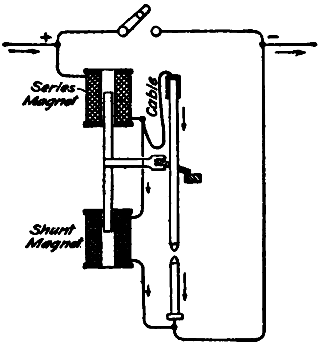


FIG. 13. DIFFERENTIAL LAMP

**Shunt Lamp.**—In a *constant-current shunt lamp*, Fig. 14, the carbons are normally held apart by the action of a spring, and, when the current is switched on, are drawn together by the coarse winding on the starting and regulating magnet. A small starting magnet in series with the arc then opens a pair of contacts in the circuit through the coarse winding of the larger magnet, thus permitting the spring to draw the carbons apart and start the arc. The shunt winding of the large magnet then acts in opposition to the spring, and the coarse winding remains out of circuit. No resistance or choke coil is needed in this lamp.

**ARC-LAMP DATA**

*(Candlepower and Power Consumption)*

| Type of Lamp | Amperes | Volts at Lamp Terminal | Total Watts | Watts Lost in Resistance or Choke Coil | Watts at Arc | Mean Spherical Candlepower | Total Watts per Mean Spherical Candlepower | Watts at Arc per Mean Spherical Candlepower |
|---|---|---|---|---|---|---|---|---|
| Open-arc, series, directc-urrent | 9.6 | 50 | 460–480 | | 450 | 375 | 1.3 | 1.2 |
| Enclosed-arc, direct-current, opalescent inner globe, no outer globe | 4.9 | 110 | 539 | 147 | 392 | 223 | 2.4 | 1.8 |
| Same, with opalescent inner globe and clear outer globe | 4.9 | 110 | 539 | 147 | 392 | 181 | 2.9 | 2.1 |
| Same, with opalescent inner and outer globes | 4.9 | 110 | 539 | 147 | 392 | 155 | 3.5 | 2.5 |
| Enclosed-arc, alternating-current, opalescent inner and clear outer globes | | 110 | 416 | 74 | 342 | 140 | 2.9 | 2.4 |
| Same with opalescent inner and outer globes | | 110 | 416 | 74 | 342 | 114 | 3.6 | 3. |

## ENCLOSED-ARC LAMP DATA

| | Alternating-Current Lamps | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | Series Lamps Constant Current | | | | | | Multiple Lamps Constant Potential | | | |
| Current, amperes.. | 6.6 | 7 | 7.5 | 6.6 | 7 | 7.5 | 4 | 6 | 4 | 6 |
| Terminal volts.... | 78 | 77 | 76 | 83 | 80 | 79 | 100 | 104 | 100 | 104 |
| Arc volts......... | 72 | 72 | 72 | 72 | 72 | 72 | 70 | 74 | 72 | 75 |
| Terminal watts.... | 435 | 465 | 490 | 430 | 455 | 485 | 260 | 405 | 270 | 405 |
| Arc watts......... | 395 | 420 | 450 | 385 | 410 | 440 | 227 | 365 | 230 | 360 |
| Watts lost........ | 40 | 40 | 40 | 45 | 45 | 45 | 33 | 40 | 40 | 45 |
| Power factor...... | .846 | .865 | .861 | .785 | .813 | .819 | .650 | .649 | .675 | .649 |
| Ratio $\frac{\text{arc watts}}{\text{terminal watts}}$ . | .908 | .905 | .918 | .895 | .901 | .908 | .873 | .901 | .852 | .889 |

**ENCLOSED-ARC LAMP DATA—(*Continued*)**

| | Direct-Current Lamps | | | | | |
|---|---|---|---|---|---|---|
| | Series Lamps Constant Current | Multiple Lamps Constant Potential | | | Multiple Series Lamps | |
| | | | | | 2 in Series on 220 Volts | 5 in Series on 550 Volts |
| Current, amperes | 6.8 (standard) | 3.5 | 5 | 2.5 | 5 | 5 |
| Terminal volts | 72 | 110 | 110 | 220 | 110 | 110 |
| Arc volts | 70 | 80 | 80 | 145 | 80 | 80 |
| Terminal watts | 489 | 385 | 550 | 550 | 550 | 550 |
| Arc watts | 469 | 280 | 400 | 363 | 472 | 472 |
| Watts lost | 20 | 105 | 150 | 177 | 78 | 78 |
| Ratio $\frac{\text{arc watts}}{\text{terminal watts}}$ | .959 | .728 | .727 | .66 | .858 | .858 |

NOTE.—Series alternating-current lamps may be adapted to currents from 4 to 7 amperes and frequencies from 40 to 140 cycles by changing the magnet windings. Series direct-current lamps can be adapted to currents from 4 to 10 amperes by changing windings.

**Open- and Enclosed-Arc Lamp.**—In the *open-arc lamp*, there is free access of air to the arc. This type, with pure carbons, is almost entirely superseded by the *enclosed-arc lamp*, in which a small enclosing globe around the arc permits only a very small quantity of air to enter. An open-arc lamp burns only from 12 to 18 hr. with one pair of carbons; an enclosed-arc lamp burns from 100 to 150 hr. with one trimming.

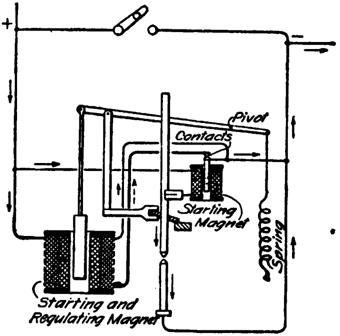


FIG. 14. SHUNT LAMP

## FLAMING-ARC LAMPS

If an arc between ordinary carbons is drawn longer than about $\frac{1}{8}$ in. when open or $\frac{3}{8}$ in. when enclosed, additional energy is required to maintain the arc and very little additional light is given off. However, if the carbons are impregnated with mineral salts, such as calcium or magnesium salt, the mineral is evaporated by the heat of the arc and makes the otherwise non-luminous flame of a long arc intensely brilliant. With the impregnated carbons, the arc can be drawn from $\frac{3}{4}$ in. to $2\frac{1}{2}$ in. long. The burning of such carbons is accompanied by the production of noxious fumes, considerable

ash, and particles of slag, or scoria, so that flaming, or luminous, arcs cannot be so completely enclosed as arcs between plain carbon electrodes.

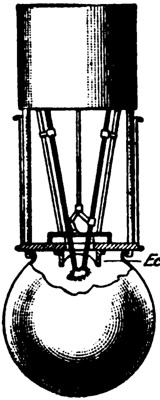


FIG. 15
FLAMING-ARC LAMP

*Flaming-arc lamps* are usually arranged so that both carbons feed downwards, as shown in Fig. 15. A magnetic field across the path of the arc causes it to bow downwards from the electrodes. In this way not only is light obtained from the flame, but also all the light from the intensely hot carbon tips is made useful. The arc is usually partly surrounded by a highly refractory chamber, called the economizer, which serves to shield the arc from drafts of air and also acts as a reflector. On account of the free access of air to flaming arcs, a set of carbons last only from 12 to 18 hr. The expense of the carbons and the frequency with which they must be renewed make the cost of maintenance high. Compared with an enclosed-arc lamp with plain carbons, the candlepower and economy of an "Excello" flaming-arc lamp are as follows:

| | Flaming Arc | Enclosed Arc |
|---|---|---|
| Mean amperes | 8 | 5.1 |
| Mean volts at the arc | 45 | 81. |
| Mean watts at the arc | 360 | 413. |
| Mean spherical candlepower | 1,020 | 282. |
| Mean hemispherical candlepower | 1,560 | 260. |
| Watts per mean spherical candlepower | .353 | 1.78 |
| Watts per mean hemispherical candlepower | .231 | 1.59 |

**Magnetite Luminous-Arc Lamp.**—The *magnetite luminous-arc lamp* made by the General Electric Company, has a positive electrode consisting of a copper bar large enough to be practically unaffected by the heat of the arc, and a negative electrode consisting of a tube of finely divided magnetite, or black oxide of iron, in which are mixed small quantities of salts of chromium, titanium, etc. These lamps are used only with direct current. Each negative electrode lasts from 150 to 200 hr.; a positive electrode lasts about 4,000 hr. The constant-current lamps consume about 320 watts and give off 400 spherical c. p.; that is, the specific consumption is .8 watt per c. p. The distribution is mostly in the lower hemisphere.

**Metallic Flame-Arc Lamp.**—The *Westinghouse metallic flame-arc lamp* also has a solid metal positive electrode and a tube filled with a mixture of powdered metallic substances for a negative electrode.

## TUBE LIGHTING

**Mercury-Vapor Tube Lamp.**—In the *mercury-vapor tube lamp*, the source of light is an arc through a tube containing vapor of mercury, which acts as a conductor and is heated to incandescence by the current. The tube is hung so that it remains normally in an inclined position (see Fig. 16), the lower, or cathode, end, containing liquid mercury. The anode is usually a piece of iron. Above the lamp is a large canopy containing resistances, inductances, and ballast to regulate the current. To start the lamp, the current is first switched on and the lamp is then tilted, either by hand or, in some types, automatically, until the mercury runs along the tube and comes in contact with the anode, thus forming a metallic conducting path, which breaks when the tube goes back to its normal position, thus starting the arc. The heat vaporizes enough of the mercury to keep the tube filled. The light given off is of a greenish cast, with an entire absence of red rays. This light is therefore useless where color selection must be made, but is a very easy, comfortable, and economical light for reading, writing, drawing, handling bales and packages of goods, machine-shop work

etc. The tube is 1 in. in diameter, and the standard lamps have light-giving portions 17½ in., 20¾ in., 28 in., and 45 in. long. The 17½-in. and 20¾-in. tubes each give off 300 c. p. at .64 watt per c. p., and the 45-in. tube gives 700 c. p. at .55 watt per c. p. The 28-in. tube is for alternating current only, and gives 425 c. p. at .64 watt per c. p.

**The Moore Light.**—The *Moore electric light* is a system of artificial lighting in which the source of light is the rarefied non-metallic gaseous contents of long glass tubes, 1¾ in.

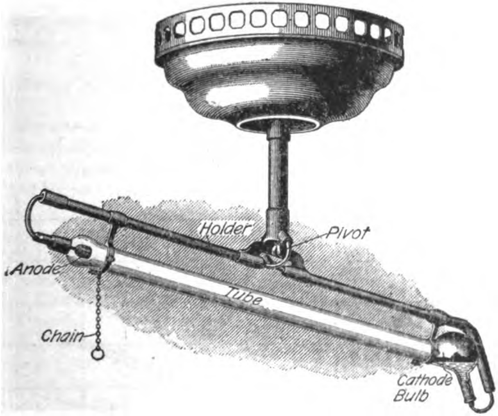


FIG. 16. MERCURY-VAPOR TUBE LAMP

in diameter, made luminous by the passage of an electric current. Only alternating current is used, because of the difficulty of raising direct current to the high pressures required. Alternating current at the ordinary pressure used for house lighting is led through the primary coil of a small step-up transformer enclosed in an iron case, in which the ends of the tube also terminate (see Fig. 17). The secondary coil of the transformer terminates in the electrodes

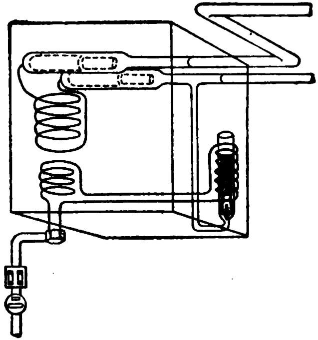

FIG. 17. MOORE LIGHT

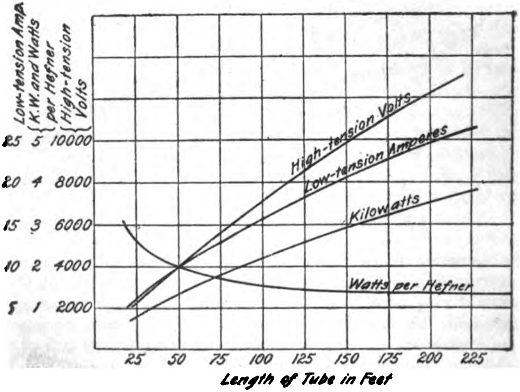


FIG. 18. DATA CURVES OF MOORE LIGHT

in the ends of the tube. In series with the primary coil is an electromagnet, which operates a small valve to admit a very minute portion of air or other gas when the vacuum in the tube becomes too high. If the contents of the tube are rarefied air, the light is orange-colored. If the tube is fed with pure nitrogen, the light is yellow, and if with pure carbon dioxide, the light is white.

The light from a Moore tube is very mild and easy on the eyes. The intrinsic brilliancy can be made anything from almost negligible up to 30 or more hefners per ft. In hefners per sq. in., the comparison with other light sources is as follows:

| | | |
|---|---|---|
| Moore light | at 6 hefners per ft. | 0.33 |
| | at 12 hefners per ft. | 0.66 |
| | at 36 hefners per ft. | 2.00 |
| Cooper Hewitt mercury tube | | 19.00 |
| Incandescent filaments | | 250.00 |
| Nernst glower | | 600.00 |
| Arc crater | | 10,000.00 |

The cost of installation is slightly less than that of a first class system of wiring for incandescent lamps together with the necessary fixtures. The life of one of the tubes, except for breakage, is almost unlimited. In economy, the Moore light ranks close with flaming-arc lamps and mercury-vapor tube lamps. Fig. 18 is a group of data curves of a Moore light, showing the variation of high-tension volts, amperes input at 200 volts, kilowatts input, and watts per hefner with length of tube.

## INTERIOR WIRING

**Underwriters' Rules.**—In order to have buildings insurable, all interior wiring must be installed according to the *National Electrical Code*, a set of rules and requirements adopted by the National Board of Fire Underwriters. Copies of this code and of the *list of approved fittings* may be obtained from any local Inspection Bureau or from the National Board of Fire Underwriters, New York or Chicago. The following are some of the requirements.

# WIRES

## CARRYING CAPACITY OF INSULATED WIRES

No wires smaller than No. 14 B. & S. can be used, except in fixtures or in flexible cords. The maximum permissible carrying capacities of various sizes of wires and cables are given in the accompanying tables. Wires selected with a view of keeping the voltage drop within practical limits are usually larger than required by the tables, but in no case must they

### WIRES FOR MARINE WORK

*(National Code Standard)*

| B. & S. Gauge Number | Area Actual Circular Mils | Number of Strands | Size of Strands B. & S. Gauge | Carrying Capacity Amperes |
|---|---|---|---|---|
| 19 | 1,288 | | | |
| 18 | 1,624 | | | 3 |
| 17 | 2,048 | | | |
| 16 | 2,583 | | | 6 |
| 15 | 3,257 | | | |
| 14 | 4,107 | | | 12 |
| 12 | 6,530 | | | 17 |
| Cables | 9,016 | 7 | 19 | 21 |
| | 11,368 | 7 | 18 | 25 |
| | 14,336 | 7 | 17 | 30 |
| | 18,081 | 7 | 16 | 35 |
| | 22,799 | 7 | 15 | 40 |
| | 30,856 | 19 | 18 | 50 |
| | 38,912 | 19 | 17 | 60 |
| | 49,077 | 19 | 16 | 70 |
| | 60,088 | 37 | 18 | 85 |
| | 75,776 | 37 | 17 | 100 |
| | 99,064 | 61 | 18 | 120 |
| | 124,928 | 61 | 17 | 145 |
| | 157,563 | 61 | 16 | 170 |
| | 198,677 | 61 | 15 | 200 |
| | 250,527 | 61 | 14 | 235 |
| | 296,387 | 91 | 15 | 270 |
| | 373,737 | 91 | 14 | 320 |
| | 413,639 | 127 | 15 | 340 |

## WIRES FOR INTERIORS OF BUILDINGS

*(National Code Standard)*

| B. & S. Gauge Number | Circular Mils | Carrying Capacity | |
|---|---|---|---|
| | | Rubber-Covered Wires Amperes | Weather-Proof Wires Amperes |
| 18 | 1,624 | 3 | 5 |
| 16 | 2,583 | 6 | 8 |
| 14 | 4,107 | 12 | 16 |
| 12 | 6,530 | 17 | 23 |
| 10 | 10,380 | 24 | 32 |
| 8 | 16,510 | 33 | 46 |
| 6 | 26,250 | 46 | 65 |
| 5 | 33,100 | 54 | 77 |
| 4 | 41,740 | 65 | 92 |
| 3 | 52,630 | 76 | 110 |
| 2 | 66,370 | 90 | 131 |
| 1 | 83,690 | 107 | 156 |
| 0 | 105,500 | 127 | 185 |
| 00 | 133,100 | 150 | 220 |
| 000 | 167,800 | 177 | 262 |
| 0000 | 211,600 | 210 | 312 |
| Cables | 200,000 | 200 | 300 |
| | 300,000 | 270 | 400 |
| | 400,000 | 330 | 500 |
| | 500,000 | 390 | 590 |
| | 600,000 | 450 | 680 |
| | 700,000 | 500 | 760 |
| | 800,000 | 550 | 840 |
| | 900,000 | 600 | 920 |
| | 1,000,000 | 650 | 1,000 |
| | 1,100,000 | 690 | 1,080 |
| | 1,200,000 | 730 | 1,150 |
| | 1,300,000 | 770 | 1,220 |
| | 1,400,000 | 810 | 1,290 |
| | 1,500,000 | 850 | 1,360 |
| | 1,600,000 | 890 | 1,430 |
| | 1,700,000 | 930 | 1,490 |
| | 1,800,000 | 970 | 1,550 |
| | 1,900,000 | 1,010 | 1,610 |
| | 2,000,000 | 1,050 | 1,670 |

be smaller. Since rubber deteriorates if constantly heated, the carrying capacity of rubber-covered wire is slightly lower than that of weather-proof wire.

**Joints.**—There should be as few *joints*, or splices (Fig. 1), in the wires as possible. The insulation should be removed from the ends to be joined without cutting into the wire; the ends should be scraped until clean and bright, then twisted together until the joint is mechanically and electrically secure, after which it should be well soldered, and finally wrapped with insulating tape until the insulation of the joint is equal to that of the wires joined. The separate strands of stranded wires must be securely soldered together before fastening the wire under terminal clamps. All stranded wires with conductivity greater than that of No. 8 B. & S. gauge must be soldered into lugs for all terminal connections.

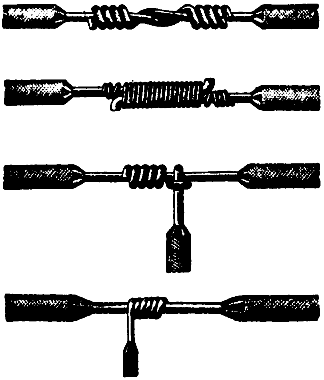

FIG. 1. WIRE JOINTS

The Code rules suggest the following formula for soldering fluid:

| | PARTS |
|---|---|
| Saturated solution of zinc chloride | 5 |
| Alcohol | 4 |
| Glycerine | 1 |

**Miscellaneous Rules.**—Unless wires are run in conduit or molding, they must be separated from all other objects by glass or porcelain insulators. Tie wires must have insulation equal to that of the wires tied. Wires must not be laid in plaster or cement, and twin wires are permitted

only in conduit. For open work in dry places, *slow-burning weather-proof wire* may be used; for all concealed work and in all damp places, the wires must be *rubber-covered*. Rubber-covered wires with an extra-heavy outside covering of tape or braid may be used in unlined metal conduit; if the conduit is lined, the outside covering on the wires is not required. Fixture wire must not be smaller than No. 18 B. & S. and must be rubber-covered.

*Flexible cord* must not be used where the E. M. F. exceeds 300 volts, nor to support clusters, nor in show windows. The cord must be protected by a bushing at the point where it enters a lamp socket. If used to support the weight of a lamp, a knot must be tied in the cord below the bushing and another above the ceiling rosette, so as to reduce the pull on the connecting screws.

## CALCULATING SIZE OF WIRE

The feeder wires entering a building are usually led to one or more *centers of distribution*, from which the distribution, or branch, circuits go to the consuming devices. In wiring for incandescent lamps, the drop from the point where the feeders enter the building to the lamps should be not over 3 to 5% of the lamp voltage. This total drop can be divided between the feeders to the centers of distribution and the distribution circuits in the proportion of say 3 and 2; that is, 3% drop in the feeders and 2% drop in the distribution circuits.

Before calculating the size of wire, it is necessary to know the current $I$ in amperes to be carried, the average distance $D$ in feet one way, and the drop $e$ in volts. So many varieties of incandescent lamps are now obtainable operating at such different economies, that it is best to calculate the current $I$ by the formula

$$I = \frac{NwC}{E},$$

in which $N$ is the number of lamps, $w$ the watts per candlepower, $C$ the candlepower per lamp, and $E$ the voltage of the lamps. The product $wC$, the watts per lamp, is often given instead of the separate factors. The distance should

## CURRENT REQUIRED BY MOTORS

| H. P. | Direct-Current Motors | | | Alternating-Current Motors | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | Single-Phase | | | Two-Phase (4 Wire) | | | Three-Phase (3 Wire) | | |
| | 110 V | 220 V | 500 V | 110 V | 220 V | 500 V | 110 V | 220 V | 500 V | 110 V | 220 V | 500 V |
| 1 | 9 | 4.5 | 2.0 | 14 | 7 | 3.1 | 6.4 | 3.2 | 1.4 | 7.4 | 3.7 | 1.6 |
| 2 | 17 | 8.5 | 3.7 | 24 | 12 | 5.3 | 11 | 5.7 | 2.5 | 13 | 6.6 | 2.9 |
| 3 | 26 | 13 | 5.6 | 34 | 17 | 7.5 | 16 | 8.1 | 3.5 | 19 | 9.3 | 4.1 |
| 5 | 40 | 20 | 8.8 | 52 | 26 | 11 | 26 | 13 | 5.5 | 30 | 15 | 6.4 |
| 7½ | 60 | 30 | 13 | 74 | 37 | 16 | 38 | 19 | 8.1 | 44 | 22 | 9.3 |
| 10 | 76 | 38 | 17 | 94 | 47 | 21 | 44 | 22 | 10 | 50 | 25 | 12 |
| 15 | 112 | 56 | 25 | | | | 66 | 33 | 15 | 76 | 38 | 17 |
| 20 | 150 | 75 | 33 | | | | 88 | 44 | 19 | 102 | 51 | 22 |
| 30 | 226 | 113 | 50 | | | | 134 | 67 | 29 | 154 | 77 | 33 |
| 40 | 302 | 151 | 66 | | | | 178 | 89 | 39 | 204 | 107 | 45 |
| 50 | 368 | 184 | 81 | | | | 204 | 102 | 45 | 236 | 118 | 52 |
| 75 | 552 | 276 | 122 | | | | 308 | 154 | 68 | 356 | 178 | 77 |
| 100 | 736 | 368 | 162 | | | | 408 | 204 | 90 | 472 | 236 | 104 |
| 150 | 1,110 | 555 | 244 | | | | 616 | 308 | 135 | 710 | 355 | 156 |
| 200 | 1,474 | 737 | 324 | | | | 818 | 409 | 180 | 940 | 470 | 208 |

be an average of the distances to the various lamps on the circuit; for example, if a circuit is 150 ft. long with lamps distributed evenly over the last 50 ft., the distance $D$ for which the wire should be calculated should be $100+\frac{50}{2}=125$ ft. The drop $e$ in volts is obtained by multiplying the lamp voltage by the per cent. drop to be allowed.

## CURRENT IN AMPERES REQUIRED BY MOTORS

The accompanying table gives the approximate current required by motors. These figures are close enough for wiring calculations, though the exact current in any case depends on the efficiency of the motor and, also, in case of alternating-current motors, on its power factor.

Knowing the distance $D$, the current $I$, and the drop $e$, the size of wire in circular mils (c. m.) is obtained from the formula

$$\text{c. m.} = \frac{21.6DI}{e}$$

Example.—Calculate the size of wire necessary to supply current to eighty-four 22-c.-p., 2-watt, 110-volt lamps distributed evenly along 100 ft. of the circuit, the nearest lamp being 50 ft. from the center of distribution; permissible drop 2%.

Solution.—The distance $D = 50+\frac{100}{2} = 100$ ft.; the current $I = \frac{84\times22\times2}{110} = 33.6$ amperes; and the drop $e = .02\times110 = 2.2$ volts.

$$\text{c. m.} = \frac{21.6\times100\times33.6}{2.2} = 33{,}000,$$

or about No. 5 B. & S. wire. Ans.

If, in the foregoing example, all the current traveled the whole length of the line, 150 ft., the size of the wire for 2% drop would be

$$\text{c. m.} = \frac{21.6\times150\times33.6}{2.2} = 49{,}500,$$

requiring the use of No. 3 B. & S. wire.

## WIRING TABLE

To use the accompanying wiring table, divide the number of amperes to be transmitted by the number of volts drop permissible and find the quotient thus obtained, or the number

# WIRING TABLE

| Size of Wire, B. & S. Gauge Number | Distance in Feet (One Way) Producing a Drop of 1 Volt for Given Currents and Given Sizes of Wire | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Amperes | 1 | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 | 50 | 60 | 70 | 80 | 90 | 100 |
| 18 | 75.2 | 15.0 | | | | | | | | | | | | | | |
| 16 | 120.0 | 24.0 | 12.0 | 8.0 | | | | | | | | | | | | |
| 14 | 190.0 | 38.0 | 19.0 | 12.7 | 9.5 | 7.6 | | | | | | | | | | |
| 12 | 302.0 | 60.4 | 30.2 | 20.2 | 15.1 | 12.1 | 10.1 | 8.6 | | | | | | | | |
| 10 | 480.0 | 96.0 | 48.0 | 32.0 | 24.0 | 19.2 | 16.0 | 13.7 | 12.0 | 10.7 | | | | | | |
| 8 | 764.0 | 153.0 | 76.4 | 51.0 | 38.3 | 30.6 | 25.5 | 21.8 | 19.1 | 17.0 | 15.3 | 12.7 | | | | |
| 6 | 1,215.0 | 243.0 | 121.5 | 81.0 | 60.8 | 48.6 | 40.5 | 34.7 | 30.4 | 27.0 | 24.3 | 20.3 | 17.4 | 15.2 | | |
| 5 | 1,533.0 | 307.0 | 153.3 | 102.0 | 76.6 | 61.3 | 51.1 | 43.8 | 38.4 | 34.1 | 30.7 | 25.6 | 21.9 | 19.2 | 17.0 | 15.3 |
| 4 | 1,933.0 | 387.0 | 193.0 | 129.0 | 96.7 | 77.3 | 64.4 | 55.1 | 48.3 | 43.0 | 38.7 | 32.2 | 27.6 | 24.2 | 21.5 | 19.3 |
| 3 | 2,437.0 | 487.0 | 244.0 | 162.0 | 122.0 | 97.4 | 81.1 | 69.6 | 60.9 | 54.0 | 48.7 | 40.6 | 34.8 | 30.5 | 27.1 | 24.4 |
| 2 | 3,073.0 | 615.0 | 307.0 | 205.0 | 154.0 | 123.0 | 102.0 | 87.9 | 76.9 | 68.3 | 61.5 | 51.4 | 43.9 | 38.4 | 34.1 | 30.7 |
| 1 | | 774.0 | 387.0 | 258.0 | 194.0 | 155.0 | 129.0 | 111.0 | 96.9 | 86.0 | 77.4 | 64.5 | 55.3 | 48.4 | 43.0 | 38.7 |
| 0 | | 978.0 | 489.0 | 326.0 | 245.0 | 195.0 | 163.0 | 140.0 | 122.0 | 109.0 | 97.8 | 81.5 | 69.9 | 61.1 | 54.3 | 48.9 |
| 00 | | 1,232.0 | 616.0 | 410.0 | 308.0 | 246.0 | 205.0 | 176.0 | 154.0 | 137.0 | 123.2 | 102.7 | 88.0 | 77.0 | 68.4 | 61.6 |
| 000 | | | 777.0 | 518.0 | 388.0 | 311.0 | 259.0 | 222.0 | 194.0 | 173.0 | 155.4 | 129.5 | 111.0 | 97.1 | 86.3 | 77.7 |
| 0000 | | | 980.0 | 653.0 | 490.0 | 392.0 | 326.0 | 280.0 | 245.0 | 218.0 | 196.0 | 163.3 | 140.0 | 122.5 | 108.9 | 98.0 |

nearest to it, in the line of amperes along the top of the table. In the column below the number thus found find the nearest distance, and to the left in the column of wires find the required size B. & S. gauge number. For example, to find the size of wire to transmit 15 amperes 140 ft. with 3 volts loss, divide 15 by 3 and find the quotient 5 (amperes per volt) in the line of amperes. In the column below, find the nearest distance 153, and to the left of this the size of wire required, which is No. 8.

---

## CIRCUIT-OPENING DEVICES

Both a *switch* and an *automatic circuit opener* (cut-out or circuit-breaker) must be placed in every supply circuit near where it enters a building, so that all the current can be cut off from the building. Except where expert supervision is maintained, circuit-breakers are not permitted unless accompanied by fuses.

Circuit-opening devices must not be installed near combustible materials nor where exposed to inflammable gases or dust. In damp places and in places where flying particles of inflammable material are present, the switches and cut-outs must be in dust- and moisture-proof cabinets, which must have self-closing doors if dust is also present.

On constant-potential circuits, the switches and cut-outs must be capable of opening all the wires of the circuit, except that single-pole switches may be used if the circuit uses less than 660 watts at not over 300 volts. No single-pole cut-out can be used to control more than 660 watts, except where all the power is used in a single device, as in a motor.

Cut-outs must be installed at each point where a change is made in the size of wire unless the cut-out in the larger wire is small enough to protect the smaller wire. The capacity of the fuses must be such that they will melt, or blow, if the current exceeds the safe carrying capacity of the wires. Circuit-breakers must be set to open if the current exceeds the safe carrying capacity of the wires by 30%, unless fusible cut-outs are also used to protect the same wires. Fuses must not be placed in the canopies or shells

of fixtures. Except on slate or marble tablet boards mounted in dust-tight, fireproof cabinets, open-link fuses are not allowed; enclosed fuses are demanded instead.

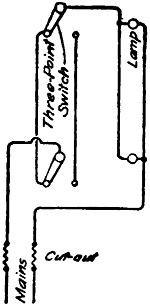


FIG. 2

A switch cabinet should be installed at each center of distribution, and in it should be placed a switch and a cut-out for each branch circuit. A convenient wall switch should also be installed for each large chandelier or group of lamps, unless the distribution cabinet is within easy reach. Two three-point switches connected, as shown in Fig. 2, so that one or more lights can be controlled from more than one point are often very convenient; for example, the hall lights in a dwelling house may be controlled from either floor by means of two three-point switches. By means of two three-point switches and one four-point switch, wired as in Fig. 3, the same lights may be controlled from three points, etc. By introducing another four-point switch for each station, this scheme may be extended.

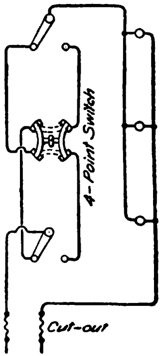


FIG. 3

Single-throw knife switches must be mounted so that gravity will tend to open them and, when possible, so that the blades will be dead when the switch is open. Double-throw knife switches may be mounted so that the throw will be either vertical or horizontal. For circuits using less than 250 volts and 30 amperes, approved snap switches are better than knife switches, but they should indicate plainly whether the current is *on* or *off*. Flush switches must be contained in a sheet-steel or cast-iron box set in the wall.

## SYSTEMS OF INSTALLATION

### OPEN WORK

**Open Work in Dry Places.**—For open work in *dry places*, or for protection against corrosive vapors, slow-burning, weather-proof insulation is permissible. The wires must be held at least ½ in. from the surfaces wired over and 2½ in.


FIG. 4. KNOB

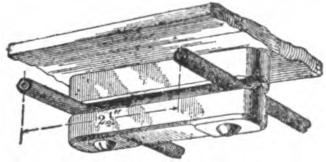


FIG. 5. CLEATS

apart for voltages up to 300, and 1 in. from the surface and 4 in. apart for voltages from 300 up to 550. The wires must be held on insulators or by cleats, Figs. 4, 5, and 6, and must be protected by insulating tubes, Fig. 7, when passing through floors or partitions, so that the wires come in contact with nothing except insulating supports or protectors. The insulators and tubes used must be non-combustible and non-absorptive; porcelain is the material nearly always used.

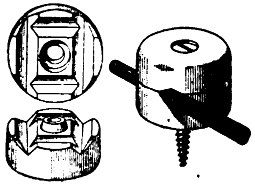

FIG. 6. KNOB CLEATS

On straight runs, the supports should be as close as every 4½ ft. In buildings of mill construction (see Fig. 8), mains of No. 8 B. & S. wire or larger, if not likely to be disturbed, may be separated about 6 in. and run across the ceiling by tying them to insulators attached to the timbers;

if likely to be disturbed or if smaller than No. 8 B. & S. wires are used, they should be run around or through the ceiling timbers, with suitable insulators or protecting tubes, or should be attached to insulators fastened to running boards.

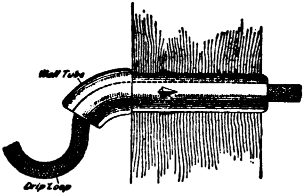


FIG. 7. WALL TUBE

When two wires cross each other, one of them should be run through an insulating tube so fastened that it cannot slide away from the crossing, as in Fig. 9. The same method may be

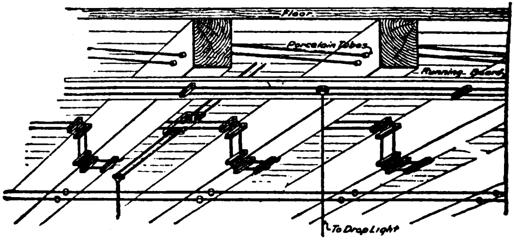


FIG. 8. METHODS OF WIRING OVER CEILING TIMBERS

used when passing near iron pipes. Wires should pass over pipes on which moisture may collect.

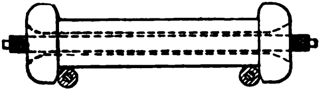

FIG. 9

**Open Work in Damp Places.**—In *damp places*, rubber-covered wire is required. The insulating supports and tubes are the same as used in dry places, but the wires must be at least 1 in. from the surface wired over.

**Constant-Current Systems.**—Wires for constant-current systems, where arc or incandescent lamps are in series, must be rubber-covered, and in each case must enter and leave the building through a switch that will cut off the current from the building and close the main circuit simultaneously. The switch must be mounted on a non-combustible base, where it will be free from moisture and within easy reach. The wires must always be in plain sight, supported on glass or porcelain insulators at least 1 in. from the surface wired over and 8 in from each other, except in the lamps, on hanger boards, etc.

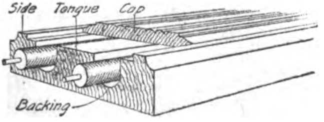


FIG. 10. WOODEN MOLDING

## CONCEALED WORK

All conductors for concealed wiring must be rubber-covered.

**Wooden molding,** Fig. 10, for encasing wires must have a backing at least $\frac{3}{8}$ in. thick under the wires; the tongue between the wire grooves must be at least $\frac{1}{2}$ in. thick, and the sides at least $\frac{1}{4}$ in. thick. The inside of the molding and the cap must have at least two coats of waterproof material, or else the whole molding must be impregnated with moisture repellant. Wooden molding must not be used in concealed or damp places, nor be placed directly against a brick wall where sweating may introduce moisture that may ultimately cause a short circuit. Wooden molding for concealing electrical conductors is prohibited by ordinances in some cities.

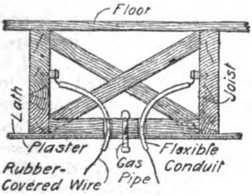


FIG. 11. CEILING OUTLET

**Concealed knob-and-tube work** is permitted by the Underwriters' rules, though forbidden by ordinances in some large cities. It is a cheap form of concealed wiring, and safe if properly installed. In running lengthwise of joists, each wire should run along a separate joist and be securely fastened to knobs not over 4½ ft. apart, and less if there is any likelihood of disturbance. Where the wires pass through joists, they should be protected by porcelain bushings projecting at least ½ in. beyond the timber on each side.

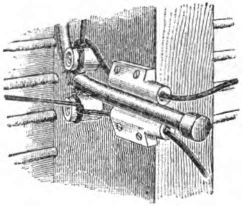
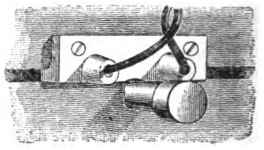

FIG. 12. WALL OUTLETS

At outlets, Figs. 11 and 12, the insulation must be particularly good. The wires should be protected by flexible conduit or insulating tubing, so that they cannot come in contact with the plaster, gas pipe, etc. Flexible circular-loom tube is very useful for this purpose. This is a woven tube without metal covering, and is treated outside and inside with insulating material that makes it hold its shape; this tube is neither weather-proof nor nail-proof, and is permitted only in dry and well-protected places. Flexible steel-armored cable, Fig. 13, makes a convenient auxiliary for the concealed knob-and-tube system, and is used throughout many large buildings where there is no risk from moisture. If moisture is present, this cable must be covered with a lead sheathing.

FIG. 13. STEEL-ARMORED CABLE

**Conduit wiring** is the best and most expensive kind of concealed wiring, and the only kind permitted in fireproof

structures. Iron pipe not less than ½ in. inside diameter, either lined with insulation or unlined, is installed wherever circuits are to run, so as to form a continuous raceway for the wires. After all the pipes are in place and all mechanical work on the building is so

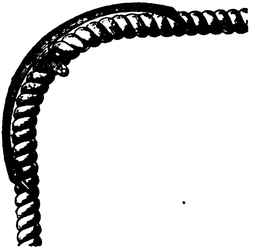

FIG. 14
STEEL-ARMORED CONDUIT

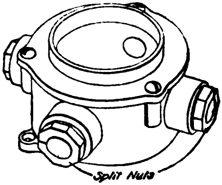


FIG. 15
OUTLET BOX

nearly complete that there is no danger of disturbing the wires, they are fished through the conduit from outlet to outlet. The two or more wires of a circuit are usually drawn through the same conduit; this is required by the Code rules if alternating current is to be used. The

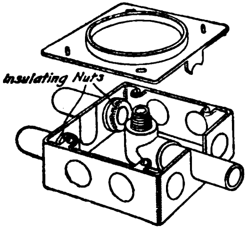


FIG. 16. OUTLET BOX

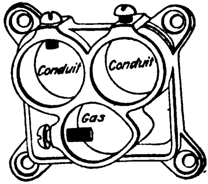


FIG. 17. OUTLET PLATE

conduit system must be in continuous electrical connection, by bonding around outlet boxes if necessary, and must be effectually grounded. The radius of the inner edge of al

bends must not be less than 3½ in. and there must not be more than an equivalent of four quarter bends from outlet to outlet, not counting the bends at the outlet.

The *fishing* is done with a long piece of steel ribbon, called a *snake;* this ribbon is about ⅛ in. wide and $\frac{1}{16}$ in. thick, and has a steel ball about ¼ in. in diameter fastened to one end. Soapstone is first blown into the conduit to overcome friction; the ball end of the snake is then pushed through

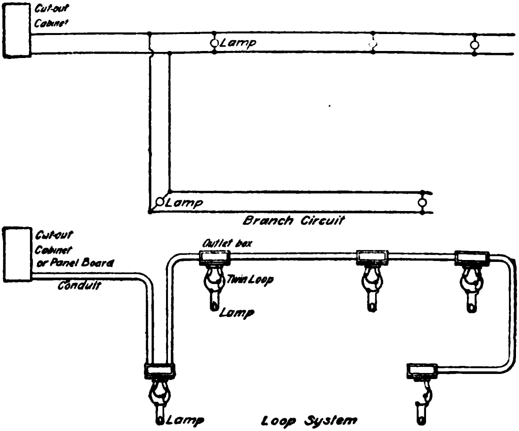


FIG. 18

from one outlet to another, a cord is attached and drawn through, and the wires are then drawn in.

Instead of iron pipe, flexible steel-armored conduit, Fig. 14, is sometimes used; this can be easily bent around curves without the necessity of screw joints. The easy curves offer little resistance to the entrance of the wires.

*Outlet* and *junction boxes* are made in great variety. In one type, Fig. 15, the conduit is held in place by split nuts,

which close around the conduit and clamp it securely when they are screwed home. In another type, Fig. 16, the conduit ends are threaded and held in place by insulating nuts outside and inside the box. Junction boxes should be installed where they are easily accessible.

*Outlet plates*, Fig. 17, are permitted where it is not feasible to use boxes.

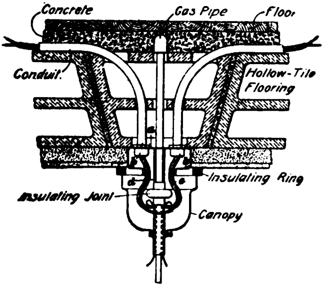

FIG. 19. CEILING OUTLET

Joints are especially objectionable in wires used in conduits, and are not made except where a junction box cannot be installed. In order to avoid branch circuits with their necessary joints, conduit wires are usually installed on the *loop system*, as shown in the lower half of Fig. 18; that is, both wires are looped to each lamp; they are then easily withdrawn from the conduit.

A *ceiling outlet* in a fireproof building, Fig. 19, may be made by bringing the conduit down through the hollow-tile flooring to the outlet box *a*, to which it is fastened with insulating nuts *b*. If gas also is used, the gas pipe is run into the same box and is covered by an insulating bushing *d*. The fixture is fastened to the end of the gas pipe with an insulating joint. The two-wire cable *e* is brought down on the loop

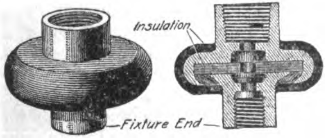

FIG. 20. INSULATING JOINT

system, and the fixture wire is connected to it. An *insulating joint* is shown in Fig. 20.

For *wall outlets* in a fireproof structure, Fig. 21, the conduit should be installed between the wooden strips to which the laths are nailed. The outlet box can be fastened to a brick wall with expansion bolts or by screwing it to wooden plugs driven into holes drilled into the brick; the fixture is then screwed into a threaded stud fastened to the back of the outlet box.

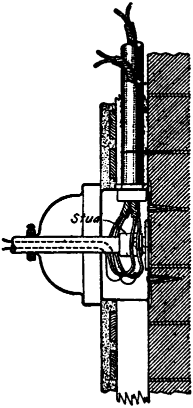


FIG. 21. WALL OUTLET

## WIRING FOR MINOR ELECTRIC APPLIANCES

The source of E. M. F. for ringing bells, operating burglar alarms, door openers, electric clocks, etc., may be primary or secondary batteries, a small dynamo, or the electric-lighting circuit. The rules for the installation of electric-light and electric-power wires do not apply to such minor systems, unless the E. M. F. used exceeds 10 volts, or unless the circuits are in electrical contact with the lighting circuits.

Primary batteries most used for such systems are of the open-circuit variety, such as the ordinary sal-ammoniac cells. Two storage batteries, each with a double-throw switch, can be arranged, as in Fig. 22, so that one can be charged while the other is in use. For bell ringing, a series of lamps can be used across the lighting circuit, as in Fig. 23. If the bell is connected in series with the lamps, its contact device makes and breaks the full lighting voltage, and is likely soon to be injured; a better plan is to connect the bell in shunt with one of the lamps.

The wire used inside buildings for bells, burglar alarms, gas lighting, etc., is usually No. 16 or No. 18 B. & S.; or in large

systems, No. 14 is used for the main-battery wire. For dry

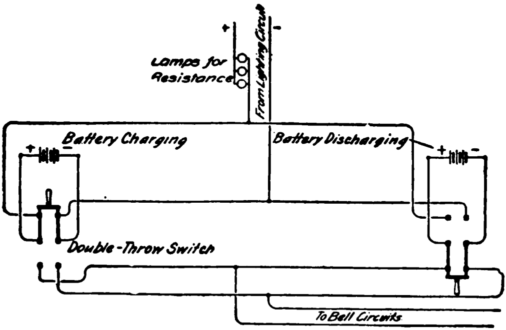


Fig. 22. Storage Batteries for Bell Ringing

places, double-cotton paraffined insulation is good enough;

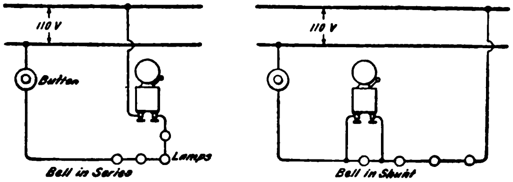


Fig. 23. Bell Ringing From Lighting Circuit

weather-proof office wire should be used in damp places, or rubber-covered wire if much moisture is present.

The wires should be neatly arranged and securely fastened. Single wires can be fastened to woodwork with bare iron staples, or, if the staples are insulated, as in Fig. 24, two or more wires may be fastened under one staple;

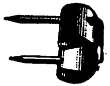

Fig. 24

but in no case should the staples be driven hard. Bell wires must not be placed less than 6 in. from any electric-light

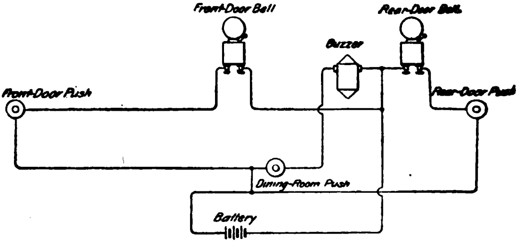


FIG. 25. SIMPLE BELL-WIRING CIRCUITS

or electric-power wire unless encased in approved tubing. When bunched together, bell wires must have a fire-proof covering or else be incased in a non-combustible tube, or

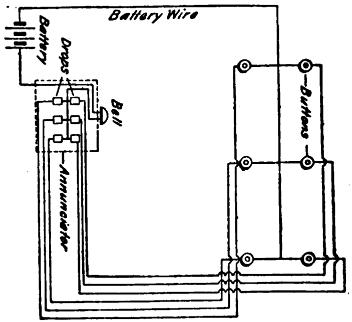


FIG. 26. ANNUNCIATOR SYSTEM

shaft. In the best class of work, all bell wires are run in conduits. Vertical wire ducts, or shafts, must be thoroughly

stopped at each floor, so that they cannot serve as flues to assist the spread of fire. Push buttons and electric-light switches must not be mounted on the same wall plate. All circuits when complete should be tested for breaks, grounds, or crosses. These tests can be made with an ordinary magneto bell.

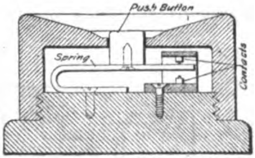


FIG. 27

## BELL CIRCUITS

In a small dwelling, the bell wires may be arranged as in Fig. 25. An annunciator is seldom necessary, since the bells at the front and back doors and the buzzer in the kitchen, to be sounded from the

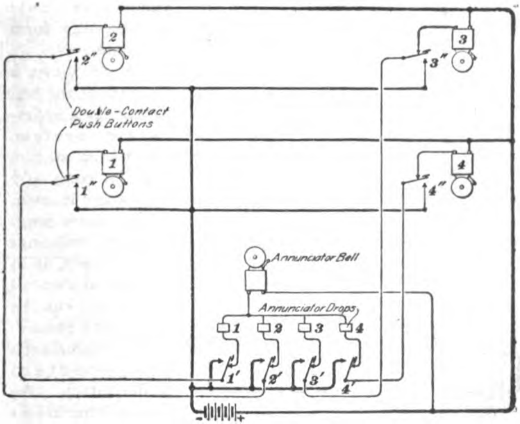


FIG. 28. RETURN-CALL SYSTEM

dining room, may be selected so that each has a distinctive sound.

In an *annunciator system*, Fig. 26, the buttons are located at convenient points, each button being arranged to close a circuit from the battery through the annunciator bell and a drop.

By means of double-contact push buttons, Fig. 27, *return-call annunciator systems* are arranged, as shown in Fig. 28. A bell in any room of a large hotel may be rung by pressing a double-contact push button in the office, or the office annunciator bell may be rung from any room.

Fig. 29 shows a diagram of the bell wiring for a *speaking-tube system*. From each station may be rung any bell except its own.

*Fire-alarm gongs* in large buildings can be wired, as in Fig. 30, so that all the gongs can be rung simultaneously by pressing a multiple-contact push button. An annunciator near each bell will at the same time reveal the source of the signal. The battery is better subdivided, with a section placed near each bell; but, if desired, it can all be placed at one location in the circuit.

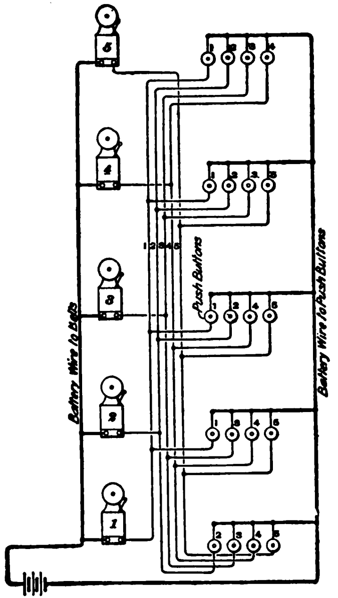

FIG. 29. SPEAKING-TUBE SYSTEM

**Wiring Flats.**—Several flats may be wired, as in Fig. 31, so that a single battery may be used to ring a bell in each flat from a push button at the front door of the building or

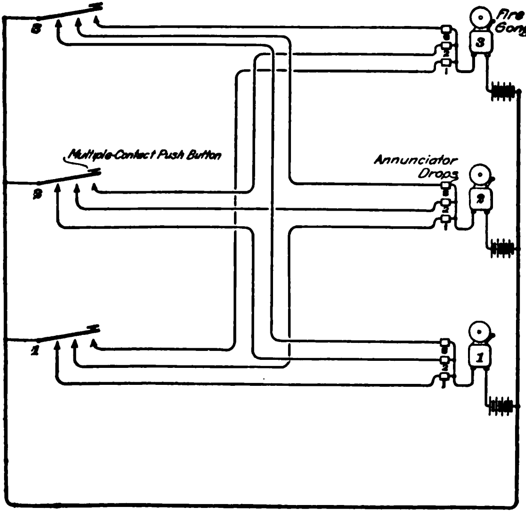


FIG. 30. FIRE-ALARM GONGS

from the hall door of the flat; also a buzzer in each flat may be rung from the rear door.

## DOOR OPENERS

In apartment houses, it is often convenient for a tenant to be able to unlock the front door of the building without leaving his own apartments. This can be done by means of a special lock operated by an electromagnet wired to a push button in each apartment.

## BURGLAR ALARMS

In the *open-circuit system* of burglar alarms, Fig. 32, several electromagnets are installed at some convenient point—one magnet for each circuit. A double-throw switch is usually provided for testing. When closing the house, this switch is placed on contact *1*, and the magnet switches $k_1$, $k_2$ are closed successively. If a window has been inadvertently left open, thus closing a contact spring, the corresponding drop will fall when the magnet switch is closed. The windows being all closed, switches $k_1$, $k_2$ are left closed, and the double-throw switch is placed on contact *2*. The raising of a window will then close a circuit, ring the bell, and cause a magnet drop to fall simultaneously; the drop closes a local circuit, so that, even if the window is closed again, the bell will continue to ring. The local circuit should contain about the same resistance as the circuit through the electromagnet.

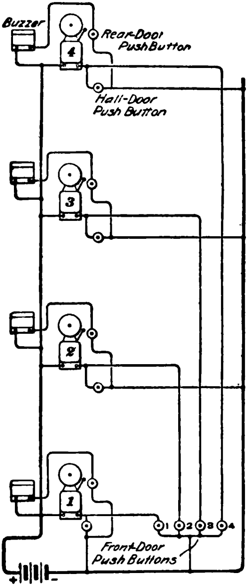


FIG. 31. WIRING FLATS

In the *closed-circuit system* of burglar alarms, Fig. 33, there is a continuous flow of current through circuits of which contacts attached to the doors and windows form a part. In these circuits are relay magnets that, when excited, hold the local bell circuit open; but if any of the main circuits are opened by opening a door or a window or by break-

ing a wire, the bell circuit is automatically closed by the release of a relay-magnet armature, and the bell rings con-

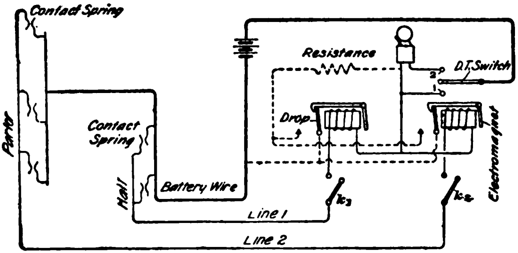


FIG. 32. OPEN-CIRCUIT BURGLAR ALARMS

tinuously until a key $k_1$ or $k_2$ is opened. These keys are left closed when the system is in use.

The two systems may be so combined that an alarm will be given if a circuit is opened or closed at a door or a window, or if the wires are broken or crossed at any point.

## ELECTRIC GAS LIGHTING

The rules for installing wires for electric gas lighting are

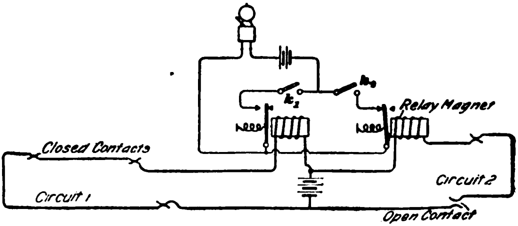


FIG. 33. CLOSED-CIRCUIT BURGLAR ALARMS

the same as for other minor appliances where the E. M. F. is not over 10 volts, except that electric gas lighting must

not be used on combination fixtures, that is, those having both gas and electric lights.

**Parallel System.**—In the *parallel system* of gas lighting, Fig. 34, one terminal of the battery is connected through a spark coil to the gas pipe, and the other through insulated circuits to the insulated terminals on the burners. Each burner has also a grounded terminal so arranged that it can be brought into contact with the insulated terminal, thus closing the circuit; when the contact is broken, a spark is caused in the jet of gas, which is thereby ignited.

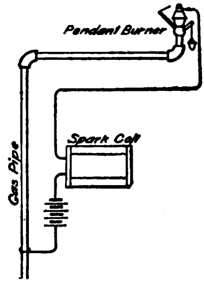


Fig. 34

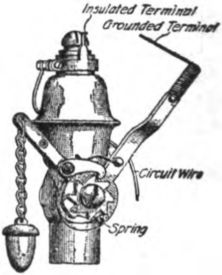


Fig. 35

**Pendant Burners.**—In the *Boston ratchet burner*, Fig. 35, which is typical of most pendant burners, one pull of the pendant turns on the gas and ignites it; on releasing the pendant, the grounded terminal is returned by a spring to its normal position, and a second pull of the pendant turns off the gas.

**Automatic burners** can be had with electromagnets so arranged that the gas can be turned on and ignited by pressing a button, and turned off by pressing another button, both buttons being located at any convenient point. Two insulated wires to each burner are required, one from each button; the gas pipe serves for the return circuit.

**Flash System.**—The *series*, or *flash, system* of gas lighting, Fig. 36, is used in large halls, churches, theaters, etc., where many lights are used in groups. Two sparking points, each insulated from the other and from the burner, are arranged at each burner, so that a spark between the points passes through the jet of gas and ignites it. A number of sparking points and the secondary of an induction coil are connected in series between two points on the gas pipe, one point being near the lighting fixture and the other near the induction coil. When the circuit through the primary of the induction coil is closed, sufficient E. M. F. is induced in the secondary to cause sparks to jump across every jet in the series. Since the E. M. F. is high, the wires must

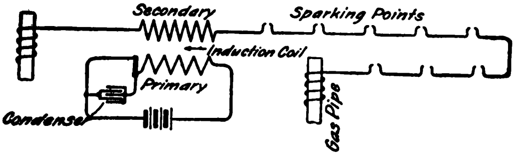


FIG. 36. SERIES GAS LIGHTING

be installed with great precaution. The wire should be enclosed in glass tubing wherever it comes within less than 1¼ in. from the gas piping, except where purposely grounded.

## ELECTRIC CLOCKS

**Watchman's Clocks.**—In a *watchman's clock*, a paper dial is caused to revolve slowly over a number of electromagnetically operated punches. Each electromagnet is connected to a station in some part of the grounds or works to which it is desired that the watchman make periodical visits. The watchman in making his rounds visits each station, and with a key closes the circuit, or, in some systems, turns a magneto located at the station, causing a punch to make a hole in the paper dial. The dial is so marked that the position of the hole indicates the station visited and the time of

the visit. An exact record of the watchman's visits to the various stations is thereby made.

Electric time clocks are used in many large establishments where uniform time is desired in several rooms or buildings. A reliable clock is arranged to close an electric circuit periodically, usually once each minute. In the circuit are the electromagnets of the several electric clocks, all in series. Each time the *master time clock* closes the circuit each electromagnet causes the hands of its clock to move forwards a space corresponding to the interval of time between successive movements. This makes all the *secondary clocks* act in unison, all dependent on the master clock.

# MISCELLANEOUS APPLICATIONS

## ELECTRODEPOSITION OF METALS

### ELECTROCHEMICAL SERIES OF ELEMENTS

Following is a list of elements arranged in such order that any one in the list will displace from its compounds any other element occurring farther on in the list, and will in turn be displaced by any element occurring earlier; that is, each element is *electropositive* to those that succeed it.

ELECTROCHEMICAL SERIES

| | | | |
|---|---|---|---|
| ELECTROPOSITIVE | Zinc | Mercury | Arsenic |
| Potassium | Iron | Silver | Selenium |
| Sodium | Cobalt | Antimony | Sulphur |
| Lithium | Nickel | Tellurium | Iodine |
| Barium | Thallium | Palladium | Bromine |
| Strontium | Lead | Gold | Chlorine |
| Calcium | Cadmium | Platinum | Oxygen |
| Magnesium | Tin | Silicon | Fluorine |
| Aluminum | Bismuth | Carbon | ELECTRONEGATIVE |
| Chromium | Copper | Boron | |
| Manganese | Hydrogen | Nitrogen | |

and *electronegative* to those that precede it. In general, the metals are electropositive and the non-metals electronegative. The farther apart any two elements appear in the series, the greater is their affinity for each other. A series of elements so arranged is called the electrochemical series or sometimes the electromotive series.

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## ELECTROPLATING

A current of electricity in passing through an electrolyte consisting of a solution of a metallic salt liberates from the solution a quantity of the metal proportional to the strength of the current and to the time—that is, to the quantity of electricity. All the liberated metal is usually deposited in a thin, even layer on the cathode. For example, since the electrochemical equivalent of silver is .001118 gram, abbreviated g. (see table, page 11), a current of 3 amperes flowing for 1 hr. and 20 min., or 4,800 sec., through a solution of silver nitrate will cause to be deposited on the cathode $4{,}800 \times 3 \times .001118 = 16.099$ g. of silver. In a copper-sulphate solution where copper enters as a monad with an electrochemical equivalent of .00065881 g., a current of 5 amperes for 1 hr. will deposit $5 \times 60 \times 60 \times .00065881 = 11.86$ g.

In case of the most commonly used plating metals, the amount of metal deposited per hour by 1 ampere is as follows: Copper, monovalent, 2.37 g.; divalent, 1.186 g.; silver, 4.025 g.; gold, 2.45 g.; nickel, 1.095 g.

### PLATING SOLUTIONS

**Copper-Acid Solution.**—Sulphate of copper, 1 lb.; sulphuric acid, 1 lb.; water, 1 gal. Dissolve the sulphate of copper in hot water, after which add the remainder of the water cold. Then add the sulphuric acid; when the solution has cooled, it is ready for use.

**Copper-Cyanide Solution.**—Carbonate of copper, 1 lb.; carbonate of potash, 6½ oz.; cyanide of potassium, 2 lb.; water, 3 gal. Dissolve the cyanide of potassium in the greater part of the water, the carbonate of copper in a

portion, and the carbonate of potash in another portion. Add to the potassium solution, first the copper solution and then the potash solution, stirring the whole thoroughly. If, on trial, the solution does not deposit freely, more cyanide or more carbonate, or both, should be added until the desired result is obtained.

**Brass Solution.**—To the preceding cyanide solution add zinc-carbonate solution made by dissolving 2 parts, by weight, of cyanide of potassium and 1 part of zinc carbonate in water. The zinc-carbonate solution is added to the copper-cyanide solution until the desired color of brass deposit is obtained.

**Nickel Solution.**—Double sulphate of nickel and ammonia (pure), 12 to 14 oz.; water, 1 gal. The nickel salts are put into a wooden tank and hot water is poured on them until they dissolve, this being hastened by stirring with a clean wooden stick. This bath is nearly neutral and has no power to remove oxide from the surface of the metal to be plated; hence, the metal must be thoroughly cleansed. For removing grease, dirt, etc., dip the articles into a hot potash bath made by dissolving ½ lb. of caustic potash in 2 gal. of water. This bath must be kept hot while in use; its strength decreases with use, so that longer time is required for cleansing an article in a bath that has been used for some time than in a fresh one. On removal from the bath, the article should be well scoured with pumice and water and then rinsed. Brass or copper articles should then be dipped for a few seconds into a solution of ½ lb. of cyanide of potassium per gallon of water, and steel or iron articles into a solution of ½ lb. of hydrochloric acid per gallon of water, using a wooden tank to hold the solution.

**Gold Solution.**—Gold baths are usually made by dissolving chloride of gold in cyanide-of-potassium solution. Gold baths are used in so many ways and for such a great variety of work that no general formula can be given. In hot baths, the higher the temperature, the darker the color of the deposit; in cold baths, the stronger the current, the darker the color. Hot baths are usually worked at 90° to 140° F., and the quantity of gold varies from 11½ to 20 gr. (grains) per

quart of solution. In cold baths, there must be at least 54 gr. of gold per quart of solution, and in some cases it may run over 300 gr.

**Silver Solution.**—Silver chloride, 3 oz.; cyanide of potassium, 10 to 12 oz.; water, 1 gal. The chloride is first made into a thin paste with some of the water, and then added to a solution of 9 or 10 oz. of the cyanide in the remainder of the water. If the solution does not work freely at first, add more cyanide.

## CURRENT DENSITIES FOR ELECTROPLATING

The following *current densities*, in amperes per 100 sq. in., are recommended: For copper electrotyping from an acid bath, best quality, tough deposit, 1.5 to 4; good and tough deposit for stereotypes, 4 to 10; good solid deposit, 10 to 25, solid deposit, sandy at edges, 25 to 40; sandy and granular deposit, 50 to 100. For copper deposit from a cyanide bath, 2 to 3; zinc (for refining), 2 to 3; silver, 1 to 3; gold, .5 to 1; brass, 3 to 3.5; iron (steel facing), .5 to 1.5; nickel, begin at 9 to 10 and gradually diminish to 1 to 2.

## VOLTAGES FOR VARIOUS PLATING BATHS

The voltage should be adjusted in each case to give the most suitable current.

| Metal | Approximate E. M. F. Volts |
|---|---|
| Copper, acid bath | .5 to 1.5 |
| Copper, cyanide bath | 3.0 to 5.0 |
| Silver | .5 to 1.0 |
| Gold | .5 to 4.0 |
| Brass | 3.0 to 5.0 |
| Iron, steel facing | 1.0 to 1.3 |
| Nickel on iron, steel, copper, with nickel anode, start deposit with 5 volts, diminishing to | 1.5 to 2.0 |
| Nickel, on iron, steel, copper, with carbon anode | 4.0 to 7.0 |
| Nickel, on zinc | 4.0 to 7.0 |
| Platinum | 5.0 to 6 0 |

# ELECTRIC HEATING

## HEATING OF CONDUCTORS

The heat generated in a conductor in which an electric current is flowing is proportional to the square of the current $I$, the resistance $R$ of the conductor, and the duration in time $t$ of the flow; that is, the work in joules converted into heat is $I^2Rt$, where $I$ is in amperes, $R$ in ohms, and $t$ in seconds. The temperature attained by the conductor depends on the readiness with which heat can escape. Heat escapes more readily from insulated wire in molding or conduit or even in air than from bare wire in still air; also, in still air, a wire with a rough, blackened surface will dissipate heat more rapidly than one with a smooth, shiny surface. A wire will carry a current with less temperature rise if suspended horizontally than if suspended vertically.

Fuse wires should never be less than 1 in. long, otherwise so much heat will escape to the terminals that the fusing of the wire at the current for which it was selected will be uncertain. Pure copper wire makes the most reliable fuse wire. The table on pages 382 and 383 gives the diameters of wires of various materials that will be fused by a current of given strength.

**Rheostats.**—In rheostats, the resistance wire is often covered with enamel or cement, to exclude the air and thus permit operating at a much higher temperature than would otherwise be safe. One manufacturer covers the resistance wire with a cement that is not injured even though the wire is white hot, and the wire itself may be operated at red heat without apparent injury. The resistance wire in rheostats is usually German silver or iron. German-silver wire is used for small current-carrying capacities, where the air can be excluded, and iron wire, mounted so as to obtain good ventilation, where large carrying capacity is desired. The circular mils per ampere for continuous safe carrying capacity of galvanized-iron wire in open air varies gradually from 1,300 for No. 0 B. W. G. to 650 for No. 18.

TINNED-IRON WIRE

| No. B. & S. | Area Circular Mils | Maximum Safe Current Continuous Duty Amperes | | Safe Current for 1 Minute | Feet per Ohm | Pounds per Foot | Ohms per Inch of .4-Inch Spiral |
|---|---|---|---|---|---|---|---|
| | | Wooden Frame | Iron Frame | | | | |
| 8 | 16,509 | 17.40 | 20.30 | 43.6 | 250.00 | .04000 | .0050 |
| 9 | 13,094 | 14.60 | 17.10 | 36.6 | 173.00 | .03300 | .0066 |
| 10 | 10,381 | 12.30 | 14.30 | 30.8 | 137.00 | .02751 | .0095 |
| 11 | 8,234 | 10.30 | 12.00 | 25.8 | 108.00 | .02182 | .0131 |
| 12 | 6,529 | 8.70 | 10.10 | 21.7 | 86.40 | .01730 | .0182 |
| 13 | 5,178 | 7.30 | 8.50 | 18.3 | 68.50 | .01372 | .0145 |
| 14 | 4,106 | 6.10 | 7.10 | 15.3 | 54.30 | .01089 | .0353 |
| 15 | 3,257 | 5.10 | 6.00 | 12.9 | 43.10 | .00863 | .0492 |
| 16 | 2,583 | 4.30 | 5.00 | 10.8 | 34.10 | .00685 | .0690 |
| 17 | 2,048 | 3.60 | 4.20 | 9.1 | 27.10 | .00543 | .0960 |
| 18 | 1,624 | 3.00 | 3.50 | 7.6 | 21.40 | .00430 | .1345 |
| 19 | 1,252 | 2.50 | 2.90 | 6.3 | 16.50 | .00341 | .1963 |
| 20 | 1,021 | 2.20 | 2.50 | 5.4 | 13.50 | .00271 | .2636 |
| 21 | 810 | 1.80 | 2.10 | 4.5 | 10.70 | .00231 | .3725 |
| 22 | 643 | 1.50 | 1.77 | 3.8 | 8.49 | .00184 | .5220 |
| 23 | 509 | 1.30 | 1.49 | 3.2 | 6.73 | .00146 | .7350 |
| 24 | 404 | 1.08 | 1.20 | 2.6 | 5.34 | .00116 | 1.0350 |

## FUSING EFFECTS OF CURRENTS

(*W. H. Preece, F. R. S.*)

| Current Amperes | Diameters in Inches | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | Copper | Aluminum | Platinum | German Silver | Platinoid | Iron | Tin | Tin-Lead Alloy | Lead |
| 1 | .0021 | .0026 | .0033 | .0033 | .0035 | .0047 | .0072 | .0083 | .0081 |
| 2 | .0034 | .0041 | .0053 | .0053 | .0056 | .0074 | .0113 | .0132 | .0128 |
| 3 | .0044 | .0054 | .007 | .0069 | .0074 | .0097 | .0149 | .0173 | .0168 |
| 4 | .0053 | .0065 | .0084 | .0084 | .0089 | .0117 | .0181 | .021 | .0203 |
| 5 | .0062 | .0076 | .0098 | .0097 | .0104 | .0136 | .021 | .0243 | .0236 |
| 10 | .0098 | .012 | .0155 | .0154 | .0164 | .0216 | .0334 | .0386 | .0375 |
| 15 | .0129 | .0158 | .0203 | .0202 | .0215 | .0283 | .0437 | .0506 | .0491 |
| 20 | .0156 | .0191 | .0246 | .0245 | .0261 | .0343 | .0529 | .0613 | .0595 |
| 25 | .0181 | .0222 | .0286 | .0284 | .0303 | .0398 | .0614 | .0711 | .069 |
| 30 | .0205 | .025 | .0323 | .032 | .0342 | .045 | .0694 | .0803 | .0779 |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 35 | .0227 | .0277 | .0358 | .0356 | .0379 | .0498 | .0769 | .089 | .0864 |
| 40 | .0248 | .0303 | .0391 | .0388 | .0414 | .0545 | .084 | .0973 | .0944 |
| 45 | .0268 | .0328 | .0423 | .042 | .0448 | .0589 | .0909 | .1052 | .1021 |
| 50 | .0288 | .0352 | .0454 | .045 | .048 | .0632 | .0975 | .1129 | .1095 |
| 60 | .0325 | .0397 | .0513 | .0509 | .0542 | .0714 | .1101 | .1275 | .1237 |
| 70 | .036 | .044 | .0568 | .0564 | .0601 | .0791 | .122 | .1413 | .1371 |
| 80 | .0394 | .0481 | .0621 | .0616 | .0657 | .0864 | .1334 | .1544 | .1499 |
| 90 | .0426 | .052 | .0672 | .0667 | .0711 | .0935 | .1443 | .1671 | .1621 |
| 100 | .0457 | .0558 | .072 | .0715 | .0762 | .1003 | .1548 | .1792 | .1739 |
| 120 | .0516 | .063 | .0814 | .0808 | .0861 | .1133 | .1748 | .2024 | .1964 |
| 140 | .0572 | .0698 | .0902 | .0895 | .0954 | .1255 | .1937 | .2243 | .2176 |
| 160 | .0625 | .0763 | .0986 | .0978 | .1043 | .1372 | .2118 | .2452 | .2379 |
| 180 | .0676 | .0826 | .1066 | .1058 | .1128 | .1484 | .2291 | .2652 | .2573 |
| 200 | .0725 | .0886 | .1144 | .1135 | .121 | .1592 | .2457 | .2845 | .276 |
| 225 | .0784 | .0958 | .1237 | .1228 | .1309 | .1722 | .2658 | .3077 | .2986 |
| 250 | .0841 | .1028 | .1327 | .1317 | .1404 | .1848 | .2851 | .3301 | .3203 |
| 275 | .0897 | .1095 | .1414 | .1404 | .1497 | .1969 | .3038 | .3518 | .3417 |
| 300 | .095 | .1161 | .1498 | .1487 | .1586 | .2083 | .322 | .3728 | .3617 |

Tinned-iron wire is much used for rheostats; the wire is usually wound on iron frames, but sometimes, if only for temporarv or experimental purposes, on wooden frames. The table on page 381 gives useful data, including safe carrying capacity both for continuous duty and for 1-min. periods, as for starting rheostats. The resistance per inch of a spiral wound on a 4-in. mandrel as close as possible without having adjacent turns in contact is also given.

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## AIR AND WATER HEATING

*(1 B. T. U. = 778 ft.-lb. = 1,055 joules)*

**Air Heating.**—To raise the temperature of 1 cu. ft. of air 1° F. requires an expenditure of about 18 joules; to make this change in 1 sec. requires an expenditure of 18 joules per sec., or 18 watts. A room 12 ft. $\times$ 14 ft. $\times$ 10 ft. high contains 1,680 cu. ft., and to raise its temperature from 0° to 70° F. in 1 hr. requires $\frac{1,680 \times 18 \times 70}{3,600} = 588$ watts, or 588 watt-hours. If the time were $\frac{1}{2}$ hr., twice as many watts would be required, but the number of watt-hours would be the same. This calculation assumes no escape of heat from the room.

At the prevailing prices of electricity, electric air heaters are not practical for warming ordinary dwelling houses, except when only a slight rise of temperature, enough to remove dampness, etc., is desired. On electric cars, however, where the cost of the energy used is not a serious consideration, electric heaters are much used. For heating an ordinary electric car in zero weather, about 12 amperes at 500 volts is required; with the outside temperature at 20° to 30° F., about 6 amperes is sufficient.

**Water Heating.**—The weight of 1 gal. of water is 8.34 lb., and to raise its temperature 1° F. requires the expenditure of 8.34 B. T. U. or $8.34 \times 1,055 = 8,798.7$ joules; $8,798.7 \div 3,600 = 2.444$ watt-hours required, provided there are no heat losses.

## ELECTRIC HEATING APPLIANCES

Some of the advantages of electric heat are as follows: Its instant availability on closing a switch; its perfect control, since heat may be obtained in almost any intensity desired; its perfect adaptability, as the application may be made at almost the exact location desired, very little heat being wasted; the absence of smoke, flame, dust, poisonous gases, etc.; the absence of fuel and ashes to be handled or fires to be maintained; the reduction of danger of fire or explosions.

**Domestic Heating Appliances.**—All *domestic heating appliances* requiring about 500 watts or less may be classified as *lighting-circuit devices*, because no special wiring or outlets are needed. Those requiring more than 500 watts should be classified as *heating-circuit devices*, for which special circuits and outlets should be provided.

Electric flat irons, coffee pots, tea pots, water heaters, chafing dishes, stoves, plate warmers, griddles, warming pads, curling-iron heaters, etc. are already finding extensive use, and reductions in the price of electric energy will not only greatly increase their popularity, but will also introduce into many homes still larger electric-heating devices. All new houses should be wired for both electric heating and electric lighting, the heating outlets being placed as near as possible to the place where the devices are to be used.

**Miscellaneous Heating Appliances.**—Among *miscellaneous heating appliances* may be classed those used in printing and book-binding establishments, electric-laundry machinery, hatter's tools, tailor's irons, glue pots, soldering irons, cigar lighters, etc.

**Power Consumption of Heating Appliances.**—The *power consumptions* of some of the ordinary electric-heating appliances are as follows: Ordinary 6-lb. flat iron, 500 watts; 3-qt. chafing dish, 500 watts; 1-qt. water heater, to boil water with an initial temperature of 60° F. in 10 min., 650 watts; 1-qt. glue pot, 440 watts; soldering iron, equivalent to a 3-lb. soldering copper, 150 watts.

## THAWING FROZEN WATER PIPES

For *thawing frozen water pipes* electrically where alternating current is available, special transformers either having large magnetic leakage or each with a choke coil in series are used. The secondary voltage is usually low, but the current is high. The large magnetic leakage or the choke coil permits the secondary of such a transformer to be short-circuited for several minutes without injury. The piece of frozen pipe is made a part of the secondary circuit, the primary terminals of the transformer are connected with the lighting circuit, and the voltage is adjusted until the desired current through the pipe is obtained. There should be the least possible resistance in the secondary circuit; that is, the secondary leads should be of ample size and as short as possible. Connections can be made at a hydrant and a faucet in a neighboring house, or to faucets in two adjacent houses, etc.

Where only direct current is available, a motor-generator or a dynamotor is necessary to reduce the voltage; a motor-generator is preferable, since the voltage is under better control. The volts, amperes, and time required to produce running water in a frozen pipe vary between wide limits and according to no fixed rule. Ordinary house piping, however, seldom requires more than 30 to 50 volts and 300 amperes.

## ELECTRIC WELDING

By joining two pieces of metal, as indicated in Fig. 1, sending through the joint enough current to heat the joined surfaces to welding temperatures, and then pressing them firmly together, they may be made to cohere like solid metal. Many welding operations are economically performed in this way. The pieces to be welded are usually clamped in a device, by means of which they may be forced together with a considerable pressure. One secondary terminal of a transformer is connected with each side of the joint, and a heavy current is sent across. The large current and the high joint resistance cause the development of enough heat to raise the temperature to welding point in a

very short time, and the pieces are then pressed together and the current shut off.

Alternating current is most convenient because of the ease of its transformation. The frequency should not be over 50; the voltage required is low, depending on the nature of the work; the current density is very high, that for copper welding sometimes being as high as 60,000 amperes per sq. in.

**Welding Transformer.**—A *welding transformer* has very few turns—often only one—of very massive secondary copper. In some cases, the secondary is a heavy copper casting slit on one side and having a clamp each side of the slit for holding the work. The current, being of very low voltage, cannot cross the slit, but goes through the work by way of the clamps. It is sometimes necessary to provide a circulation of cold water through the clamps to keep them from overheating.

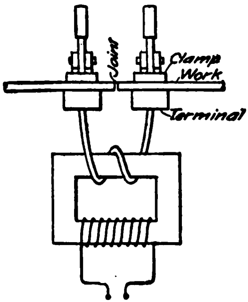


FIG. 1. ELECTRIC WELDING

**Power Required for Electric Welding.**—The power required for electric welding varies inversely as the time consumed in making weld, and to some extent on the metals welded. Copper, brass, tool steel, and other metals that are deteriorated by high temperatures must be heated quickly, pressed together with sufficient force to push all the injured metal out from the weld, and allowed to cool at once. Comparatively little heat is wasted, 70 to 75% of the power applied being utilized. In the table on page 388, the areas are in square inches, the power is in watts supplied to the primary of the welding transformer, and the time is in seconds. The distance between terminal clamps was twice the diameter of the iron pieces being welded, three times the diameter for brass. and four times the diameter for copper

## POWER AND TIME REQUIRED FOR ELECTRIC WELDING

| Iron and Steel | | | Brass | | | Copper | | |
|---|---|---|---|---|---|---|---|---|
| Area Square Inches | Power Watts | Time Seconds | Area Square Inches | Power Watts | Time Seconds | Area Square Inches | Power Watts | Time Seconds |
| .5 | 8,550 | 33 | .25 | 7,500 | 17 | .125 | 6,000 | 8 |
| 1.0 | 16,700 | 45 | .50 | 13,500 | 22 | .250 | 14,000 | 11 |
| 1.5 | 23,500 | 55 | .75 | 19,000 | 29 | .375 | 19,000 | 13 |
| 2.0 | 29,000 | 65 | 1.00 | 25,000 | 33 | .500 | 25,000 | 16 |
| 2.5 | 34,000 | 70 | 1.25 | 31,000 | 38 | .625 | 31,000 | 18 |
| 3.0 | 39,000 | 78 | 1.50 | 36,000 | 42 | .750 | 36,500 | 21 |
| 3.5 | 44,000 | 85 | 1.75 | 40,000 | 45 | .875 | 43,000 | 22 |
| 4.0 | 50,000 | 90 | 2.00 | 44,000 | 48 | 1.000 | 49,000 | 23 |

**Rail Welding.**—Welding rails electrically is accomplished by placing an iron plate on each side of the rail joint, pressing the plates against the rails by means of massive clamps constituting the secondary terminals of a special welding transformer, and sending a powerful electric current through the plates and the rail ends until all are firmly welded together. A joint welded in this manner will stand a stress of 279,000 lb., whereas the maximum stress caused by temperature variations is only about 150,000 lb. The conductivity is as great as that of the solid rail. The welding outfit consists of a rotary converter and a welding transformer. The converter is fed by 500-volt direct current from the trolley and delivers 300-volt alternating current for the transformer. The average alternating current supplied the primary of the transformer during a welding operation is about 650 amperes.

## ELECTRIC ANNEALING

*Electric annealing* is a process by which parts of steel plates or castings may be softened. The terminals of the annealing transformer are pressed against the casting on each side of the spot to be softened as in Fig. 2, and enough current is sent through to heat the part until it is softened, so that it can be drilled or machined. Surrounding parts are not affected.

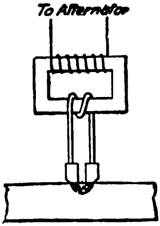


FIG. 2. ANNEALING

## ELECTROLYTIC FORGE

An *electrolytic forge*, or *tempering bath*, consists of a metallic-lined vessel containing water or a tempering solution. One terminal of an electric circuit is connected with the metallic lining, and the other with an insulated bar near the edge of the vessel. By resting the piece of metal to be tempered across the bar and thrusting the end in the liquid, the circuit is completed through the bar and the liquid, and by adjusting the strength of current and the time, the metal can be heated to any desired temperature, even to melting point.

When the desired heat is attained, the current is shut off, leaving the metal in contact with the tempering bath. Great accuracy in tempering may be thereby attained. Suitable insulating shields may be made to protect the parts not to be tempered. This process was discovered by Lagrange and Hoho, and is often called the *Hoho process.*

### ELECTRIC FURNACE

In an *electric furnace*, a temperature of about 3,500° C. is produced by means of an electric arc between carbon electrodes. A supply of air in the crucible is not necessary, and the heat may be generated in a very confined location. Fig. 3 shows a small furnace, but the principle is the same in all. In some types, the crucible itself is made one of the electrodes. By means of the electric furnace, many electrochemical and electrometallurgical processes not otherwise possible can be performed.

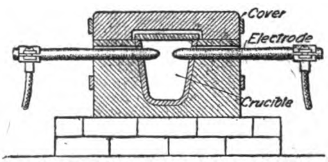


FIG. 3. ELECTRIC FURNACE

## ELECTROMAGNETS

### LIFTING MAGNETS

Since the lifting power of a magnet depends on the nature of the contact surface between the poles and the armature or object to be lifted, the only practical way to design an electromagnet for general lifting purposes, where the contact surfaces may be very rough and uneven, is to design the core of ample size, put on an experimental coil, and adjust the current until the desired lifting power is obtained. The product of the number of turns in the experimental coil and the number of amperes found necessary gives the

number of ampere-turns for which the permanent coil must be designed.

If $P$ is the pull in pounds, $\mathbf{B}$ the magnetic density at the pole faces in lines per square inch, and $A$ the polar area in square inches, their relation is expressed by the following formula:

$$P=\frac{\mathbf{B}^2A}{72,134,000}$$

If any two of the quantities are known, the other can be found by this formula.

If both poles of the magnet are made effective, the total pull is the sum of the pulls of the two poles. In order to obtain maximum tractive effort per pound weight of magnet, the magnetic circuit should be as short as possible; the sectional area of the magnetic circuit should be uniform and large enough to avoid an approach to magnetic saturation at any point; the iron or steel used should have high magnetic permeability; and the magnetic density at the contact surfaces should not be much over 100,000 lines per sq. in.

**Heating Effect in Lifting Magnetic Coils.**—The *heating effect* of the current in a lifting-magnet coil depends on the continuity of the service, the depth of the winding, and the effectiveness of the ventilation. Lifting magnets are seldom in continuous use for more than a few moments at a time. The winding, however, is usually so deep that the heat developed in the inside layers reaches the outer surface slowly. The ventilation is often very poor, the magnets being used in hot locations, in engine rooms, rolling mills, etc., and the windings being entirely enclosed. Lifting-magnet coils should usually be designed so that the watts lost in each coil divided by the square inches outside cylindrical surface (not including the ends) of the coil shall be not over 1, and for the most unfavorable conditions—continuous service, deep winding, and poor ventilation—not over .25.

**Lifting Magnet Calculations.**—The calculation of lifting magnets is based very largely on experimental data. Knowing the lifting power and assuming the densities, the areas

of cross-sections are calculated. The winding space necessary is estimated, the lengths of various parts of the magnetic circuit calculated, and the ampere-turns $IT$ determined by reference to a magnetization curve for the material (steel or iron). The mean length of turn $l$ is estimated, and the size of copper wire calculated by the formula

$$a = \frac{lIT}{E},$$

in which $a$ is the cross-section in circular mils, and $E$ the voltage on which the coil is to be used. Enough wire of the size calculated must be used to keep the watts per square inch down to a safe value.

When preliminary determinations of the size and quantity of wire have been made, all the calculations are checked

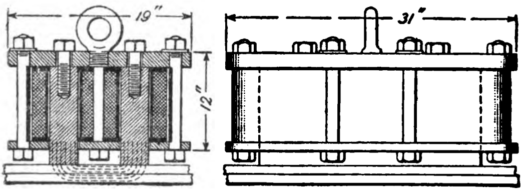



FIG. 4. BIPOLAR LIFTING MAGNET

to see if sufficient winding space, etc., has been allowed. Two or three trials may be necessary to obtain the best results. These calculations are fairly accurate only where the magnetic circuit is closed; where part of the magnetic circuit is through air, as with solenoids having movable plungers, the calculations are somewhat different. In horseshoe-type magnets the leakage between legs may modify results considerably, and a liberal allowance should accordingly be made.

The *bipolar rectangular lifting magnet*, shown in Fig. 5, weighs 1,200 lb. and is capable of lifting 6 tons. The two coils in series take $6\frac{1}{4}$ amperes at 250 volts. Lifting magnets are much used in handling steel and iron plates, for which

purpose rectangular magnets are better than those of circular section. A very satisfactory design for such work is a *multipolar magnet*, Fig. 5, adjacent poles being of opposite polarity. Each pole consists of a rectangular bar of iron;

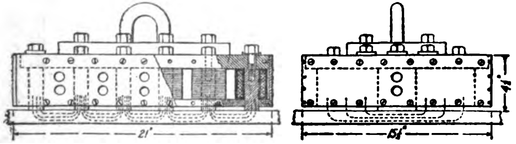



FIG. 5. MULTIPOLAR LIFTING MAGNET

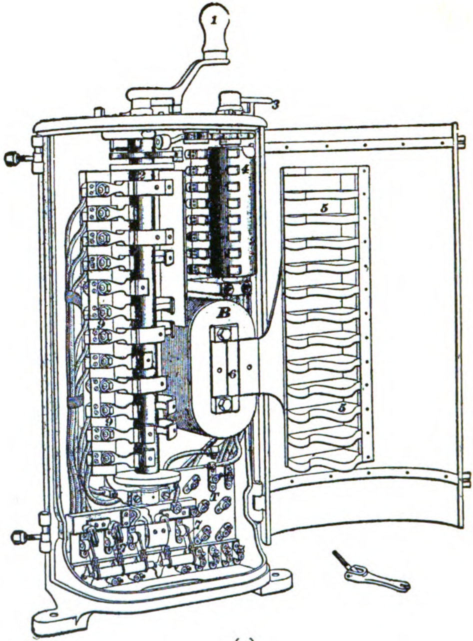
each is provided with a magnetizing coil, and all are bolted to a steel plate that forms the magnet yoke. The whole is surrounded and protected by a metal casing.

# ELECTRIC-CAR CONTROLLERS

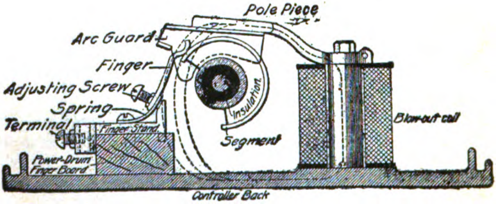
## SERIES-PARALLEL CONTROL

Practically all electric railway cars have at least two motors. In starting and when running at slow speeds, these motors are connected in series, so that each has one-half the total voltage of the system across its terminals; at high speeds, the motors are connected in parallel so that upon each is impressed the full voltage of the system. An adjustable resistance is also used in starting and when changing from series to parallel connections. The resistance adjustments and the changes in the connections are made by means of controllers.

Fig. 1 (*a*) shows a typical electric-railway, *series-parallel car controller* with the cover, or door, thrown back so as to expose the interior to view. The operating handle *1* turns the main control cylinder *2*, on which are fastened insulated copper contact segments; the segments slide under the




(a)




(b)

**Fig. 1. Series-Parallel Controller** 394

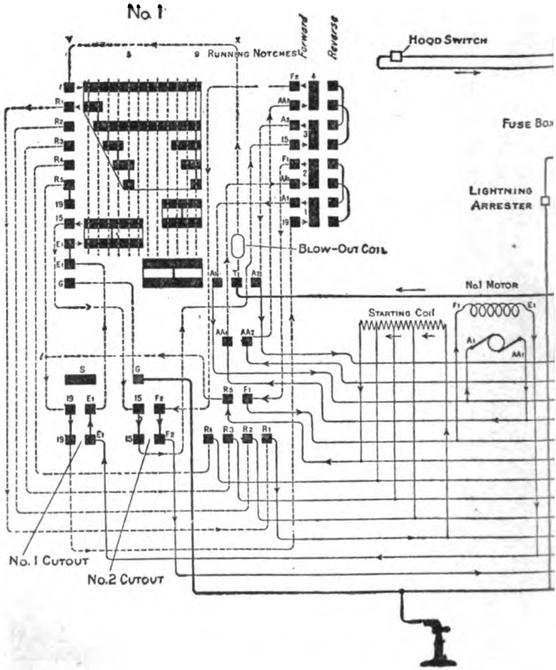



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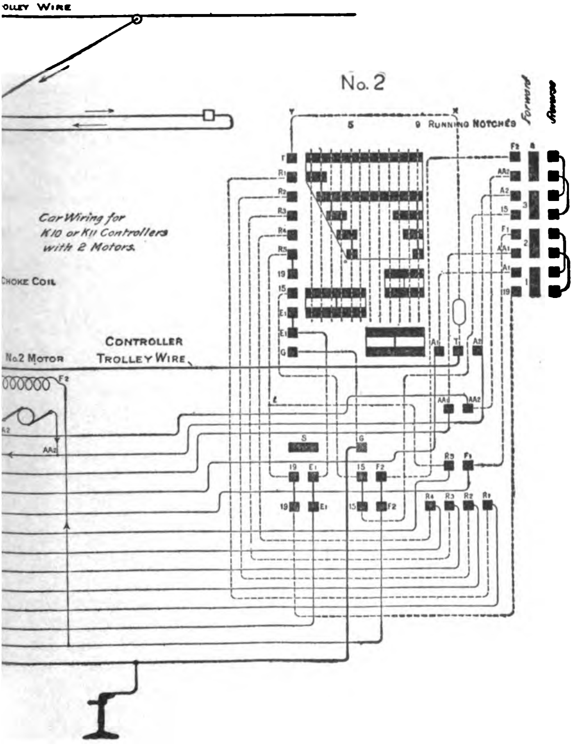
Fig. 2. Controller Dia




AM—TWO-MOTOR EQUIPMENT

fingers attached to the stationary finger board *9*, and to the fingers are made the various connections. Handle *3* controls the reverse cylinder *4*, by means of which the direction of the current through the motor armatures can be reversed. The two cylinders interlock, so that it is impossible to move the reverse cylinder unless the operating cylinder is at the off-position; the reverse cylinder and contact fingers are not designed to interrupt the current. The connections are made so that the car runs forwards when handle *3* points forwards and vice versa. Handle *3* cannot be removed unless both cylinders are at the off-position. A series of vulcabeston arc guards *5* are attached to a frame hinged at *6*, so that the guards when closed in position for service are between the fingers; arcs are thereby prevented from forming between two adjacent fingers or segments. A strong magnetic field set up by the blow-out coil *B* is made to pass across the space between a finger and its segment when they break a circuit, and thus extinguish the arc. The hinged piece to which the arc guards are attached forms one pole piece, and the back of the controller the other, the path of the magnetism being as shown by the dotted lines in the end view of the controller, Fig. 1 (*b*). The wires leading into the controller are connected to binding posts on the terminal board *T*, Fig. 1 (*a*), on which are also switches *7*, by means of which either motor may be cut out of service. An interlocking device prevents the operating cylinder from being turned beyond the series position when either motor is cut out.

In the following controller diagrams the heavy black bands represent the developed segments on the controller cylinders, and the vertical dotted lines across the segments represent the various notches. In most controllers, there are only two running notches—one with the motors in series and one with the motors in parallel, all resistance being cut out in each case.

Fig. 2 shows a diagram of the connections of a ***two-motor equipment*** with two-series-parallel controllers. In tracing the current paths on a controller diagram, conceive the reverse cylinder to be turned until the contact segments

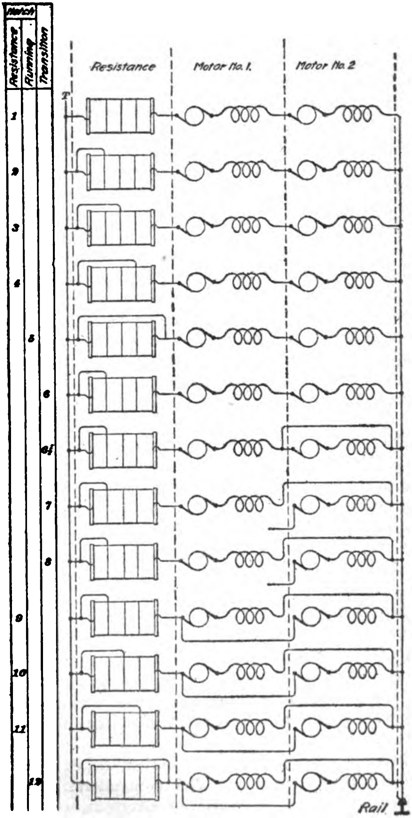
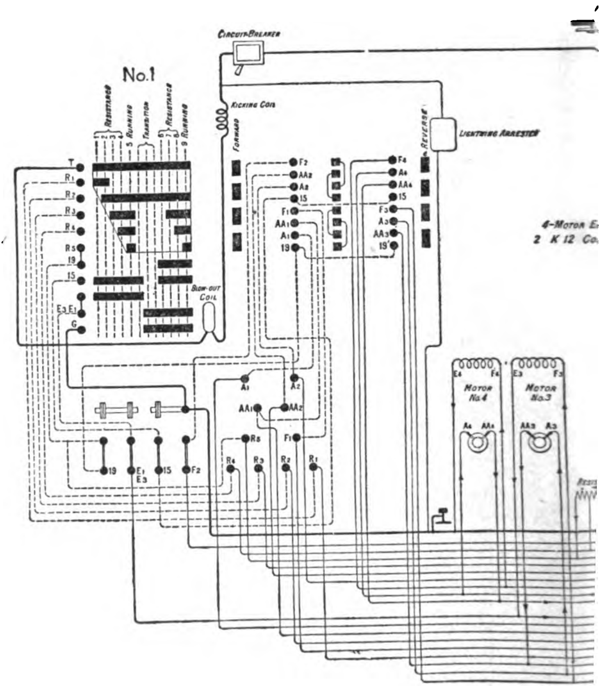
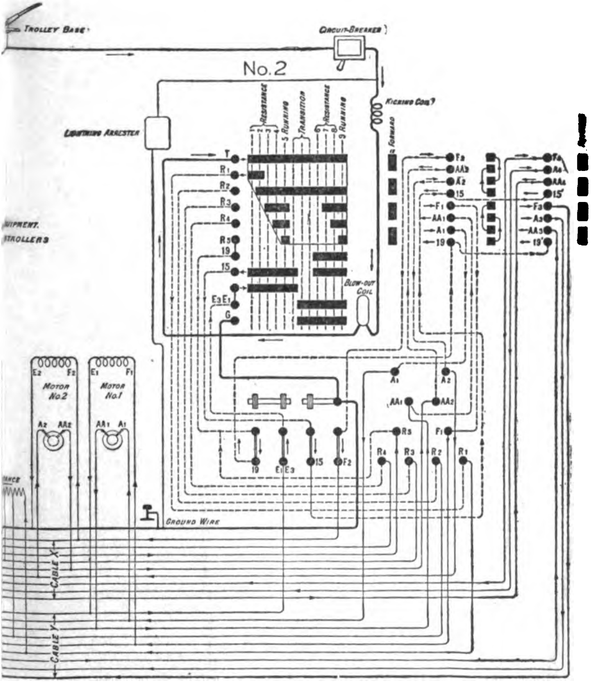



**Fig. 3. Combinations—Two-Motor Equipment**




p. 397

**Fig. 4. Controller Diag**




AM—FOUR-MOTOR EQUIPMENT

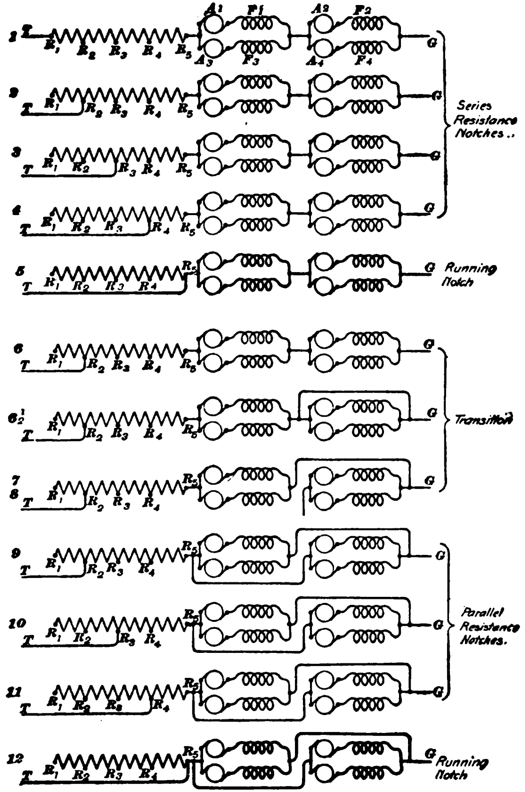



FIG. 5. COMBINATIONS—FOUR-MOTOR EQUIPMENT

are under a row of fingers on one side, and then conceive the main control cylinder to be rotated so that its segments slide under their fingers from notch to notch. In Fig. 2 the segments on the main control are in four castings; the electrical connections between segments are represented by the short vertical lines. The motors start in series, with all resistance in circuit; as the cylinder is turned, the resistance is cut out step by step until the first running notch *5* is reached, where all resistance is cut out; then follow four transition steps, leaving the motors in parallel with resistance in series, and three resistance steps, leaving the controller on the second running notch *9*, with the motors in parallel and all resistance cut out. The arrowheads show the path of the current when controller No. 1 is on the first notch and the reverse cylinder is in the forward position. Fig. 3 shows the combinations effected on the various notches.

In a *four-motor equipment*, the motors are permanently connected in pairs; the two motors of each pair are in parallel, and each pair is treated in the connection changes as a single motor. Fig. 4 shows a diagram of connections. The reversing cylinder carries three rows of segments and there are two sets of fingers, so that the current in the four armatures can be reversed. Motors *1* and *3* form one pair, and *2* and *4* the other. On heavy equipments, circuit-breakers are used in place of hood switches. The arrows and arrowheads indicate the paths of the current with controller No. 2 on the first notch and the reversing switch in the forward position. The various combinations are shown diagrammatically in Fig. 5. The principles of all series-parallel controllers are the same as those already described.

## INSTALLATION OF CAR WIRING

The expression *car wiring* is usually understood to mean the interconnections under the car between the two controllers, the motors, the resistances, and ground. The connections to the car heaters and lamps are known respectively as *heater wiring* and *lamp wiring*, and the portion of the

circuit between the current collector and the controllers is known as the *trunk connections*. The trunk connections include the trunk wire, or actual wire part of the circuit; the motor switches (also called hood switches, canopy switches, overhead switches, and bonnet switches); and circuit-breakers, fuse boxes and lightning arresters (when used). The relative locations of many of these parts are shown in the diagrams, Figs. 2 and 4.

The parts of the trunk wire are: the *roof wire*, which connects the trolley stand with the hood switches or circuit-breakers; the *wall wire*, which runs down between the end walls of a closed car or through a corner post of an open car; and the *controller trolley wire*, which extends the whole length of the car under the floor and connects the trolley posts of the two controllers. When hood switches are used instead of circuit-breakers, a fuse box is installed in the trunk wire under the car floor, near the point where the wall wire comes down through the floor. The lightning arrester, when one is used, is connected in the trunk wire near its junction with the controller trolley wire.

The controller resistance is suspended under the car floor. On cars of moderate weight, the wires under the floor usually run in a canvas hose or in conduit extending from one controller to the other, with openings at intervals for the taps to motors, resistance, etc. On heavier cars, the wires are run separately in grooved insulating material, and are then covered with insulation.

The following general directions relating to car wiring are given by the General Electric Company:

1. Protect all wires underneath or inside the car from mechanical injury through strains due to racking of the car body or from chafing with car doors, motor frames, trucks, wheels, brake rigging, rheostat frames, or other parts.

2. Locate the cables and wires so that the heat from rheostats and heaters will not seriously affect the insulation.

3. Avoid placing wires of extreme difference of potential in close proximity.

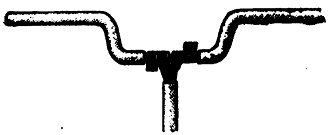
4. Solder all permanent taps and splices, and thoroughly tape them.

5. Securely support cables and wires so as to minimize vibration.

6. Tighten well all setscrew or clamp connections at controllers, rheostats, fuses, circuit-breakers, and other points.

In general, the cables should be located in dry out-of-the-way positions. It is sometimes preferable to run them in a boxed compartment carried along the car sills from end to end of the car. On open cars, the cables should be located under the car bodies and platforms, as near the center line of the car as possible; they should pass around bolsters, so that water and mud thrown by the wheels will not reach them. Care should be taken on all double-truck cars to see that the cables are so placed that the wheels will not cut them in rounding sharp corners. In closed cars, with seats running lengthwise, cables should be carried under the seats and supported on brackets or with leather cleats to keep them off the floor of the car. The cables should be located so as not to interfere with car heaters, and it is preferable to run the cables under rather than over the heaters. All taps from the cables to the motors should be as short as possible, with sufficient slack to prevent strains, and on double-truck equipments, a coiled metal armor should preferably be used. All connections between these taps and the motor leads should be thoroughly made. In order to prevent breakage, nothing but stranded wire should be used for connecting the field and armature leads with the cable wires. In making a tap joint, about 4 in. of the end of the stranded

(a)



(b)

FIG. 6

CONNECTING STRANDED-WIRE TAP

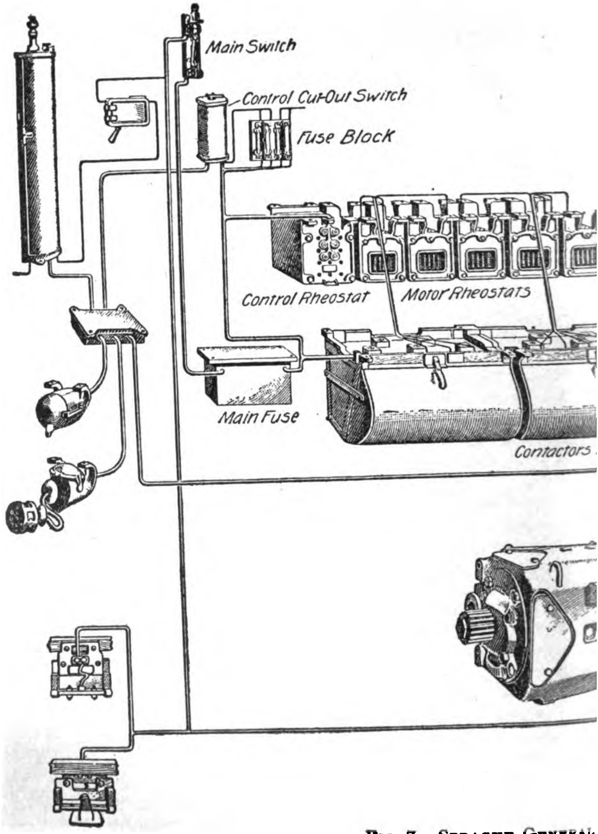



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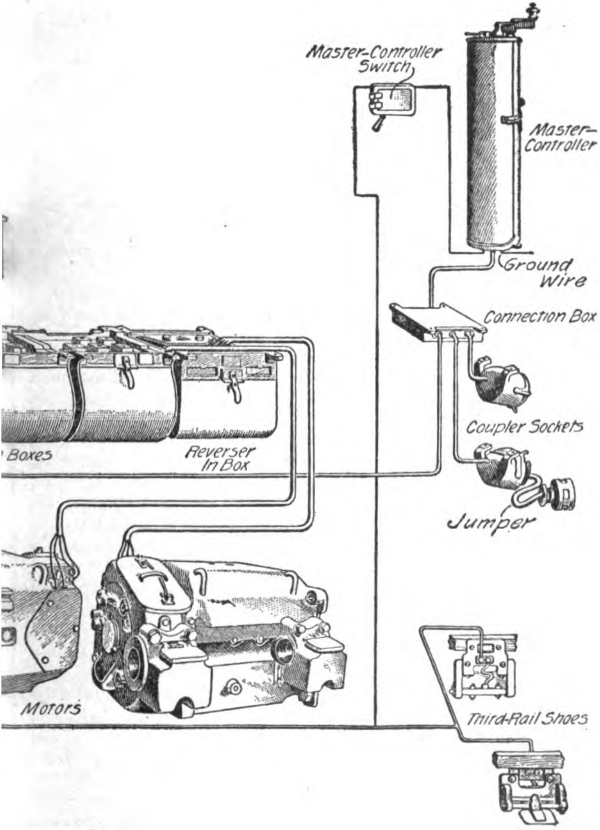
FIG. 7. SPRAGUE GENERAL


Master-Controller Switch
Master-Controller
Ground Wire
Connection Box
Coupler Sockets
Jumper
Boxes
Reverser In Box
Motors
Third-Rail Shoes

lead, or tap, is cleaned of insulation, and the strands are divided into halves, as shown in Fig. 6 (*a*). The halves are twisted around the bared portion of the cable in opposite directions, as in Fig. 6 (*b*), and the ends pressed down compactly. Melted solder is then poured over the joint from one ladle to another, and when the solder is cool the joint is smoothed with a file, covered with friction tape, and given a coat of thick shellac. Splicing compound is sometimes applied before taping.

## MULTIPLE-UNIT SYSTEMS

In the *multiple-unit system* of operating electric trains, each car is equipped with two master controllers—one at each end of the car in the motorman's compartment. By means of these master controllers, changes are made in the connections of the wires of a cable, or train line, that runs from car to car through couplers and sockets; the current in this train line operates the main controllers located under each car. The current through the master controller is therefore small, being only enough to excite the electromagnetic switches of the main controllers. These switches effect the necessary changes in the motor and resistance connections, and are made large enough to carry the very strong currents required by the motors. Several cars are coupled together to form a train, which is operated from the master controller on the front end, all the other master controllers then being automatically locked. The movement of the operating controller effects the same motor and resistance combinations on all the cars.

A general view of the apparatus used on a car equipped with the *Sprague General Electric system of control* is shown in Fig. 7. The current from the third-rail shoes or from the trolley passes through a main switch and fuse box before reaching the groups of electromagnetically operated switches, or contractors, forming the main controller. Taps lead from the main wire through switches to the master controllers. From the controller, the various wires of the control cable pass to the connection boxes, in which changes

of a permanent nature in the connections are made. From the connection boxes, the wires of the control cable pass through the control cut-out switch, a fuse block, and a rheostat, the resistance of which limits the control current to the required strength. From the rheostat, the control current traverses the operating coils of the contractors and reverser, returns to the master controller, and passes thence to ground. The reverser consists of a rocker operated by two solenoids—one to throw it to the forward position, and the other to the reverse. The contacts are provided with blow-out coils, so that all arcs are immediately suppressed.

Seven train wires are necessary to carry the control current from car to car, and these are covered with different colored insulation, to make them readily distinguishable, and are formed into cables called *jumper cables*. Connections are made to the car control cable by means of couplers and coupler sockets. The contactors on all the cars operate simultaneously. The motor rheostats, made of cast-iron girds, limit the motor-starting current and thereby secure smooth acceleration. In modern controllers, an automatic device prevents too rapid acceleration, so that even if the motorman moves the master-controller handle around to the full-on position at one stroke, the contactors will close in regular consecutive order, each delaying until the starting current has fallen to a predetermined limit before closing.

The *Westinghouse multiple-unit control apparatus* accomplishes the same object as that just described. In the Westinghouse system, the control current is taken from a storage battery at low pressure (about 15 volts), and is thus entirely independent of the main circuit or the current-collecting devices. The main controller consists of a group of *unit switches* operated by compressed air. The switches are grouped radially around a blow-out coil that is common to all. In more recent control equipments the unit switches are grouped in a rectangular box, the blow-out coils being placed between the switches. The air valves are operated by electromagnets under the control of the operator. A *unit-switch control equipment* is shown in Fig. 8. The limit switch

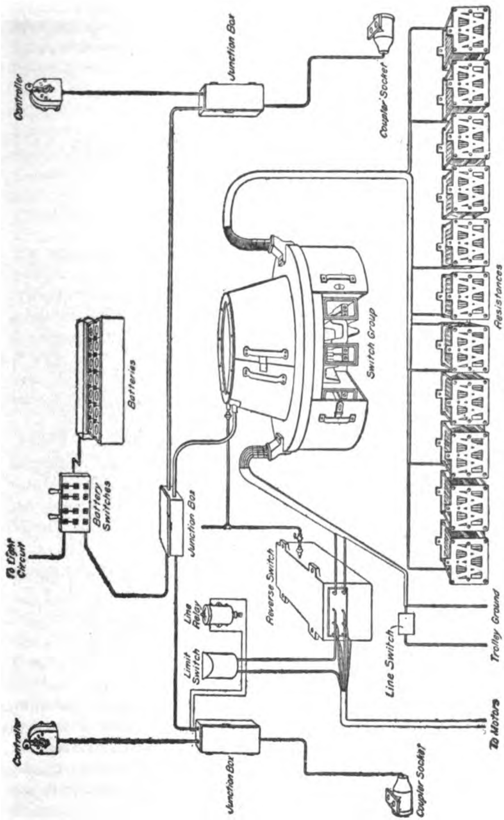



FIG. 8. WESTINGHOUSE UNIT-SWITCH CONTROL APPARATUS

regulates the rate of acceleration by stopping the progressive action of the unit switches whenever the motor current exceeds a predetermined value. The line relay opens all the switches except the line switch when the power goes off the line.

# FIRST AID TO THE INJURED

## PREPARATION

In every place where a large number of persons are employed and where accidents are liable to occur, a supply of articles needed to render first aid should be available. These should include one or more stretchers, bandages, absorbent cotton, carron-oil (equal parts of raw linseed-oil and lime-water), splints, soap, towels, blankets, aromatic spirits of ammonia, etc. The necessary quantity of any of these or other articles depends on the nature and size of the works.

**Sterilizing.**—Many disease germs may be killed by heat; others by chemicals called disinfectants, such as bichloride of mercury, carbolic acid, etc. The solutions used in washing wounds should be made up of about the following strengths: Bichloride of mercury 15 gr. to 1 qt. of water; or, liquid carbolic acid, 2 teaspoonfuls to 1 qt. of water. The substances should be thoroughly dissolved before the solution is used.

# ACCIDENTS AND INJURIES

## FAINTING

*Fainting*, or *swooning*, with loss of sensation, motion, and consciousness, may result from a severe blow or wound, from loss of blood, from great emotion (extreme fear or joy), from electric shock, etc. The patient becomes pale, inanimate, and is in a condition of apparent death; if not soon relieved, death may result.

The patient should be laid with the head lower than the feet, and ligatures or bands of some sort should be tied around the arms and legs close to the body, so as to confine the circulation to the trunk and head. The tongue should be kept out of the throat, in order to allow free access of air, and the respiration may be helped by pressing in and down on the ribs and chest and allowing the chest to expand by its own elasticity.

**Artificial Respiration.**—The process just described is one form of *artificial respiration*, and may in some cases be effective. If the desired results are not soon obtained, place the patient on his back with a pad (a roll of clothing will do) under the back just below the shoulders, so

FIG. 1

as to raise the pit of the stomach. The patient's tongue should be drawn out and held by an assistant, or, it should be fastened against the lower teeth by a rubber band passing under the chin or clasped between the patient's teeth, the lower jaw being held up by a bandage tied over the head. Grasp the forearms half way between the elbows and wrists, and draw the arms back rather quickly but steadily in vertical planes until they meet above the patient's head, as in Fig. 1, and hold them thus for 2 sec. This motion draws the ribs up, expands the chest, and air enters. Now bring the arms back to the sides of the body, and press firmly on the sides and front of the chest over the lower ribs, as in Fig. 2; the object of this movement is to

contract the chest and force the air out of the lungs. If enough assistants are present, one can stand astride the patient and press firmly against the sides and top of the chest while the arms are held down along the sides. This series of movements, constituting one inspiration and one expiration, should be repeated about once every 4 sec., or fifteen times per min., for 1½ or 2 hr. if necessary, unless in the meantime a physician pronounces life extinct. While working over the patient prevent unnecessary crowding of persons, avoid rough usage, and do not allow the patient

FIG. 2

to remain on his back unless his tongue is secured. Under no circumstances should the patient be held up by his feet, nor should he be placed in a warm bath unless under medical direction.

## TRAUMATIC SHOCK

Severe injuries may sometimes result in *traumatic shock* (*trauma* meaning a wound), in which the victim appears confused and listless and perhaps stupefied, but not unconscious. The pulses and respiration are perceptible, though feeble and irregular. Sometimes the bowels move involuntarily. Intelligence is not usually wholly lost, and the patient can be made to respond to questions if repeatedly urged. This condition may last a few moments or several hours, and may terminate in death.

Place the patient in a horizontal position with head lowered, and warm him by rubbing and by using warm linen or blankets. Let him inhale the odor from dilute ammonia water. If he can swallow, give a little hot brandy and water with a few drops of ammonia water added; 1 teaspoonful of aromatic spirits of ammonia in a wineglassful of water is also good. From 2 to 4 teaspoonfuls of turpentine in a quart of water, as hot as may be used without discomfort, may be injected into the bowels, often with good results.

Wounds consisting of severe bruises are sometimes characterized by numbness, coldness, and absence of bleeding until reaction begins. In such cases, use stimulants and antiseptics and keep the injured part as quiet as possible and protected by warm dressing.

## HEMORRHAGE, OR BLEEDING

*Hemorrhage*, or *bleeding*, may come from the arteries, the veins, or the capillaries. The arteries are the channels through which blood flows from the heart to the various parts of the body, and the veins are the channels through which the blood returns to the heart. The capillaries form the network of very minute tubes through which the blood passes from the arteries to the veins and by which all the tissues of the body are nourished.

*Arterial hemorrhage* is usually distinguished by the bright red color of the blood and the regular pulsations with which it issues from the blood vessels; *venous hemorrhage* can be known by the dark-blue tint of the blood and the steadiness of its flow; in *capillary hemorrhage*, the blood has a reddish tint and exudes from the tissues or wells up from the surface of the wound. *Internal hemorrhage* may exist without any external flow of blood.

After excessive loss of blood, the patient's face and lips turn pale; he experiences chills, cold sweats, nausea, frequent vomiting, irregular respiration, feeble pulse, dizziness, buzzing in the ears, and finally unconsciousness, terminating either in death or in the cessation of the bleeding. In the latter case, consciousness may soon return, but very often the tendency to fainting fits persists for a time.

Capillary hemorrhage is arrested by bathing the wounded part in cold sterilized water and bandaging it with a pad, or compress of sterilized gauze or lint.

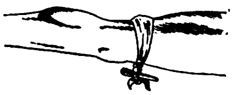


Fig. 3

Venous hemorrhage is more serious and cannot always be stopped by binding a pad over the wound; in this case, the limb must be bandaged on the side of the wound away from the heart. The limb should be raised and held above the rest of the body and the patient should be made to lie perfectly quiet.

Arterial hemorrhage is more serious than either of the others. If a large artery or a number of small ones are ruptured, the blood may escape so rapidly that death occurs almost at once. Pressure enough to stop the flow should be applied to the artery where it passes over a bone

(a) (b)

Fig. 4

between the wound and the heart. The location of the artery is revealed by the distinct pulsations. Pressure

applied with the fingers will answer temporarily, and this method affords a way of finding the proper spot on which to press. A knot or any hard substance, in a handkerchief or a bandage may then be placed on the spot, tied loosely

FIG. 5

FIG. 6

around the limb, and twisted with a stick, as in Fig. 3, until bleeding ceases. The stick may then be fastened with another bandage.

The course of the main (brachial) artery in the arm is well indicated by the inner sleeve of a man's coat; this artery

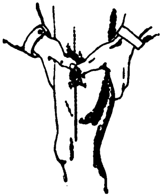


FIG. 7

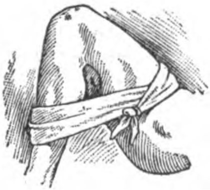


FIG. 8

can be compressed by grasping the arm by either method (*a*) or (*b*), Fig. 4. The pressure should always be downwards against the bone and not against soft muscle. The subclavian artery supplying blood to the arm may be closed

by applying pressure in the hollow just above the collar bone, as shown in Fig. 5. The temporal artery runs up the side of the forehead, and may be closed by applying a pad, as in Fig. 6. The femoral artery runs from the groin down a little inside of the front of the leg about one-third the distance to the knee, then passes through the muscles and approaches the surface again behind the knee. Pressure applied as at *P*, Fig. 7, may stop bleeding from a wound above the knee, and a pad applied as in Fig. 8 is applicable for a wound below the knee.

## ELECTRIC SHOCK

*Electric shock* may produce severe burns, unconsciousness, or death, depending on the strength of the current through the body as well as on its duration and flow. If the skin is thin and moist and the contacts with the conductors good, comparatively low voltage, 220 or possibly less, may be sufficient to send considerable current through the body. On the other hand, a person with thick, dry skin, as on the palms of the hands, may sometimes make slight accidental contact with a circuit of several thousand volts without serious results. A very small current through the region of the heart may paralyze its action and cause death; currents of greater density stimulate the heart to increased action, but paralyze the nerve centers controlling respiration and may cause death by suffocation, the same as in drowning.

Accidental contact with an electric conductor should be broken as quickly as possible; if maintained until heart action ceases, as a result of suffocation, death invariably results. In breaking the contact (provided, of course, the power cannot be immediately turned off the circuit), use the feet to push the victim and the conductor apart—never the hands. Current passing from one foot through the legs and the other foot to ground does comparatively little injury, since the important nerve centers and the heart are not in its path. As soon as the contact is broken, the victim, if he has not lost consciousness, soon recovers. If the victim is unconscious but has not ceased breathing, an effort should be made to revive him, the same as in an ordinary

fainting fit. If respiration has ceased, artificial respiration should be tried and continued for some time, even though the heart action is so feeble as to be almost imperceptible. The first and most important requirement in producing respiration by artificial means is to hold the tongue so that it cannot obstruct the throat.

Burns caused by contact with electric conductors should be protected with sterilized gauze. Such burns are generally deep, sometimes even carbonizing the bones, especially those of the fingers. They heal quickly, however—ordinarily in from 3 to 6 weeks.

## WOUNDS

Before being used on a wound, all instruments, bandages, etc., should be sterilized by heating in steam or boiling water or by baking or by treating with a germ-destroying solution. The water used in washing a wound should first be boiled, in fact nothing unsterilized should be permitted to come in contact with the wounded surface. The germs entering a wound from the skin of the patient or from the object that produced the wound may be removed by thoroughly washing with sterilized water, and the sterilized dressings will prevent further infection.

The first treatment of a wound includes checking the bleeding; the removal of all foreign matter and a thorough washing; drawing the lips of the wound together or gently straightening bruised or torn flesh; applying several layers of sterilized gauze, with absorbent cotton next the wound if it is likely to bleed or discharge, and holding all in place with a suitable bandage. Sterilized adhesive strips are sometimes necessary to hold the wound together.

## FRACTURES

The signs of *fracture* are: (1) Loss of power in the limb, or part, injured. (2) Pain and swelling at the seat of the injury. (3) Distortion of the injured limb—it will be longer or shorter than the other or will lie in some unnatural position. By gentle pulling, the limb may be brought back to its natural shape, but on being released will immedi-

ately return to the distorted position. (4) On gently moving the limb, a grating sensation (crepitation) may be felt where the ends of the broken bone rub against each other. (5) If near the surface, the break may be felt from

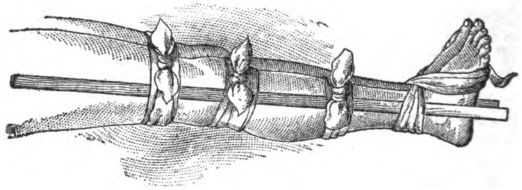


FIG. 9

the outside. A fracture should be handled with extreme gentleness; rough usage may do much harm.

Before attempting to move a patient suffering from fracture, the injured part should be supported in a rigid position by tying on splints. Almost any substance stiff enough to support the injured part will answer for a temporary splint; for example, a stocking leg or a coat sleeve filled with earth, sand, moss, hay, chaff, or paper and securely tied at each end, a barrel stave, a piece of board, a roll of paper, etc. If hard substances are used for splints, the leg should be padded. If feasible, the splints should extend past the nearest joints, and should be securely bandaged so that both the fracture and the joints are held rigid, as in Fig. 9.

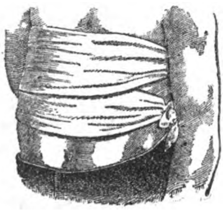


FIG. 10

Until the physician comes, a fractured jaw should be held in place by a bandage passed under the chin and over the head. If the collar bone is broken, the arm should be raised gently, and a pad made by

tightly rolling a handkerchief or a piece of cloth should be placed in the armpit; the forearm should be supported horizontally across the chest by a large arm sling, and the arm and sling should be held firmly in position by a broad bandage placed around the body and just above the elbow. Fractured ribs may be temporarily treated by fastening broad bandages around the body, tying the knot on the side opposite the fracture, as in Fig. 10.

## DISLOCATIONS AND SPRAINS

A *dislocation* is the displacement of the bones of a joint. Ordinarily, a physician is needed, and little can be done before his arrival except to make the patient as comfortable as possible.

A *sprain* should be kept very quiet. If possible, keep the injured member in water as hot as can be borne for 1½ hr. or more; then bandage with moderate firmness in such a manner as to prevent any movement of the joint, using splints for this purpose if necessary.

## EFFECTS OF HEAT

**Burns.**—The general treatment of a burn consists in relieving the pain, in combating the depression, and increasing the warmth of the patient. The pain may usually be relieved by excluding the air from the burned portion, stimulants should be given, if necessary, to relieve the depression. A covering of flour may be spread over the burned surface; or bicarbonate of soda, either in the form of paste or powder, can be used; any oil, such as sweet-oil, raw linseed-oil, or carron-oil, or a dressing, such as vaseline, cold cream, etc., is effective.

In removing the clothing from over a burn or in dressing it, the blisters should not be broken. If any clothing adheres, it should be saturated with oil and allowed to remain. The patient should not be exposed to cold.

**Heat exhaustion** is generally accompanied by weakness, cool skin, pale face, weak voice, rapid and feeble pulse, increased respiration, dim vision, and possibly by unconsciousness. The patient should be placed in a horizontal

position with the head low, and stimulants and hot applications should be administered. Occasional doses of brandy should be given, also a teaspoonful of aromatic spirits of ammonia in a little hot milk or water every half hour. If the patient cannot swallow, these remedies may be injected into the rectum.

Sunstroke, which may occur in any hot, moist temperature, is accompanied by high fever. In a few cases, unconsciousness and death come very quickly; but usually the progressive symptoms are intense headache, dizziness, oppression, nausea, vomiting, occasionally diarrhea, and unconsciousness with delirium and restlessness. The face is flushed, the eyes bloodshot, the skin very hot and dry (temperature from 107° to 112° F.), the breathing labored and sometimes noisy, and the pulse frequent and full.

Both the symptoms and the treatment are directly opposite those for heat exhaustion. In cases of sunstroke, every effort should be made to reduce the excessive bodily temperature. Rubbing with ice, a cold bath, a cold pack, and cold rectal injections are all good.

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